An Analytical Model for Flame Quenching Distance in Aluminum Dust Suspensions

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Abstract: A new analytical model is presented in order to derive the quenching distance in laminar dust flame that is based on common hypotheses in the science of combustion. It must be said that incomplete combustion due to quenching distance in a narrow confinement has been a major problem for the realization of a reliable micro combustion device. Quenching is mainly caused by heat loss to combustor walls; it can be used to determine characteristics of flame. In present work, conservation equations are considered to formulate the thermal transmission between quenching plates and derive the quenching distance. So, in this mathematical model, we find influences of heat transmission by radiation, changes on temperature quenching surfaces and influence of first temperature of entrance gases on quenching distance. We have attained minimum quenching distance equal with almost 3mm for aluminum dust-air mixture which its particle diameter is \( 5.4 \mu m \). Consequently, the mathematical results show good agreement with experimental data.

Key words: Dust Combustion, Quenching Distance, Analytical Modeling, Laminar Flame, Radiation

INTRODUCTION

Combustion of suspended solid-fuel particles in a gaseous oxidizer medium premixed dust flame is important for many areas of modern technology (e.g., coal combustion, dust explosion hazards, propulsion systems, etc) (Trent S. et al. 2006) (Shoshin, Y. and E.L. Dreizin, 2006) Aluminum, because of its high enthalpy of combustion has been added to propellants and explosives, its extensive use as an energetic material, several research efforts have been directed to understand the mechanism and model the oxidation of aluminum particles (Eapen, et.al. 2004. Seshadri et.al. 1992) Thus, models describing combustion of metals are necessary to design new and control existing propulsion devices (Shoshin, Y. and E. Dreizin, 2003). For specifying the parameters of combustion, a simple model is needed to analyze and take into account the efficient parameters of combustion. Theoretically, the combustion of fine solid particles is yet to be improved in comparison with the combustion of gases and liquids. This could be mostly attributed to the complexity of such phenomena. However, an equally great obstacle is the special experimental difficulties in generating a uniform and stable dust suspension with repeatable dust concentration (Goroshin et.al. 1996. Dreizin, E.L., 2003) The experimental and theoretical modeling for aluminum dust cloud combustion, have been presented by (Goroshin et al. 1996; Goroshin et al. 2000; Goroshin et al. 1996 and Bidabadi, M., 1995) and reactive particle-laden flows and parameters have been estimated. One of the important parameters that have a special role in micro combustion design is quenching distance beside the other parameters of the flame. The specific length of the combustor plays an important role in flame propagation when the size of a combustor decreases to a scale of the flame thickness. Heat loss to the wall and radical adsorption on the surface are two major effects of the surface on the flame (Sejin Kwon., 2006). In most micro combustors, the effects of flow aren’t considered in the quenching because the flow is laminar and no severe stretch is present. (Kuo, K.K., 1986) assumed that quenching occurs when heat loss to a wall is larger than the heat generated in the reaction zone. A number of numerical studies regarding wall quenching were conducted by using complex reaction mechanisms (Westbrook, et.al. 1981; Daou, J., M. Matalon, 2002) When the size of the combustor is relatively large, the surface-to-volume ratio is small and the effect of flame-surface interaction is negligible (Sejin Kwon., 2006). A simplified theory of dust flame quenching in narrow channels (Bidabadi, M., 1995) (Yarin, L.P. and G. Hetsroni, 2004) predicts that flame quenching distance is proportional to a square root of characteristic combustion time \( L_{\text{q}} \propto \frac{t_{\text{comb}}}{\sqrt{\tau}} \).
In the present work, a mathematical model of combustion of solid particles will be developed. In this article, conservation equations are considered to formulate the thermal transmission between quenching plates and derive the quenching distance. Quenching distance decreases with the plate surface temperature as the temperature difference between the reaction zone and the plate decreases. Changes as the plate temperature vary from $300$ to $1100K$. The other effective parameter on quenching distance is temperature of entrance gases to quenching zone. So, in this mathematical model, we find influences of heat transmission by radiation, changes on temperature quenching surfaces and influence of first temperature of entrance gases on quenching distance.

**MATHEMATICAL MODELING**

As the gap between the two plates decreases, the flame became unstable. Due to the contact with the plate surface and heat loss, the flame flickered and experienced partial extinction, particularly in the region near the center of the plates. Complete extinction occurred when the gap between the plates was further reduced. The plates played a major role as a heat sink by removing heat generated in the reaction zone (Bidabadi, M., 1995).

In the analysis, gradients of all dependent variables including that of the temperature and concentration in the direction parallel to the flame front are neglected (Goroshin, et al. 1996).

Combustion of fine metal particles is paid attention in this model. It is expressed that the combustion takes place in the gas phase where evaporated aluminum reacts with oxidizer (Merzhanov, et.al. 1977). The near-wall analysis is based on a one-dimensional, steady-flow and the analysis confines the space bounded by the walls. The initial concentration and the initial particle size are presumed to be known. All external forces including gravitational effects are assumed to be negligible. The conservation equations were solved analytically between walls.

In this model, a mathematical solution is considered to formulate the heat transfer equation between quenching plates, as shown schematically in figure (1). The assumed flame structure consists of a preheat zone, flame zone, and post-flame zone for lean mixtures. In this model, plate's temperature is supposed fixed throughout length of channel ($T_i = T_e$). As seen, the flame doesn’t cover complete mass between the plates when it passes through the quenching plates (Goroshin et al. 1996. Bidabadi, M., 1995) It existed only in an area near the longitudinal axis (2$l_1$) is width of the flame area. Finally, in order to determine an energy conservation equation between these plates, setting the generation term equal to zero in a small area near the wall.

Finally, in order to write an energy conservation equation between these plates, the generation term should be equal to zero in sides of partition (Goroshin et al. 1996; Bidabadi, M., 1995).

The gas-phase governing equations for mass and energy conservation can be written for width of flame area across quenching plate. Where as there is no flame in the sides of plates, therefore there is no generation term. Because of being symmetric between quenching plates, equation will be solved in half channel. Also, point of, $(0,0)$ will be considered such as channel center point on flame zone (Goroshin et al. 1996).

Formulation of thermal transmission is determined by following equation:

For area of combustion:

$$\rho_u C_p \frac{T - T_e}{\delta_x} = \lambda \frac{\partial^2 T}{\partial y^2} + W_f Q$$

(1)

$$W_f = \frac{\rho}{\tau}$$

(2)

Where $\rho, u, T_e, C_p$ are gas density, velocity, and temperature, respectively. The subscript $u$ denotes the value related to the unburned mixture. $C_p$ the specific heat of gas at constant pressure, $Q$ the reaction rate characterizing consumption of fuel, $Q$ heat of reaction per unit mass of fuel, $\delta_x$ is mass fuel content in the $x$ direction and $\tau$ is the combustion time of an individual particle, which is assumed to have a weak dependence on temperature (Goroshin et al. 1996; Goroshin et al. 2000; Bidabadi, M., 1995).

The thermal conductivity of the gas which is supposed to be ideal and isobaric is a linear function of
temperature \( T_q \) (Goroshin et al. 1996. Goroshin et al. 2000. Bidabadi, M., 1995) and is the quenching plates distance.

For side of quenching plate:

\[ \rho_{y} C_{y} \frac{T - T_{q}}{\tau} = \lambda \frac{\partial^{2} T}{\partial y^{2}} \]  \hspace{1cm} (3)

If there is frontier of these two areas, energy conservation formulations should be written as follow:

\[ y = 0 \rightarrow \frac{\partial T}{\partial y} = 0 \]  \hspace{1cm} (4)

\[ y = y_l \rightarrow \frac{\partial T}{\partial y} \Big|_{y_l} - \frac{\partial T}{\partial y} \Big|_{y_l} = T_{p} - T \]

\[ y = d \rightarrow T = T_{q} \]

**Dimensionless Form of Equations:**

Conservation equations can be written in a non dimensional form by using the following non dimensional parameters:

\[ \theta = \frac{T}{T_{q}}, \hspace{1cm} \zeta = \frac{y}{d_{/2}}, \hspace{1cm} \Phi = \frac{\dot{B}}{B_0} \]  \hspace{1cm} (5)

\[ \Phi_{1} = \frac{\rho C_{d} d_{/2}^{2}}{4 \tau \lambda}, \hspace{1cm} \Phi_{2} = \frac{\Phi_{Q} \rho d_{/2}^{2}}{4 \tau \lambda T_{q}} \]

By inserting dimensionless parameters in conservative equation, the form of equations in combustion zone can be written as follows:

\[ \frac{\partial \theta}{\partial \zeta} - \beta_{1} \Phi_{1} = -(\Phi_{2} + \Phi_{4}) \]  \hspace{1cm} (6)

For side of quenching plate, it can be written as follows:

\[ \frac{\partial \theta}{\partial \zeta} - \beta_{2} \Phi_{1} = -\beta_{4} \]  \hspace{1cm} (7)

Boundary conditions of above differential equations are as follows:

\[ \zeta \to 0 \rightarrow \frac{\partial \theta}{\partial \zeta} = 0 \]  \hspace{1cm} (8)

\[ \zeta = \zeta_{l} \rightarrow \Phi_{4} + \Phi_{5} = \frac{\partial \theta}{\partial \zeta} \]

\[ \zeta = 1 \rightarrow \Phi_{5} = 1 \]

Algebraic formulation of the temperature profile across these two areas can be derived by solving equations (7) and (8) and considering boundary conditions. The flame appears in areas where temperature is more than ignition temperature of particles (Goroshin et al. 1996. Bidabadi, M., 1995). Therefore, frontier between these two parts is placed where the temperature of mixture will be equal to the ignition temperature of particles. After that, the temperature of mixture decreases and consequently there is no flame in this area.

Temperature profile can be obtained for area of flame by considering the presented model. Temperature profile is shown in area of combustion and quenching of plate sides figure (2).

In order to temperature profile in gas fuel mixtures, we should substituted generation term of gases for generation term dust cloud in equation (1):

\[ \dot{W} = \rho \dot{A}_{y} \exp(-\frac{E}{RT_{q}}) \]  \hspace{1cm} (9)

Where \( A, B, R, \), are pre exponential factor of the Arrhenius law of the reaction, activation energy and constant gas, respectively (Kuo, K.K., 1986).
We eliminate dimension from energy equations by under non-dimensional parameters:

\[ \theta = \frac{T}{T_i}, \quad \zeta = \frac{y}{\delta/2}, \quad Y = \frac{y}{y_i}, \quad \beta = \frac{R}{R_F} \]

\[ \beta_i = \frac{\rho v C_i d^2}{4kT}, \quad \beta_1 = \frac{\rho A V \exp(-\beta) Q d^3}{4kT} \]

We find temperature profile for gas fuels between quenching plates by replace non-dimensional rates, \( \beta_i \), \( \beta_1 \), \( \beta_2 \), in equations (6) and (7) and so by using boundary conditions defined in equation(8), (figure (3)).

By comparing figures (2) and (3), we find that the flame ignition temperature for fuels of dust cloud is more than that of flame for gas fuels.

**Fig. 1:** Schematic diagram of the quenching plates.

**Fig. 2:** Profile temperature as quenching plates width (\( d = 5.4 \mu m, \tau = 10^{-3} s \)).

**Fig. 3:** Profile temperature as quenching plates width for gases.
Model Presented in Order to Account Quenching Distance:

In order to account quenching distance, a formulation of energy generation in area of flame is as following:

\[ \frac{E}{Q} \cdot QV = \dot{m}_1 \cdot C_v \cdot \left[ (T - T_f) + \dot{m}_2 \cdot C_p \left( \frac{T + T_f}{2} - T_f \right) - \lambda \cdot A \frac{\partial T_f}{\partial r} \right]. \]

Left term shows energy generation in flame area and first right term is formulation of energy increment in area where there is flame, second right term is energy increment in sides of quenching plates and finally, third term is the heat loss to the quenching plates. Some of the above parameters are defined as follows:

\[ \dot{V} = \frac{y}{\delta} \times 1, \quad A = \frac{\delta}{\delta} \times 1, \quad \delta = \nu \cdot \tau, \]

\[ \dot{m}_1 = \rho \cdot v \cdot y, \quad \dot{m}_2 = \rho \cdot v \left( \frac{d}{2} - y \right) \]

Where \( V, A, \delta \), are volume, surface area and thickness of combustion zone, respectively. \( \dot{m}_1 \) is the mass flow rate in area of combustion and \( \dot{m}_2 \) is the mass flow rate in side of quenching plate. \( T_f \) is the ignition temperature and \( \partial T_f / \partial r \) is the temperature gradient in side of quenching plate.

Influence of Radiation on Quenching Distance:

The radiation term will be added to conductive terms and Energy increment and its effects on the combustion of fine aluminum particles will be investigated. Due to the significant effect of radiation in two-phase combustion which is composed of fine particles, further examination of the radiation term is needed (Yarin, L.P. and G. Hetsroni, 2004).

Radiation in dust-air mixture is one example of a wide class of problems in which the radiative heat transfer occurs in an absorbing medium. It must be realized that scattering and absorbing by real aluminum particles is much more complex than what can actually be calculated. In general, the system in combustors consist of non uniform, emitting, absorbing, reflecting, solid surfaces that are in complex geometrical configurations and surround a multi phase, non-uniform, emitting, absorbing, transmitting, scattering particles laden fluid that generates heat. In addition to the mathematical difficulty, one of the more serious problems in this study is the insufficient of knowledge on the radiative properties of the solid particles involved in the system.

The complete solution to Maxwell's equations discussed so far (Mie theory) is only applicable to spherical particles of constant composition. It is obvious that many simplifying assumptions have to be made to be able to include radiative heat transfer in a dust combustion application (Siegel, R. and J.R. Howell, 1981). In practice, any kind of dust contains particles which of different sizes over a specific size range, and are also different in terms of shape, and geometry. Due to many difficulties, both experimental and theoretical, obtaining the role of radiation as a mechanism of dust flame propagation is not yet completely elucidated. For heat transfer by radiation in dust combustion, several coefficients and parameters must be defined.

Now, we study calculation of radiation intensity.

The general equation of radiation transfer is:

\[ \frac{dT}{dX} = K_a l + K_s l - K_a l - \frac{K_s}{4\pi} \int l(\Omega)P(\theta, \phi) \, d\Omega \]  \hspace{1cm} (13)

The terms on the right of equation (13) are radiation intensity caused by absorption, scattering, emission, and incoming scattering brought about by other particles respectively. \( K_a, K_s, l \), and \( P(\theta, \phi) \) are scattering coefficient, absorption coefficient and radiation intensity respectively. \( P(\theta, \phi) \) is phasic function of scattering (Siegel, R. and J.R. Howell, 1981).

The absorption coefficient may be related to the particles size (of diameter \( d_p \)) and to particles density (by number \( \dot{n}_p \)); we assume that the fraction absorption of radiation passing through a very thin element of the cloud is the ratio of project solid area of particles to the total area of the containing element. We then have for the absorption coefficient (Bidabadi, M., 1995. Bohren, C.F. and D.R. Huffman, 1983).
And if we supposed that the scattering of light is done only by particles then:

$$K_s = \frac{3}{2} Q_s \frac{B}{\rho \lambda_p}$$  \hspace{1cm} (14)

where $Q_a, Q_s$ are absorption efficiency and scattering efficiency, respectively. The integral term on the right of equation (13) introduces the contribution of multiple scattering. If we assume negligible multi-scattering contribution along the path length within a very thin element of the cloud, general equation of radiation transfer is reduced to a differential form.

In according to above explanations, the equation will be such lower part:

$$\frac{\partial j}{\partial x} = +K_a j + K_s j - K_s j$$  \hspace{1cm} (16)

Where the parameters, $K_a, K_s, j$, in the equations are defined as:

$$K_a = K_a + K_s$$

$$j = \frac{\alpha}{\pi} (T^4 - T_e^4)$$  \hspace{1cm} (17)

$\alpha$ is Stefan-Boltzman constant.

By solving equation (16) such by using above boundary conditions:

$$y = 0 \quad \rightarrow \quad j = \frac{\alpha}{\pi} T_e^4$$  \hspace{1cm} (18)

Radiation intensity in flame zone will be attained:

$$I = (I_e - I_f) \frac{K_a}{K_s} \exp(K_s y) + \frac{K_s}{K_a} I_f$$  \hspace{1cm} (19)

Some non dimensional parameters are defined as follow:

$$B = \frac{\alpha c_r^2}{\rho \lambda_p} \frac{\alpha c_r^2}{\rho \lambda_p} \frac{\alpha c_r^2}{\rho \lambda_p}$$  \hspace{1cm} (20)

By inserting dimensionless parameters in conservative equation we can write the find form of equation reaction zone.

$$\frac{E}{(e - 1)} = \frac{C_{(e)}}{C_{(e-1)}} + \frac{d}{4} + 2B_1 + \frac{B_1 (\theta^2 - \frac{c}{K} (\theta^2 - 1))}{(e - 1)} \exp(K_s y) + \frac{B_1 (\theta^2 - 1)}{(e - 1)}$$  \hspace{1cm} (21)

where $\theta$ is the non-dimensional ignition temperature of solid particle.

Quenching distance for different particles is obtained by solving equation (21).

RESULTS AND DISCUSSIONS

According to the new analytical model, reaction zone length obtained as a function of the quenching plates distance, which is shown in figure (4). Flame quenching happens where direction of curve alters. After this point When distance between plates decrease, losses term will be more than reaction term and flame will be
Figure (4) shows that changes of length of combustion zone drawn in proportion to distance between quenching plates, (in two forms: accompaniment with radiation and without that) so it is attained that radiation cause increase of quenching distance.

Some of the parameters effective on quenching distance are influence of temperature of quenching plates and influence of temperature of entrance gases form preheat zone to flame zone. We can enter increase of temperature of plate in terms of conductive and radiation in equation (21). It is told these equations are solved in two forms \( T'_p = 300k \) \( T'_p = 900k \), and we see this increase cause decrease of quenching distance and event corresponds to experimental results. As we know in figure (5), decrease of heat loss due to increase of temperature of quenching plates cause decrease of quenching distance.

![Fig. 4: Effect of radiation on changes of Non dimensional reaction zone length per plates distance.](image)

![Fig. 5: Effect of plate temperature on changes of Non dimensional reaction zone length per plates distance.](image)

We can examine influence of increase of temperature of entrance gases from preheat zone to reaction zone in above equation in according to increases term of energy. So increases of temperature of entrance gases cause increase of temperature of quenching plates and being same temperature of this plates with that of entrance gases (figure (6), such these rates will be increased to terms of radiation and conductive.

Quenching distance is defined as a function of ignition temperature and particles diameter, which are shown in figures (7), (8). These parameters have a direct relation with quenching distance (Bidabadi, M., 1995). Because, ignition temperature increase cause temperature difference increase between flame and wall. This event result losses increase and finally quenching distance increase. So, it has seen that in figure (7), concentration increase causes quenching distance decrease on different ignition temperature. It has seen that in figure (8), there is a direct relation between particles diameter and quenching distance (Bidabadi, M., 1995).

Figure (9) shows that the quenching distance is a function of dust concentration for particles with different diameters. For a constant value of dust concentration, smaller particles have more surface area per unit volume than larger particles, thus more fuel is available in the reaction zone and consequently the value of the flame
temperature is higher. The quenching distance is roughly proportional to the heat loss term, and therefore increasing the value of the heat loss term corresponds to a decrease in the quenching distance (Goroshin, et.al. 1996). As well, the burning time of a large particle is greater than that of a smaller one (Goroshin, et.al. 1996). Also, a large value of burning time corresponds to a thicker flame, and as a result a larger value of quenching distance. This figure shows that for a given value of concentration the quenching distance decrease with decreasing value of particle size.

![Fig. 6: Effect of temperature of entrance gases to reaction zone on changes of Non dimensional reaction zone length per plates distance \( (\phi_p = 5.4 \, \mu m) \)](image)

![Fig. 7: Quenching distance as a function of ignition temperature.](image)

![Fig. 8: Particles diameter as a function of quenching distance.](image)
Fig. 9: Quenching distance per non-dimensional dust concentration in different particles diameter (with radiation).

Figure (10) shows the quenching distance as a function of dust concentration. It can be seen that the quenching distance and dust concentration have an inverse relation with each other due to increased heat generation per unit volume by increasing dust concentration. This figure shows that the present work results are in good agreement with corresponding experimental result (Goroshin, *et al.* 1996).

Consequently, in a specific concentration which is named lean limit of flame propagation is one of the most argumentative questions in dust combustion [20]. This research expresses that quenching distance increases by decreasing in concentration.

Fig. 10: Experimental and theoretical results of quenching distance \( (d_p = 5.4 \mu m) \).

**Conclusion:**

The present results show that the flame temperature for fuels of dust cloud is more than that of flame for gas fuels. Also, an algebraic equation for the quenching distance was obtained for particle-laden flows. It is shown that heat losses play major roles in affecting quenching distance. The dust cloud quenching distance depends also on the temperature of quenching plates and influence of temperature of entrance gases form preheat zone to flame zone. By increasing quenching plates temperature and gases temperature on preheat zone, quenching distance will be decreased. Also, it can be seen that the quenching distance and dust concentration have an inverse relation with each other. This model shows that for a given value of concentration the quenching distance decrease with decreasing value of particle size.

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