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# Adaptive Robust Tracking Control of Robot Manipulators in the Task-space under Uncertainties

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**Abstract:** In this paper, a novel control approach for tracking control of robot manipulators in the task space is developed which not only parametric uncertainties but also unstructured uncertainties such as friction, disturbances and un-modeled dynamics are considered. Disregarding the unstructured uncertainties may cause an unstable closed loop control system. A robust controller is designed based on Lyapunov method, using robot physical properties and known bounds of uncertainties. It is then proven that the closed loop system has global exponential stability. The bounds of unstructured uncertainties are estimated by adding an adaptive controller to the robust controller. It is verified that the proposed control system has global asymptotic stability. A case of study is a two links elbow robot where the analytical works and simulation results show a good performance of the control system.

**Key words:**

## INTRODUCTION

It is well known that the kinematics and dynamics of robots are highly nonlinear with coupling existing between joints. To cope with the nonlinear and uncertainty of the robot dynamics, it has been shown in (Takogaki, M. and S. Arimoto, 1981; Arimoto, S., 1996) that a simple joint space controller such as the PD or PID feedback is effective for setpoint control. However, in some applications, it is necessary to specify the motion in much more details than simply stating the desired final position.

In trajectory tracking control, a model-based robot controller that is tuned or calibrated to work perfectly using exact models of the system may give rise to very good control performance (Hollerbach, J.M., 1980; Luh, J.Y.S., M.H. Walker, 1980; Craig, J.J., 1986). However, the assumption of having exact models of the robot system also means that the robot is not able to adapt to any changes and uncertainties in its models and environment. For example, when a robot picks up several tools of different dimensions, unknown orientations or gripping points, the overall dynamics of robot changes and is therefore difficult to derive exactly.

The way by which human manipulates his arms easily and skillfully shows that we do not need the exact knowledge of the lengths and dynamics of our arms, the desired joint angles to reach for an object and the exact geometric relationship between our eyes and arms.

In most of the robot applications, a desired position for the end effector is usually specified in task space or Cartesian space. In order to move the robot end-effector to the desired position, the exact knowledge of the kinematics is required to solve the inverse kinematics problem to generate the desired position in joint space (Cheah, C.C., M. Hirano, 2003; Dixon, W.E., 2004; Cheah, C.C. and H.C. Liaw, 2005). When the control problem is formulated directly in task space, the need to solve the inverse kinematics problem is eliminated (Cheah, C.C., M. Hirano, 2003; Dixon, W.E., 2004; Cheah, C.C. and H.C. Liaw, 2005).

To overcome the problem of parameter uncertainty several setpoint controllers (Cheah, C.C., M. Hirano, 2003; Dixon, W.E., 2004) were proposed in the task space recently. Using the proposed controllers, other open problems such as force control with uncertainties (Cheah, C.C., S. Kawamura, 2003) and control of robot fingers with uncertain contact points (Cheah, C.C., H. Han, 1998) can be resolved in a unified formulation. However, the results in (Cheah, C.C., M. Hirano, 2003; Dixon, W.E., 2004) are focusing on parameter uncertainty in setpoint control of robot.

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Recently, an adaptive jacobian controller was proposed for trajectory tracking control of robot manipulators in the task space (Cheah, C.C., C. Liu, 2004; Cheah, C.C., C. Liu, 2005). The controller dose not requires the exact knowledge of jacobian matrix and dynamic parameters. However in dynamics of robot manipulators, there are unstructured uncertainties such as friction, disturbance and un-model dynamics that may cause an unstable closed loop control system.

In this paper, by use of physical properties of robot manipulator through direct Lyapunov method, robust nonlinear control and adaptive robust control have been proposed for trajectory tracking of robot with presence of structured and unstructured uncertainties in the task space. In section 2, robot dynamic in the joint space, its physical properties and the assumptions necessary for deign of controllers are expressed. In section 3, with the assumption of defined boundary of structured and unstructured uncertainties, design of robust nonlinear controller and proof of global exponential stability of closed-loop system are shown. In section 4, the bounds of unstructured uncertainties are estimated by adding an adaptive controller to the robust controller. It is verified that the proposed control system has global asymptotic stability. The performance of the proposed control strategy is illustrated through computer simulation for two link elbow robot in section 5.

***Dynamics of Robot Manipulator in the Joint Space:***

The joint space dynamics of an $n$ -link rigid-body robot manipulator can be described by the following second order nonlinear vector differential equation, so-called Euler-Lagrange equation (Qu, Z. and D. Dawson, 1996):

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d = \tau(t) \tag{1}$$

Where $q(t) \in R^n$ denotes the joint angles of the manipulator, $\dot{q}(t)$ and $\ddot{q}(t)$ are the vectors of joint velocity and joint acceleration, respectively. $M(q) \in R^{n\times n}$ is the inertia matrix which is symmetric and positive definite, $V_m(q,\dot{q})\dot{q} \in R^n$ is a vector function containing coriolis and centrifugal forces, $G(q) \in R^n$ is a vector function consisting of gravitational forces. $F_d \in R^{n\times n}$ is a diagonal matrix of viscous and dynamic friction coefficients, $F_s(\dot{q}) \in R^n$ is the vector of unstructured friction effects such as static friction terms. $T_d \in R^n$ s the vector of any generalized input due to disturbances or un-modeled dynamics and $\tau(t) \in R^n$ is the vector function consisting of applied generalized torques.

According to (Cheah, C.C., C. Liu, 2004; Cheah, C.C., C. Liu, 2005), the robot dynamics described above has the following properties:

***Property 1:***

The inertia matrix $M(q)$ is symmetric and positive definite for all $q \in R^n$ and $M(q)$ is uniformly bounded above and below. That is

$$\mu_1 I \le M(q) \le \mu_2 I \quad or \quad \mu_1 \le \left\| M(q) \right\| \le \mu_2 \tag{2}$$

Where $\left\| \circ \right\|$ stand for the Euclidean norm, $\mu_1$ and $\mu_2$ are positive constant.

***Property 2:***

The matrix $\dot{M}(q) - 2V_m(q,\dot{q})$ is skew-symmetric. That is

$$y^T \dot{M}(q) y = 2y^T V_m(q,\dot{q}) y \quad , \quad \forall y,q,\dot{q} \in R^n \tag{3}$$

***Property 3:***

The left side of (1) can be linearly parameterized. This property may be expressed as

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) = W(q,\dot{q},\ddot{q})P \tag{4}$$

Where $p \in r^m$ s a parameter vector and $W(q,\dot{q},\ddot{q})$ s a known matrix of robot function depending on the joint variables, joint velocities and joint accelerations.

In most applications of robot manipulators, a desired path for the end-effector is specified in task space such as visual space or Cartesian space. Let $X \in R^n$ be a task space vector defined by:

$$X = h(q) \tag{5}$$

Where $h(\circ) \in R^n \to R^n$ is generally a nonlinear transformation describing the relation between the joint space and task space. According to (Cheah, C.C., C. Liu, 2004; Cheah, C.C., C. Liu, 2005), the task space velocity $\dot{X}$ is related to joint space velocity $\dot{q}$ as:

$$\dot{X} = J(q)\dot{q} \tag{6}$$

Where $J(q) \in R^{n \times n}$ is the Jacobian matrix from joint space to task space. For design of robust nonlinear controller, the following assumptions should be established.

***Assumptions:***

1. $\left\| F_d y + F_s(y) \right\| = \xi_{f_0} + \xi_{f_1} \left\| y \right\| \ , \forall y \in R^n$

2. $\left\| T_d \right\| \leq \xi_t$

3. $\left\| \tilde{p} \right\| = \left\| p - \hat{p} \right\| \leq \rho$

Where $\xi_{f_0}$ , $\xi_{f_1}$ $\xi_t$ and ρ denote some positive constant and $\hat{P}$ denotes a estimate parameter vector of $P$ . From Eq (6) we have:

$$\dot{q} = J^{-1}(q)\dot{X} \tag{7}$$

Where $J^{-1}(q)$ is the inverse of the Jacobian matrix. In this paper, we assume that the robot is operating in a finite task space such that the Jacobian matrix is full rank. Now, the question is to synthesize a robust control τ (t) such that the robot dynamic (1) is stable in the presence of the structured and unstructured uncertainties.

***Robust Control in Task Space:***

Let us define a vector $\dot{X}_p \in R^n$ as:

$$\dot{X}_p = \alpha\left(X_d - X\right) + \dot{X}_d \tag{8}$$

Where α is positive constant, *X* is measured from a position sensor and $X_d \in R^n$ s a desired trajectory specified In task space and $\dot{X}_d \in R^n$ s the desired velocity specified in task space. Many commercial sensors are available for measurement of *X*, such as vision systems, electromagnetic measurement systems, position sensitive detectors or laser tracking systems. To design of robust control, we define a task space sliding vector as:

$$S_X = \dot{X}_p - \dot{X} = \alpha\left(X_d - X\right) + \left(\dot{X}_d - \dot{X}\right) \tag{9}$$

We define the task space position error $X_d - X = e\left(t\right)$ therefore we have:

$$S_X = \alpha e\left(t\right) + \dot{e}\left(t\right) \tag{10}$$

From Eq (7) we have:

$$\dot{q}_p = J^{-1}\left(q\right)\dot{X}_p \tag{11}$$

The derivative of Eq (11) respect to time can be written as:

$$\ddot{q}_p = J^{-1}\left(q\right)\ddot{X}_p + \dot{J}^{-1}\left(q\right)\dot{X}_p \tag{12}$$

We define a joint space sliding vector as:

$$S_q = \dot{q}_p - \dot{q} \tag{13}$$

Jacobian matrix is multiplied by both side of (13) and from Eq.(6) and Eq.(9), we have:

$$J\left(q\right)S_q = J\left(q\right)\dot{q}_p - J\left(q\right)\dot{q} = \dot{X}_p - \dot{X} = S_X \tag{14}$$

Eq. (14) can be expressed as:

$$S_q = J^{-1}\left(q\right)S_X \tag{15}$$

According to (Cheah, C.C., S. Kawamura, 2003), the torque τ (t) is related to force *f* (t) as:

$$\tau\left(t\right) = J^{-1}\left(q\right)f\left(t\right) \tag{16}$$

From (16), we propose a robust control as:

$$\tau(t) = \hat{M}(q)\ddot{q}_p + \hat{V}_m(q,\dot{q})\dot{q}_p + \hat{G}(q) + \gamma \hat{M}(q) J^{-1}(q) S_X + u + u_r \tag{17}$$

Where $\hat{M}(q)$ , $\hat{V}_m(q,\dot{q})$ and $\hat{G}(q)$ can be simplified versions of the known parts of $M(q)$ , $V_m(q,\dot{q})$ and $G(q)$ respectively, γ is positive constant, $u$ and $u_r$ are new input controls. From Eq. (15), Eq. (17) can be expressed as:

$$\tau(t) = \hat{M}(q)\ddot{q}_p + \hat{V}_m(q,\dot{q})\dot{q}_p + \hat{G}(q) + \gamma \hat{M}(q) S_q + u + u_r \tag{18}$$

Eq (18) is substituted into Eq (1) and we define $\Delta A = F_d\dot{q} + F_s(q) + T_d$ thus:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + \Delta A = \hat{M}(q)\ddot{q}_p + \hat{V}_m(q,\dot{q})\dot{q}_p + \hat{G}(q) + \gamma \hat{M}(q) S_q + u + u_r \tag{19}$$

From Eq. (13) and Eq. (19), the following equation can be expressed as:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + \Delta A = \hat{M}(q)(\dot{S}_q + \ddot{q}) + \hat{V}_m(q,\dot{q})(S_q + \dot{q}) + \hat{G}(q) + \gamma \hat{M}(q) S_q + u + u_r \tag{20}$$

Eq. (20) can be simplified as:

$$\begin{aligned}\hat{M}(q)\dot{S}_q + \hat{V}_m(q,\dot{q})S_q = {} & \left(M(q) - \hat{M}(q)\right)\ddot{q} + \left(V_m(q,\dot{q}) - \hat{V}_m(q,\dot{q})\right)\dot{q} + \left(G(q) - \hat{G}(q)\right) \\ & -\gamma \hat{M}(q) S_q + (\Delta A - u_r) - u\end{aligned} \tag{21}$$

From Eq. (4), Eq. (21) can be expressed as:

$$\hat{M}(q)\dot{S}_q + \hat{V}_m(q,\dot{q})S_q = W(q,\dot{q},\ddot{q})\left(P - \hat{P}\right) - \gamma \hat{M}(q) S_q + (\Delta A - u_r) - u \tag{22}$$

Let us define $P - \hat{P} = \tilde{P}$ as parameter error, therefore we have:

$$\hat{M}(q)\dot{S}_q + \hat{V}_m(q,\dot{q})S_q = W(q,\dot{q},\ddot{q})\tilde{P} - \gamma \hat{M}(q) S_q + (\Delta A - u_r) - u \tag{23}$$

***Stability Proof:***

To prove the stability of system (23), the Lyapunov function candidate is presented as:

$$V(t) = \frac{1}{2} S_q^T \hat{M}(q) S_q \tag{24}$$

The derivative of Eq (24) respect to time can be written as:

$$\dot{V}(t) = S_q^T \hat{M}(q)\dot{S}_q + \frac{1}{2} S_q^T \dot{\hat{M}}(q) S_q \tag{25}$$

Eq (23) is substituted into Eq (25):

$$\dot{V}(t) = S_q^T\left(-\hat{V}_m(q,\dot{q})S_q + W(q,\dot{q},\ddot{q})\tilde{P} - \gamma\hat{M}(q)S_q + (\Delta A - u_r) - u\right) + \frac{1}{2}S_q^T\dot{\hat{M}}(q)S_q \tag{26}$$

According to properties in section 2, Eq (26) can be expressed as:

$$\dot{V}(t) = -2\gamma V + S_q^T W(q,\dot{q},\ddot{q})\tilde{P} + S_q^T(\Delta A - u_r) - S_q^T u \tag{27}$$

According to Assumption in section 2.1, $\Delta A$ can be written as:

$$\begin{aligned} \|\Delta A\| &\le \xi_{f_0} + \xi_{f_1}\|\dot{q}\| + \xi_t \\ \eta &= \xi_{f_0} + \xi_{f_1}\|\dot{q}\| + \xi_t \end{aligned} \tag{28}$$

According to Assumption in section 2.1 and Eq. (28), we define $u$ and $u_r$ to the following forms:

$$\begin{aligned} u &= \begin{cases} \dfrac{WW^T S_q \rho^2}{\|S_q^T W\|\rho + \varepsilon\|S_q\| e^{-\beta(t-t_0)}} & \|S_q\| \ne 0 \\ 0 & \|S_q\| = 0 \end{cases} \\ u_r &= \begin{cases} \dfrac{Z\eta}{\left(\|Z\| + \lambda e^{-\beta(t-t_0)}\right)} \\ Z = \eta S_q \end{cases} \end{aligned} \tag{29}$$

Where έ, λ and β are positive constant. Eq. (29) is substituted into Eq. (27) and can be simplified as:

$$\dot{V}(t) \le -2\gamma V + \frac{\|Z\|\lambda e^{-\beta(t-t_0)}}{\|Z\| + \lambda e^{-\beta(t-t_0)}} + \frac{\varepsilon\|S_q\|\|S_q^T W\|\rho e^{-\beta(t-t_0)}}{\|S_q^T W\|\rho + \varepsilon\|S_q\| e^{-\beta(t-t_0)}} \tag{30}$$

We have:

$$0 \le \frac{XY}{X+Y} \le Y \qquad , \forall\ X, Y \tag{31}$$

From Eq. (30) and Eq. (31), we have:

$$\dot{V}(t) \leq -2\gamma V + \left(\lambda + \varepsilon \left\| S_q \right\|\right) e^{-\beta(t-t_0)} \tag{32}$$

According to Eq. (32), since $t \to \infty$, exponential section will be converged to zero therefore $\dot{V}(t) \leq 0$, thus $S_q$ and according to Eq. (14), $S_x$ will be converged to zero. Therefore, the closed loop system with proposed controller has global asymptotic stability.

***Global exponential Stability Proof:***

Let us define a scaler function φ (t) to the following form:

$$\varphi(t) = \dot{V}(t) + 2\gamma V(t) - \left(\lambda + \varepsilon \left\| S_q \right\|\right) e^{-\beta(t-t_0)} \tag{33}$$

From Eq. (33), the Lyapunov function is presented as:

$$V(t) = V(t_0) e^{-2\gamma(t-t_0)} + \int_{t_0}^{t} e^{-2\gamma(t-Q)} \left[ \left(\lambda + \varepsilon \left\| S_q \right\|\right) e^{-\beta(Q-t_0)} + \varphi(Q) \right] dQ \tag{34}$$

From Eq. (32) and Eq. (33), it is concluded that $\varphi(t) \leq 0$ Thus:

$$V(t) \leq V(t_0) e^{-2\gamma(t-t_0)} + \left(\lambda + \varepsilon \left\| S_q \right\|\right) \int_{t_0}^{t} e^{-2\gamma(t-Q)} e^{-\beta(Q-t_0)} dQ \tag{35}$$

Eq. (35) is simplified as:

$$V(t) \leq V(t_0) e^{-2\gamma(t-t_0)} + \frac{\left(\lambda + \varepsilon \left\| S_q \right\|\right)}{2\gamma - \beta} \left( e^{-\beta(t-t_0)} - e^{-2\gamma(t-t_0)} \right) \tag{36}$$

From Eq. (2), Eq. (36) and by uniform continuity and the barbalat lemma [14], consequently the closed loop system (23) with (29) is globally exponentially stable in the presence of structured and unstructured uncertainties. Hence, the proposed controller is presented as:

$$\tau(t) = \hat{M}(q)\ddot{q}_p + \hat{V}_m(q,\dot{q})\dot{q}_p + \hat{G}(q) + \gamma \hat{M}(q) J^{-1}(q) S_X + u + u_r$$

$$u = \begin{cases} \dfrac{W W^T S_q \rho^2}{\left\| S_q^T W \right\| \rho + \varepsilon \left\| S_q \right\| e^{-\beta(t-t_0)}} & \left\| S_q \right\| \neq 0 \\ 0 & \left\| S_q \right\| = 0 \end{cases} \tag{37}$$

$$u_r = \begin{cases} \dfrac{Z\eta}{\left( \left\| Z \right\| + \lambda e^{-\beta(t-t_0)} \right)} \\ Z = \eta S_q \end{cases}$$

***Adaptive Robust control:***

As has been shown, proposed control ensures robust stability for robot systems. Robust controls are in terms of some bounding function, and determination of the bounding function requires information on the size of the uncertainties, such as maximum load variation. Without specifying applications, it may be difficult to know this size information. While underestimation is not permitted when considering robustness, overestimating the maximum size of uncertainties can potentially give robust control an unnecessarily large magnitude and gain, and consequently put too many requirements on the actuators.

According to assumptions in section 2.1, the structured and the unstructured uncertainties is specified, therefore, we can solve the problem by adding an adaptive control to the robust control in section 3. Because, the adaptive control can estimate the bound of uncertainties in any time. Therefore, the size of adaptive robust control input is determined by the estimated bound. Thus, we can specify the required actuator through the approximate size of unstructured uncertainties. Therefore, According to assumptions section 2.1 and Eq. (29), we have:

$$\eta = W_1^T\left(\dot{q}\right)\phi \tag{38}$$

Where $\phi \in R^l$ is a parameter vector and $W_1^T\left(\dot{q}\right)$ is a known vector of function. By adding an adaptive control to the proposed control Eq. (37), we can estimate unknown parameter. Therefore Eq. (37) is changed to the following form:

$$\begin{aligned}
&\tau\left(t\right) = \hat{M}\left(q\right)\ddot{q}_p + \hat{V}_m\left(q,\dot{q}\right)\dot{q}_p + \hat{G}\left(q\right) + \gamma\hat{M}\left(q\right)J^{-1}\left(q\right)S_X + u + u_r \\
&u = \begin{cases} \dfrac{WW^TS_q\rho^2}{\left\|S_q^TW\right\|\rho + \varepsilon\left\|S_q\right\|e^{-\beta(t-t_0)}} & \left\|S_q\right\| \neq 0 \\ 0 & \left\|S_q\right\| = 0 \end{cases} \\
&u_r = \begin{cases} \dfrac{\hat{Z}\hat{\eta}}{\left(\left\|\hat{Z}\right\| + \lambda e^{-\beta(t-t_0)}\right)} \\ \hat{Z} = \hat{\eta}S_q \end{cases}
\end{aligned} \tag{39}$$

Where $\hat{Z}$ and $\hat{\eta}$ are estimation of $Z$ and $\eta$ respectively.

***Stability Proof:***

We define $\phi - \hat{\phi} = \tilde{\phi}$ as error estimation. To prove the stability of system (23), the Lyapunov function candidate is presented as:

$$V\left(t\right) = \frac{1}{2}S_q^T\hat{M}\left(q\right)S_q + \frac{1}{2}\tilde{\phi}^T\tilde{\phi} \tag{40}$$

The derivative of Eq (40) respect to time can be written as:

$$\dot{V}(t) = S_q^T \hat{M}(q)\dot{S}_q + \frac{1}{2} S_q^T \dot{M}(q) S_q - \tilde{\phi}^T \dot{\hat{\phi}} \tag{41}$$

Eq (23) is substituted into Eq (41) and it is simplified as:

$$\dot{V}(t) = -\gamma S_q^T \hat{M}(q) S_q + S_q^T W(q,\dot{q},\ddot{q})\tilde{P} + S_q^T(\Delta A - u_r) - S_q^T u - \tilde{\phi}^T \dot{\hat{\phi}} \tag{42}$$

According to assumptions of section 2.1 and Eq. (28), Eq. (39) and Eq. (42), we have:

$$\dot{V}(t) \leq -\gamma S_q^T \hat{M}(q) S_q + \left\|S_q^T W(q,\dot{q},\ddot{q})\right\|\rho + \left(\|Z\| - \left\|\hat{Z}\right\|\right) + \left\|\hat{Z}\right\| - S_q^T u_r - S_q^T u - \tilde{\phi}^T \dot{\hat{\phi}} \tag{43}$$

Eq. (39) is substituted into Eq. (43) and it is simplified as:

$$\dot{V}(t) \leq -\gamma S_q^T \hat{M}(q) S_q + \frac{\|Z\|\lambda e^{-\beta(t-t_0)}}{\|Z\| + \lambda e^{-\beta(t-t_0)}} + \frac{\varepsilon\left\|S_q\right\|\left\|S_q^T W\right\|\rho e^{-\beta(t-t_0)}}{\left\|S_q^T W\right\|\rho + \varepsilon\left\|S_q\right\| e^{-\beta(t-t_0)}} + \left\|S_q\right\| W_2^T \tilde{\phi} - \tilde{\phi}^T \dot{\hat{\phi}} \tag{44}$$

From Eq. (31) and Eq. (44), we have:

$$\dot{V}(t) \leq -\gamma S_q^T \hat{M}(q) S_q + \left(\lambda + \varepsilon\left\|S_q\right\|\right) e^{-\beta(t-t_0)} + \left\|S_q\right\| W_2^T \tilde{\phi} - \tilde{\phi}^T \dot{\hat{\phi}} \tag{45}$$

In order to $\dot{V}(t) \leq 0$ we should equivalent the third and the fourth part on the right side of Eq. (45), thus:

$$\dot{\hat{\phi}} = W_2 \left\|S_q\right\| \tag{46}$$

From Eq. (45), Eq. (46), we have $\dot{V}(t) \leq 0$ consequently the closed loop system (23) with (39) is globally asymptotically stable in the presence of structured and unstructured uncertainties. Hence, adaptive robust controller is presented as:

$$\begin{aligned}
&\tau(t) = \hat{M}(q)\ddot{q}_p + \hat{V}_m(q,\dot{q})\dot{q}_p + \hat{G}(q) + \gamma \hat{M}(q) J^{-1}(q) S_X + u + u_r \\
&u = \begin{cases} \dfrac{WW^T S_q \rho^2}{\left\|S_q^T W\right\|\rho + \varepsilon\left\|S_q\right\| e^{-\beta(t-t_0)}} & \left\|S_q\right\| \neq 0 \\ 0 & \left\|S_q\right\| = 0 \end{cases} \\
&u_r = \begin{cases} \dfrac{\hat{Z}\hat{\eta}}{\left(\left\|\hat{Z}\right\| + \lambda e^{-\beta(t-t_0)}\right)} \\ \hat{Z} = \hat{\eta} S_q \end{cases} \\
&\hat{\eta} = W_1^T \hat{\phi} \quad , \quad \dot{\hat{\phi}} = W_2 \left\|S_q\right\|
\end{aligned} \tag{47}$$

***Case Study of Two-Link Elbow Robot Manipulator:***

In order to verify the performance of proposed control schemes, as an illustration, we will apply the above presented controllers to a two-link elbow robot manipulator as shown in Fig.1. the dynamic of the two-link elbow robot manipulator can be described in the following differential equation:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+\begin{bmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{bmatrix}=\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{48}$$

$$M_{11}=\left(l_2{}^2 m_2+2l_1l_2\cos(q_2)+l_1{}^2(m_1+m_2)\right) \tag{49}$$

$$M_{12}=M_{21}=\left(l_2{}^2 m_2+l_1l_2m_2\cos(q_1)\right) \tag{50}$$

$$M_{22}=l_2{}^2 m_2 \tag{51}$$

$$h_1(q,\dot{q})= \begin{Bmatrix} -m_2l_1l_2\sin(q_2)\dot{q}_2{}^2-2m_2l_1l_2\sin(q_2)\dot{q}_1\dot{q}_2+m_2l_2g\cos(q_1+q_2)+(m_1+m_2)l_1g\cos(q_1) \\ +F_{d_1}\dot{q}_1+F_{s_1}(\dot{q}_1)+T_{d_1} \end{Bmatrix} \tag{52}$$

$$h_2(q,\dot{q})= \left\{\left(m_2l_2g\cos(q_1+q_2)\right)+\left(m_2l_1l_2\sin(q_2)\dot{q}_1^2\right)+F_{d_2}\dot{q}_2+F_{s_1}(\dot{q}_2)+T_{d_2}\right\} \tag{53}$$

Where, $l_1$ and $l_2$ are lengths of the first and second links respectively, $m_1$ and $m_2$ are masses of the first and second links respectively, $g$ is the gravitational force, $F_d$ is dynamic friction. $F_s(\dot{q})$ is static friction and $T_d$ is disturbance and un-modeled dynamics. $u_1$ and $u_2$ are input torques of the first and second links respectively. Robot parameters which have been used in this simulation are given in table. 1.

**Table 1:** parameters of two link elbow robot

| | | | |
|---|---|---|---|
| $l_1=0.38\ m$ | $l_2=0.29\ m$ | $m_1=1.4\ kg$ | $m_2=0.8\ kg$ |
| $g=9.8$ | $T_{d_1}=T_{d_2}=0.5$ | $F_{d_1}=F_{d_2}=0.5$ | $F_{s_1}(\dot{q}_1)=F_{s_2}(\dot{q}_2)=0.5$ |

Parameters of Controller are shown in table. 2.

**Table 2 :** Parameters of Controller

| | |
|---|---|
| $\hat{l}_1=0.48\ m$ | $\hat{l}_2=0.4\ m$ |
| $\hat{m}_1=1.5\ kg$ | $\hat{m}_2=0.9\ kg$ |
| $\beta=0.01$ | $\gamma=100$ |
| $\varepsilon=0.5$ | $\alpha=250$ |
| $\hat{g}=9$ | $\rho=0.1$ |
| $\eta=2$ | $\lambda=15$ |

Physical parameters of adaptive control are expressed in the following form:

$$\hat{P} = \left[ \hat{l}_2^2 \hat{m}_2 \quad \hat{l}_1 \hat{l}_2 \hat{m}_2 \quad \hat{l}_1^2 \left( \hat{m}_1 + \hat{m}_2 \right) \quad \hat{m}_2 \hat{l}_2 \hat{g} \quad \left( \hat{m}_1 + \hat{m}_2 \right) \hat{l}_1 \hat{g} \right]^T$$

Regression matrix is shown in table 3.

**Table 3 :** Regression Matrix

| | |
|---|---|
| $W_{11} = \ddot{q}_1 + \ddot{q}_2$ | $W_{12} = 2\cos(q_2)\ddot{q}_1 + \cos(q_2)\ddot{q}_2 - \sin(q_2)\dot{q}_2 - 2\sin(q_2)\dot{q}_1$ |
| $W_{13} = \ddot{q}_1$ | $W_{14} = \cos(q_1 + q_2)$ |
| $W_{15} = \cos(q_1)$ | $W_{21} = \ddot{q}_1 + \ddot{q}_2$ |
| $W_{22} = \cos(q_2)\ddot{q}_1 + \sin(q_2)\dot{q}_1$ | $W_{23} = 0$ |
| $W_{24} = \cos(q_1 + q_2)$ | $W_{25} = 0$ |

Desired path in the task space and initial condition are expressed in table. 4.

**Table 4:** Desired Path and Initial Condition

| | |
|---|---|
| $X_d = 0.284 + 0.05\sin(3t)$ | $Y_d = 0.403 + 0.05\cos(3t)$ |
| $X_d(0) = 0.255$ | $Y_d(0) = 0.48$ |

In this paper, simulation is performed in two sections. In the first section, according to Fig. 2 and Fig. 3, the robust control performs appropriately and trajectory errors converge to zero in the presence of structured and unstructured uncertainties. Desired path and actual path in the task space are shown in Fig. 4. Input controls of the first and second joint are presented in Fig. 5 and Fig. 6 respectively. According to the figures,

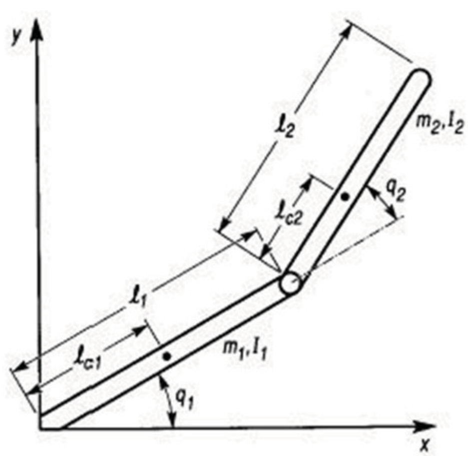


**Fig. 1:** Two-Link Elbow Robot Manipulator

***Simulation resulting from application of the robust control***

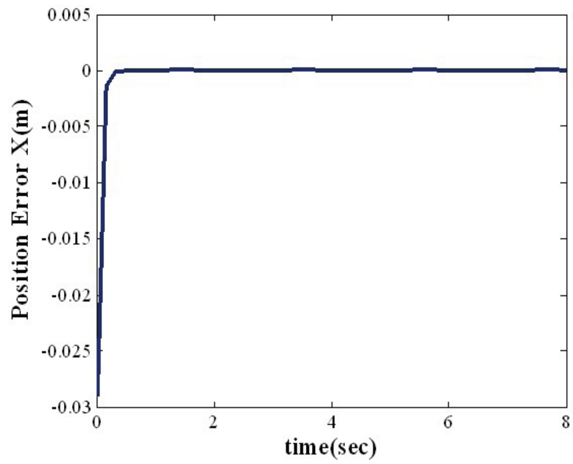


**Fig. 2:** Tracking error of $X_d$ in the task space

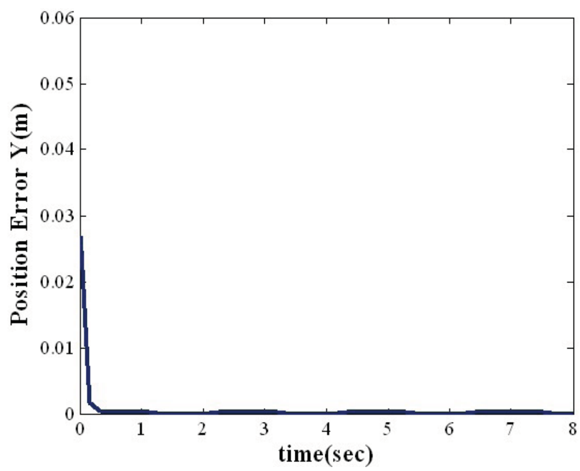


**Fig. 3:** Tracking error of $Y_d$ in the task space

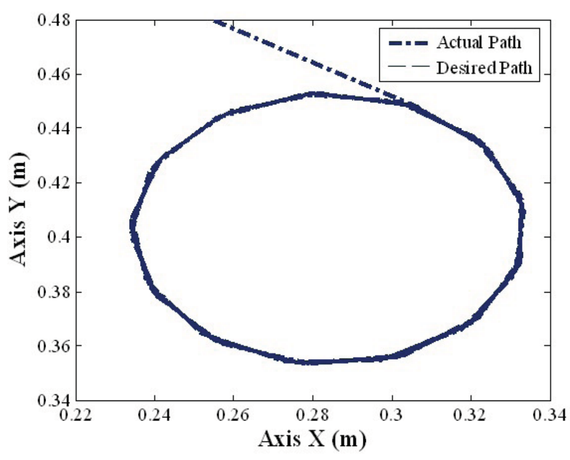


**Fig. 4:** Trajectory of end effector in the task space

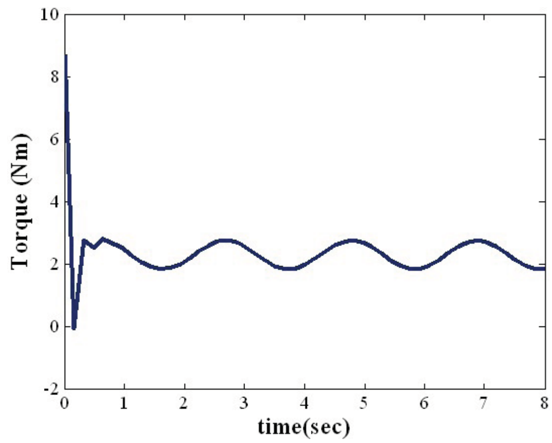


**Fig. 5:** Input control of joint 1

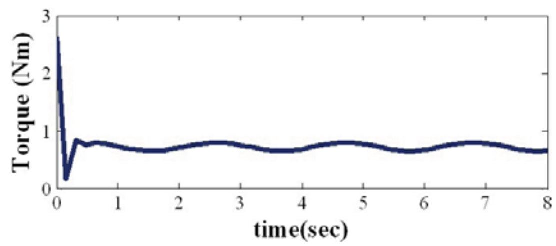


**Fig. 6:** Input control of joint 2

input controls are appropriate. In the second section of simulation, adaptive robust control is applied to the two-link robot manipulator. According to Fig. 7 and Fig. 8, the adaptive robust control performs appropriately and trajectory errors converge to zero. Input controls are shown in Fig 9, Fig 10 respectively. Unstructured uncertainty estimation is presented in Fig. 11.

***Simulation resulting from application of the adaptive robust control***

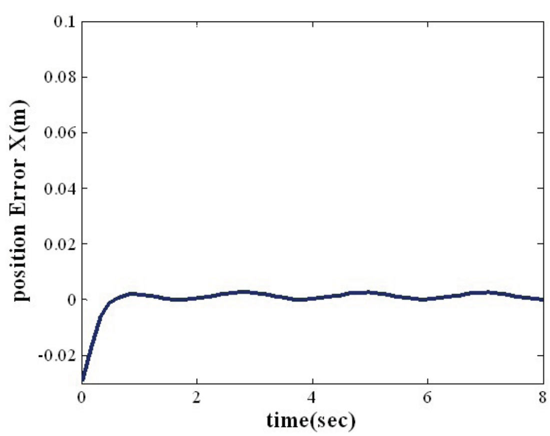


**Fig. 7:** Tracking error of $X_d$ in the task space

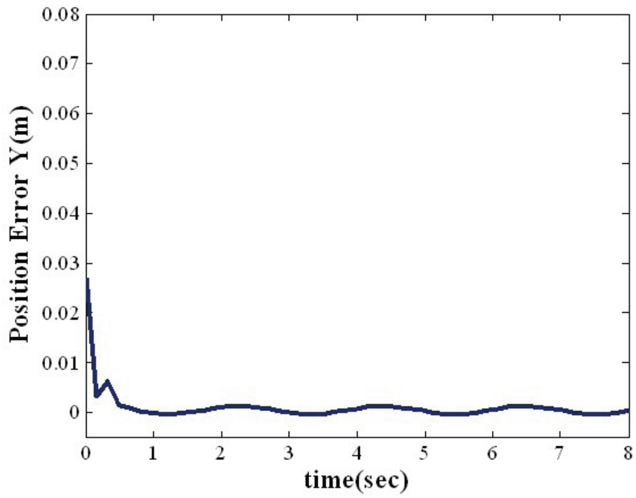


**Fig. 8:** Tracking error of $Y_d$ in the task space

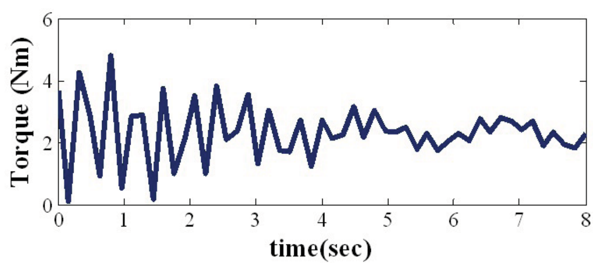


**Fig. 9:** Input control of joint 1

***Conclusion:***

In this paper, based on the physical properties of the robot manipulator, the robust control was designed in which the bounds of structured and unstructured uncertainties are known. It was proven that the closed loop system had global exponential stability. Since, in most applications of the robot, the bounds of structured uncertainties like load changing are known but those of the unstructured uncertainties are unknown. In the next stage, an adaptive control was added to the robust control so that it could estimate the bounds of such uncertainties in any time. Consequently, the problem of unknown uncertainties was solution in spite of the adaptive robust control and it was proven that the closed loop system had global asymptotic stability.

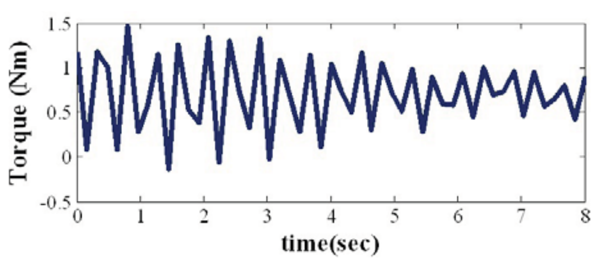


**Fig. 10:** Input control of joint 2

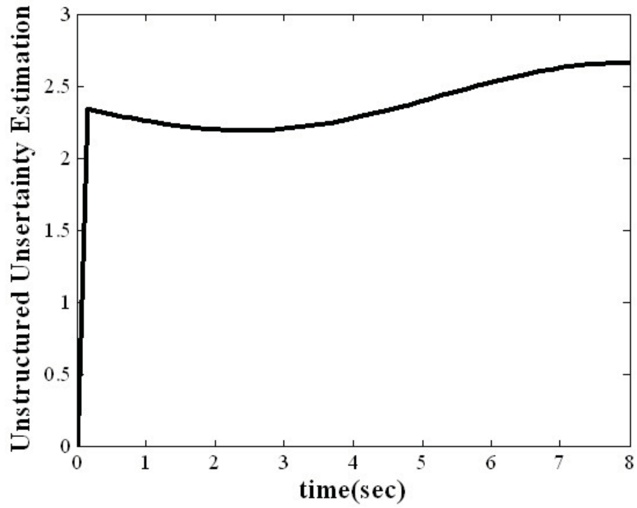


**Fig. 11:** Estimation of unstructured uncertainty

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