Modelling and Control of Flexible Wind Turbines Without Wind Speed Measurements

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Abstract: Modelling and simulation of wind turbines were investigated from the control point of view in order to analyse the effect of flexibility and aero elastic coupling on power extraction rate efficiency. Descriptions of aerodynamic interaction and aero elastic mechanical coupling were performed by means of simplified effective models. The main effect of aero elastic coupling was modelled by replacing, in equations giving the classical aerodynamic forces, the incident wind speed by the relative wind speed that takes into account velocity of the dominant structural modes having large axial components in the wind direction. A torque based controller was considered in the below rated power regime under the assumption that wind speed measurements enable exact prediction of the captured aerodynamic torque. Analysis of power rate sensitivity to structural flexibility has shown that, for the particular wind turbine investigated in this work, structural flexibility acts like small disturbances such that H-infinity technique could be introduced to design an enhanced wind turbine controller. Since in real wind turbines the captured aerodynamic torque could not be predicted from single point wind speed measurements as this quantity varies stochastically over the whole rotor plane, a special estimator for wind speed was constructed in order to perform control without speed sensing. This has been shown to yield for the flexible wind turbine a robust nonlinear controller that takes into account directly wind perturbations and structural disturbances. The obtained results in terms of power efficiency have shown that this controller is more performing than classical control techniques.

Key words: Wind energy, turbine, structural flexibility, aeroelasticity, power extraction, control, speed estimator

INTRODUCTION

Wind energy is the issue currently of an unrecorded growth as the cost price of this energy form has become competitive and considerable technological progress has been achieved in the field of wind turbines. More intelligence is being introduced in control of these machines (Burton, T., D. Sharpe, N. Jenkins, 2001; Santoso, S. and H.T. Le, 2007; Jauch, C., S.M. Islam, 2007; Calderaro, V., V. Galdi, 2008; Sainz, E., A. Llombart, 2009; Galdi, V., A. Piccolo, 2009). The objective is to optimize power efficiency and quality and to enhance operating conditions.

Modelling and simulation of wind turbines aim at analysing and optimizing the power extraction rate. But, these tasks are complex since they evolve descriptions of aerodynamic interaction, elastic mechanical coupling, electrical subsystem, electric grid and pitch actuator subsystem.

Aerodynamic forces acting on wind turbines are turbulent in nature. Wind speed is known to vary stochastically, (Larsen, T.J., 2004). As a result, it is impossible to predict the captured aerodynamic torque from single point wind speed measurements. Therefore, one is lead to estimate it for control purposes.

For wind turbines which are designed to operate at variable rotational speeds, a common practice in power control is that the generator torque demand is set proportional to the square of the measured generator speed.
The value of the coefficient of proportionality is often determined from performance code simulations (Wright, A., 2003). It can be identified experimentally by accomplishing tests when the wind turbine could be operated at constant rotational speeds over a range of wind speeds. Tracking the rotor speed along this trajectory would yield maximum power coefficient values for all wind speeds.

Another approach that eliminated the need to a priori knowledge of the turbine’s performance and enabled to deal with slow changes that could affect aerodynamic properties was introduced in (Johnson, K.E., 2004). It consists of a simple empiric gain adaptation algorithm that was shown to improve capture in below-rated wind speeds compared to the standard generator torque controller. The adaptive controller begins by changing the gain by some increment value. At the end of the adaptation period which is of the order of few minutes to an hour, the controller evaluates the turbine’s performance. The average power coefficient for the actual adaptation period is compared to the average power coefficient for the previous adaptation sequence. The process is iterated and it has been shown to converge to a value that resulted in maximum power capture. Variations that could affect wind turbines properties could never be stabilized and the adaptive controller had to be permanently moved to track ideal convergence conditions for maximising power extraction. This introduces some delay in wind turbine operation because of the necessary period of adaptation.

Other extensions to classical techniques based on PI or PID controller have been used to improve the performance, for example the use of non linear gains to penalize large peaks appearing in control variables; the gain of a PI pitch controller can be increased as the power or speed error increases (Burton, T., D. Sharpe, 2001). Advanced controller design methods have been investigated in the context of wind turbine systems such as LQG, H-infinity, fuzzy logic controllers and neural network methods (Burton, T., D. Sharpe, 2001).

As stated in (Bemporad, A., M. Morari, 2002), model predictive control has become the accepted standard for complex constrained multivariable control problems and had been implemented for wind turbines (Martinez, J., 2007). But, model predictive control requires on-line computational effort, which limits its applicability to relatively slow and/or small problems.

Optimal or robust adaptive nonlinear controls have also been introduced (Abdin, A., 2000) and (Boukhezzar, A., 2006).

Classical control laws were often elaborated by neglecting structural flexibility and performing linearization of system equations around an operating point. These control laws assumed a priori rigid body motion of the drive train and neglected without any advanced justification aero elastic coupling and structural flexibility. Modern approaches to wind turbine power control incorporated these elastic phenomena, (Wright, A., 2003) and (Bianchi, F.D., H. De Battista, 2007). However, comprehensive studies that deal with control in the presence of elastic effects associated to real wind turbines and stochastic variations that characterise wind speed have not been largely reported in the literature.

In this work, modelling of wind turbine dynamics is considered to include structural flexibility with a reasonable number of elastic degrees of freedom. A torque controller is introduced at first under the assumption that wind speed measurements enable exact prediction of the captured aerodynamic torque. The interest of this hypothesis is that it has enabled to assess that structural flexibility and aero elastic coupling could be considered, under torque control protocol, to act like small disturbances. This has opened the way to consider a robust adaptive nonlinear controller based on H-infinity algorithm in order to take into account directly wind perturbations and structural disturbances. A special wind speed estimator was developed to perform wind turbine control. Simulations performed by (Vivas et al., 2008) have shown that the proposed wind estimation methodology, tracks the mean tendency of the wind profile, discarding the high turbulent components. An excellent performance is obtained in terms of extracted power from the wind with this estimator without excessively increasing peaks in generator torque control variable.

2 Aerodynamic Interaction Model for Variable Speed, Variable Pitch Angle Wind Turbine:

Air mass flowing across the wind turbine rotor plane suffers sudden drop of momentum which is gradually transformed to kinetic energy of rotor blades. The captured aerodynamic torque can be written in terms of the power coefficient $C_p(\lambda, \beta)$ as, (Camblong, H., 2003),

$$T_s = \frac{\rho \pi R^2 V^3}{2 \omega_s} C_p(\lambda, \beta)$$

with $\lambda$ the tip speed ratio defined as
where $\omega_a$ is the rotational rotor speed, $V$ the effective wind speed, $\rho$ the air density and $R$ the blades rotor radius.

The effective wind speed is related to the upstream wind speed at large of the rotor plane, but an explicit relation does not exist between these two parameters. There is no mean in practice to get the value of the effective wind speed from measurements and it must be estimated for control purposes.

The power coefficient $C_p(\lambda, \beta)$ could be estimated using aerodynamic data (obtained from wind tunnel measurements) and performing BEM calculations or obtained directly from wind tunnel testing. It is generally represented under the form of an analytical formula which gives $C_p(\lambda)$ for various values of parameter $\beta$. Linear interpolations are performed between two adjacent discrete values of parameter $\beta$ in order to obtain any $C_p(\lambda, \beta)$ value associated to any arbitrary $\beta$. In the literature, (Moriatory, P.J. and A.C. Hansen, 2004), one finds the following approximation

$$C_p(\lambda, \beta) = \left(\frac{c_1}{\lambda} - c_2 \beta - c_3\right) e^{-\frac{\beta^2}{\lambda}} + c_5 \lambda$$

with

$$\frac{1}{\lambda} = \frac{1}{\lambda + c_6 \beta} - \frac{c_7}{\beta^3 + c_8}$$

where coefficients $c_i, i = 1, \ldots, 8$ are identified from real $C_p$ curves.

Another way to represent $C_p(\lambda, \beta)$ consists in giving directly $C_p$ as a look-up table as function of discrete values of parameters $\lambda$ and $\beta$.

3 Modelling Flexibility of Wind Turbines:

Flexibility of wind turbines of modern scale has major influence on the control design (Wright, A., 2003). For below rated torque control design, as considered in this work, modelling flexibility of the whole wind turbine system is not really needed. One should only consider the most flexible parts of the wind turbine system which could interact significantly with the other subsystems and modify their behaviour in the low frequency domain, (Slootweg, J.G., S.W.H. Haan, 2003), that is to say the driving train which is compound of the low and high speed shafts. Even if flexibility of blades is more important in the general dynamic behavior, considering generator torque control results in that shaft torsion is the most important flexibility in the system. Other flexibilities cannot be omitted, but they are shown to remain small and to act as disturbances that could be tackled in a systematic way by H-infinity control.

When only the high speed shaft torsion deformation is considered, a flexible model of wind turbine having two degrees of freedom, figure 1, is obtained as
where the point \((\cdot)\) designates the first time derivative, \(T_g, \theta_g, \omega_g = \dot{\theta}_g, J_a, J_g, K_c, D_c, D_a, D_g\) are the generator torque, the azimuthally position of the high speed shaft, the rotational speed of the high speed shaft, the moment of inertia of rotor side masses, the moment of inertia of generator side masses, the coupling stiffness, the coupling damping, the mechanical damping in the rotor side and the mechanical damping in the generator side respectively. But \(n\) is the gear ratio between the primary and the secondary shafts.

\[
\theta_s = n\theta_a - \theta_g \quad \text{and} \quad \omega_s = n\omega_a - \omega_g
\]

are the elastic torsion angle and its velocity given in terms of the azimuthally rotor position, \(\theta_a\), and the rotor rotational speed, \(\omega_a\).

One should notice that by letting \(\omega_s = 0\) in equations system (5), the rigid model with only one degree of freedom is obtained. For this model, \(\omega_g = n\omega_a\) is satisfied. Using then the resulting equations from system (5) yields

\[
J_t \ddot{\omega}_g = T_s^* + T_g^* - D_t \omega_g
\]

where \(J_t = J_a + n^2 J_g\), \(D_t = D_a + n^2 D_g\), \(T_s^* = nT_a\) and \(T_g^* = n^2 T_g\).

Wind turbine model defined by equation (6) has largely been used in practice in order to design control laws. This model is however rigid and is far from modern wind turbines control concerns.

4 Modelling Aeroelastic Coupling in Wind Turbines:

Aeroelastic coupling in wind turbines is very complex in reality. But, it could be separated into a major effect which acts directly on the effective wind speed and a secondary effect which modifies the wake structure. The main aeroelastic effect can be expressed by replacing, in the relations giving the aerodynamic
forces, the incident wind speed, $V$, by the relative wind speed, $V-W$, where $W$ is the velocity of the dominant structural mode or a combination of the first structural modes. Elastic modes with large axial components that are parallel to the wind direction (fore-aft motion) are in general those which have significant contributions on aero elastic coupling, (Bianchi, F.D., H. De Battista, 2007). One could verify that the first tower fore-aft flexural mode frequency is generally well below that associated to blade fore-aft flexure. So, aero elastic coupling will be more sensitive to tower motion than to out of plane blades oscillations. If one retains as an essential elastic degree of freedom the tower top displacement denoted $Y$, the dynamical equation describing the fore-aft tower motion according to its first mode writes, (Bianchi, F.D., H. De Battista, 2007)

$$M\ddot{Y} + D\dot{Y} + Ky = F_T$$  \hspace{1cm} (7)

where $M$, $D$ and $K$ are respectively the modal mass, modal damping and modal stiffness of the first fore-aft tower flexural mode. $F_T$ is the thrust force which is applied by the wind on the tower.

Considering the elastic tower top velocity defined as $W = \dot{Y}$ and the high speed shaft torsion angular speed $\omega_T$, the aerodynamic thrust and the captured aerodynamic torque are given by, (Bianchi, F.D., H. De Battista, 2007)

$$F_T = \frac{1}{2} \rho \pi R^2 (V-W)^2 C_T (\lambda, \beta)$$

$$T_a = \frac{1}{2} \rho \pi R^3 (V-W)^2 C_p (\lambda, \beta)$$  \hspace{1cm} (8)

where $C_p (\lambda, \beta)$ is the above mentioned power coefficient and $C_T (\lambda, \beta)$ is the thrust coefficient.

The coefficient $C_T (\lambda, \beta)$ is obtained, from wind tunnel testing, as coefficient $C_p (\lambda, \beta)$. It is generally tabulated directly as function of parameters $\lambda$ and $\beta$.

When aero elastic coupling and torsion strain of the high speed shaft are taken into account, the tip speed ratio defined in relation (2) becomes

$$\lambda = \frac{R (\omega_0 + \omega_s)}{\Omega (V-W)}$$  \hspace{1cm} (9)

5 Control Strategy:

It is useful to define a control strategy by indicating desired variations of wind turbine velocity and torque as well as pitch angle through a curve represented on the $(T_a, \omega_T)$ plane. Among the common used strategies one finds those presented in (Burton, T., D. Sharpe, 2001). In this work, use is made of the particular strategy depicted in figure 2 and recalled in (Camblong, H., 2003).

Region 1 denotes the starting zone that is the lower wind speed for which operating the wind turbine becomes beneficial. It servers to reach at constant rotor speed the operating point which is located on the maximum efficiency curve and had the velocity $\Omega_{\lambda, \text{max}}$. 

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Region 2 corresponds to tracking the maximum efficiency curve where the objective is to adjust the rotor speed to wind speed such that the captured aerodynamic torque is always optimal. The pitch angle is kept then constant at its optimal value $\beta_{\text{opt}}$. Region 3 is a transition zone between the below rated power region (region 2) and the nominal power rate region (region 4). This is performed by maintaining at constant value $\Omega_{\text{a,nom}}$ the rotor speed. Limiting the wind turbine speed is so performed in order to reduce mechanical loads acting on the wind turbine structure. Finally, region 4 corresponds to the maximum load zone (rated nominal power region). In this region the pitch angle is permanently adjusted according to the actual value of wind speed in order to reduce the captured aerodynamic torque allowing then for continuous nominal power generation, (Burton, T., D. Sharpe, 2001).

In the $(T_a, \Omega_{\text{a}})$ plane, the captured aerodynamic torque will depend, for a given wind speed, on the rotor speed and on the actual pitch angle $\beta$. The dashed point curve in figure 2 is associated to $\beta_{\text{opt}}$ which allows maximum power efficiency, whereas the discontinuous curve is associated to a pitch angle whose value is determined in such a way that the associated wind speed curve passes through the nominal operating point in region 4.

**6 Nonlinear Control of Wind Turbine:**

One should understand here by nonlinear control of wind turbine control that takes into account the previously presented simplified non-linear aerodynamic modeling without considering additional non-linear flexible dynamics, like gravity or centrifugal stiffening.

In the subsequent, torque control in region 2 will be analysed, (Camblong, H., 2003). Assuming that the wind speed $V$ is perfectly known, the control goal is to track the maximum efficiency curve by forcing the aerodynamic torque and the generator speed to be equal to their optimal values given as function of $V$ by
Synthesis of control needs then to specify the control variable $T$, the output variables $\omega_s$ (measured) and $T_s$ (exogenous input).

Since, in reality, it is impossible to know exactly the effective wind speed in order to calculate $T$, one is led to estimate it by an observer structure. But, when the objective is only to analyse effects on power efficiency resulting from incorporating structural flexibility and aeroelastic coupling, the aerodynamic torque could be assumed to be well determined from the effective wind speed. This will enable making more comprehensive comparisons in order to assess these special effects on the generated power. Two kinds of control laws will then be considered in the following:

- At first, an elementary control law is considered. The instantaneous value of wind speed $V$ and $T$ are assumed to be exactly known.
- Secondly, estimation of wind speed will be performed via a dynamic filter in a robust adaptive H-infinity control in order to simulate real operating conditions where wind speed is uncertain. Wind fluctuations as well as flexibility and aeroelastic coupling are considered as finite energy disturbances to be rejected by H-infinity controller.

6.1 Nonlinear Torque Control:

Using equilibrium conditions associated to wind turbine equations (5) and to the operating point defined by equations (10) yield the generator torque and torsion angle to be expressed as

$$
\bar{T}_g = -\frac{\rho \pi R^3 \nu^2}{2 \lambda_{opt}} C_{p, opt} + \frac{\lambda_{opt} V}{\nu R},
$$

$$
\bar{\theta}_s = \frac{n \rho \pi R^3 \nu^2}{2 K_c \lambda_{opt}} C_{p, opt} - \frac{n D_a \lambda_{opt} V}{R K_c}.
$$

By imposing a first order dynamic to the generator torque with a time constant $\tau_g > 0$ and omitting $\beta_{opt}$ from $C_p$ argument, relations (5), (10) and (11) yield the following nonlinear control equations

$$
T_g = \frac{1}{\tau_g} (\bar{T}_g - T_g)
$$

$$
\dot{\omega}_s = \frac{n^2 \rho \pi R^2 (\nu - w)^2}{2(\omega_s + \omega_s) J_1} C_p \left( \frac{R (\omega_s + \omega_s)}{n(\nu - w)} \right) - \frac{1}{J_1} T_g - \left( \frac{D_a}{J_a} - \frac{D_c}{J_c} \right) \dot{\omega}_g - \left( \frac{D_a + D_c + D_c}{J_a + n^2 J_c} \right) \dot{\omega}_s,
$$

$$
\dot{\theta}_s = -\frac{1}{J_1} T_g - \frac{D_a}{J_a} \dot{\omega}_g + \frac{D_c}{n^2 J_c} \omega_s + \frac{K_c}{n^2 J_c} \theta_s.
$$

$$
\dot{\omega}_s = \omega_s - \dot{\omega}_s
$$

$$
\dot{\mathbf{w}} = \frac{n \rho \pi R (\nu - w)^2}{2(\omega_s + \omega_s) M} C_T \left( \frac{R (\omega_s + \omega_s)}{n(\nu - w)} \right) - \frac{D}{M} \mathbf{w} - \frac{K}{M} \mathbf{y}.
$$
The only parameter to be tuned for torque controller design, in equations system (12), is the characteristic
time \( T_g \). This must be sufficiently small to enable tracking the mean wind speed variations.

### 6.2 Robust Adaptive H-infinity Control:

Robustness considered here results form H-infinity controller features. This control guaranties asymptotic
stability of the closed loop system as well as some attenuation for exogenous inputs effects on the augmented
system. This last incorporates the system itself and the introduced weighting filters. Inputs are either reference
points or eventually disturbances associated to system environment. H-infinity synthesis gives a robust control
against these perturbations. Moreover, if filters are designed to take into account uncertainties present in the
system, H-infinity control is also robust against these disturbances. H-infinity control considers naturally a trade
off between robustness and performance.

Let denote by \( X \) the deviation of rotor speed from its optimal value on the maximum efficiency curve. Using
the second equation of system (10) one arrives at

\[
X = \omega_{\text{opt}} - \omega = \frac{\lambda_{\text{opt}}(V - W)}{R} - \omega
\]

Differentiating equation (13) with respect to time and using equations system (12) one arrives at

\[
\dot{X} = \frac{\lambda_{\text{opt}}(\dot{V} - \omega)}{R} - \frac{n \rho \pi R^2 (V - W)^3}{2(\omega + \omega_s)} C_p \left( \frac{R}{n(V - W)} \right) \left( \frac{1}{n} \right) T_g + \frac{1}{n} \left( \frac{D_s + D_e}{J_s} \right) \frac{1}{n} \left( \frac{1}{n^2 J_s} \right) \theta_s
\]

Denoting \( u = T_e - T_g \), \( \dot{V} = V - \nabla \), \( \dot{\theta} = \theta - \rho \), \( k_x = -1 + \frac{D_s}{J_s} \), \( k_u = \frac{1}{n J_s} \), and expressing the equilibrium condition stating that \( \dot{V} = \dot{X} = \dot{X} = \omega = \omega \), equation (14) yields

\[
\dot{X} = -k_x X + k_u u + \varphi
\]

with

\[
\varphi = \frac{\lambda_{\text{opt}}}{R} (\dot{V} - \omega) - \frac{1}{n} \left( \frac{D_s}{J_s} + \frac{D_e}{J_s} \right) \left( \frac{n \lambda_{\text{opt}} \dot{V}}{R} - \omega_s \right) + \left( \frac{D_s + D_e}{n J_s} + \frac{D_e}{n J_s} \right) \left( \frac{1}{n^2 J_s} \right) \dot{\theta}_s + K_c \left( \frac{1}{n J_s} + \frac{1}{n^2 J_s} \right) \dot{\theta}_s
\]

\[
+ \frac{n \rho \pi R^2}{2 J_s} \left[ \frac{C_p \left( \frac{\dot{V} \omega_s}{n \dot{V}} \right) \left( (\dot{V} - \omega_s)^3 \right)}{(\omega_s + \omega_s + \dot{\omega}_s)} \left( \frac{R}{n(V - W)} \right) \left( \frac{1}{n(V - W)} \right) \right]
\]

It has been shown in (Vivas, V.C.F., F.C. Castaño, 2008) that a robust adaptive nonlinear H-infinity
control law can be designed for the nonlinear affine equations system (15) under the following condition:

\[
0 < h \leq k_x + \frac{1}{4} \left( \frac{1}{\rho} - \frac{1}{\gamma^2} \right); \quad \rho > 0
\]
where $h$ and $\rho$ define the control error and control input parts in the performance index, $\gamma$ is the disturbance $H$-infinity norm.

The adaptive $H$-infinity control law is written as follows

$$u = -\frac{1}{k_u} \left( \dot{\varphi} + \frac{1}{2\rho} \varphi \right), \quad \dot{\varphi} = \rho \varphi$$

(18)

With this control law it can be proved that when $\varphi - \varphi^* \in L_2$ where $\varphi^*$ is the nominal disturbance, the overall system is bounded with $\varphi \to 0$ and $\dot{\varphi} \to \varphi^*$.

If both flexibility and aeroelastic coupling terms are dropped from equation (16), it has been demonstrated in (Vivas, V.C.F., F.C. Castaño, 2008) that a dynamic filter structure can be designed to estimate wind speed. Thus, given a wind turbine system described as in (15), with an adaptive control law as in (18), with $\varphi \in R$ bounded, the dynamic filter

$$\sigma \dot{\varphi} = -k_x \dot{\varphi} + k_o \dot{\lambda} - k_{\phi} \dot{\phi} - k_{\phi} \dot{k} \kappa H(\nu, \alpha)(\dot{\nu} - \alpha)^2$$

(19)

where $\lambda = \frac{\omega R}{\dot{\nu}}, \quad k_x = \frac{1}{2\rho \pi R^3 C_{p,\text{opt}}}, \quad k_o = \frac{\lambda_{\text{opt}}}{R}, \quad \alpha = \frac{k_x k_o}{2k_x k_T}$ and the function $H$ defined such that $H(a, b) = \begin{cases} 1 & \text{if } a \geq b \\ 0 & \text{if } a < b \end{cases}$, asymptotically tracks wind speed $\dot{\nu} \to \nu$ as $\dot{\varphi} \to \varphi$.

Use is made here of the dynamic filter defined by equation (19) in order to estimate wind speed when the small disturbances resulting from flexibility and aeroelastic coupling are present in equation (16). It has been shown heuristically, through various simulations, that this dynamic filter tracks here again the real wind speed.

7 Simulation and Results:

Simulation models had been built under Matlab and Simulink in order to solve the two nonlinear control problems defined by system of equations (12) and (15). A particular case of the problem defined by equations (12) was chosen. In this problem only wind turbine flexibility resulting from the high speed shaft torsion without any aeroelastic coupling was considered. Equations system (12) reduced then to the first four equations. Numerical simulations had been performed in the case of wind turbine CART, (Stol, K.A., 2004), with the following parameters:

$$\tau_g = 5 s, \quad \beta_{\text{opt}} = -1^\circ, \quad \lambda_{\text{opt}} = 7.5, \quad \rho = 1.225 \text{ kg.m}^{-3}, \quad R = 21.38 \text{ m}, \quad n = 43.165,$$
$$J_a = 3.25 \times 10^5 \text{ kg.m}^2, \quad J_{\phi} = 34.4 \text{ kg.m}^{-2}, \quad K_c = 2.4 \times 10^7 \text{ N.m}^{-1}, \quad D_g = 37 \text{ Ns.m}^{-1},$$
$$D_a = 15 \text{ Ns.m}^{-1}, \quad D_c = 10 \text{ Ns.m}^{-1}.$$  

The chosen mean wind speed was set at $V_m = 12 \text{ m.s}^{-1}$. It corresponds to the below rated zone where control attempts to maximize energy capture. The operating point in $(T_a, \omega_a)$ plane is given by:
The instantaneous wind speed profile used in simulations is shown in figure 3 over the time interval $[0, 600]$ S. It was generated by a turbulent wind model. The turbulent component of the wind speed is described by a power spectral density where the turbulence intensity and the roughness length of the site are specified. A time series is then derived from the spectral density according to Shinozuka formula, (Shinozuka, M. and C.M. Jan, 1972), where the phase angle of each of the frequency components is determined by randomizing the initial phase angle in the interval from 0 to $2\pi$. In the application considered here the roughness length is set equal to 0.05 and the turbulence intensity is 0.18.

Figure 4 gives a comparison of the obtained results between rigid and flexible models in terms of the power efficiency ratio which is defined as

$$\eta = \frac{P_e}{P_{\text{max}}} = \frac{\omega_g T_g}{(\omega_g, \text{opt} T_g, \text{opt})},$$

that is, the fraction of actual extracted power with respect to the possible theoretical maximum. It could be noticed that in case of CART wind turbine, flexibility has only a limited effect on the generated power efficiency.

It should be mentioned here that the CART machine is small and quite rigid compared to modern wind turbines which could exhibit more flexibility effects. But, when torque control is considered in the below rated zone, the aeroelastic coupling is expected to have a limited effect on the power efficiency ratio. Hence, flexibility and aeroelastic coupling effects could be treated as disturbances in an adaptive H-infinity like control procedure.
Simulations performed in (Vivas, V.C.F., F.C. Castaño, 2008) have shown that the proposed wind estimation methodology, equation (19), tracks the mean tendency of the wind profile, discarding the high turbulent components. An excellent performance is obtained in terms of extracted power from the wind. Figure 5, shows the power efficiency ratio.

It can be observed that, except the very few initial seconds of convergence, efficiencies above 90% are obtained most of the time, with an average of 93.8%.

Other simulations have been performed in (Ayyat, B., 2008) where comparison of control performances had been conducted. Control indirect speed control (ISC), (Boukhezzar, A., 2006), was compared to the present torque control using the wind speed estimator. The effect on loads was analysed. Figure 6 gives the real wind speed and the estimated one by means of equation (19). Figure 7 gives the generator torque for ISC control. Figure 8 gives the generator torque for the actual proposed method with wind speed estimator.

ISC control uses wind speed measurements to estimate torque whereas the actual method does not use this information. Comparison of the efforts and power efficiency is summarized in table 1.

From figures 7 and 8 and from table 1, it could be noticed that the mean value of generator torque is almost the same for the two strategies even if it is a little bit higher for ISC control. The actual control method shows a higher efficiency but also higher torque variations.
8 Conclusions:

Effects resulting from wind turbine flexibility and from aero elastic coupling were considered in this work through pertinent simplified models. Aerodynamic forces acting on the wind turbine structure were assumed to be turbulent and wind speed was supposed to vary stochastically. The aero elastic coupling was taken into account by modifying, in the aerodynamic equations, the incident wind speed by the relative wind speed as it could be affected by the structural velocity associated to the most significant modes. Equations describing torque control law protocol in the below rated regime of the flexible wind turbine were derived and simulations had shown that, for the particular investigated wind turbine in this work, power rate sensitivity to structural flexibility is rather small. Structural flexibility is likely to act like small disturbances such that H-infinity technique could be advantageously used to design wind turbine controller. Since, in the presence of turbulence, the captured aerodynamic torque could not be predicted from single point wind speed measurements, a special

\[
\begin{array}{cccc}
T_{\text{isc}} \quad (10^4 \text{ N.m}) & T_{\text{Actual}} \quad (10^4 \text{ N.m}) & \text{Mean} \quad T_g \quad (10^4 \text{ N.m}) & \text{Efficiency } (\%) \\
\hline
\text{ISC} & 17.72 & 11.42 & 14.27 & 85.58 \\
\text{Actual} & 18.52 & 9.53 & 13.81 & 93.80 \\
\end{array}
\]
estimator for wind speed was constructed. Exact convergence of this estimator was proven to take place for a rigid wind turbine. Heuristic tracking of real wind speed was shown to occur for a flexible wind turbine. Associating this estimator with the flexible wind turbine model had yielded a robust nonlinear controller that takes into account directly wind perturbations and structural disturbances and having the special feature to don't need wind speed measurements. Performance of this controller was stated by comparison with the classical indirect speed control technique.

ACKNOWLEDGMENTS

The authors would like to thank financial support from the Spanish organism AECI under grants: A/7313/06 and A/9518/07.

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