Application of a Fractal Method Relating Power Spectrum and Area for Separation of Geochemical Anomalies from Background

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Abstract: Anomaly separation plays an important role in mineral exploration. The fractal based method presented in this study provides an anomaly separation method which accomplishes anomaly separation in frequency domain. It is based on the Fourier transformation, which can be compared to a glass prism. The prism separates light into various components, each depending on its wavelength (or frequency) content. Fourier transform can be viewed as a "mathematical prism" that separates a function into various components based on frequency content. The fractal filtering technique is able to separate the anomaly from background or extract the other meaningful patterns from the geochemical map using both the frequency and spatial information. A power spectrum-area plot (S-A) have been applied to separation of patterns on the basis of distinct self-similarity in frequency domain. The S-A method can decompose the geochemical patterns into anomalies and background. This filter can be constructed on the basis of the separate power spectrum levels. Conversion of signals from the frequency domain back to the space domain using 2-D Inverse Fourier Transformation with these filters applied, provides the decomposed patterns reflecting anomalous and background values. This paper briefly introduces the theory of the fractal filtering technique. A case study of 107 regional geochemical data of stream sediment from the western Iran, Kurdistan, Shurab Haji will be used to illustrate application of this technique.

Key words: Geochemical anomaly; Fractal Filter; Fourier transform; power spectrum-area method.

INTRODUCTION

A regional geochemical map normally is constructed from a set of geochemical data sampled over an area. For the past years, the regional geochemical mapping has drawn a great attention of geochemists researchers because the map usually contains a large amount of information essential to mineral exploration. With respect to the prediction of mineral deposits, the geochemical map usually is needed for distinguishing anomalies and characterizing background.

Separation anomaly from background is a fundamental issue in mineral exploration. The methods and techniques for separation of anomalies from background have been the subject of research for many years (Grunsky, 1997; Grunsky and Smee, 1999; Harris et al., 1999; Cheng et al., 1994).

For the past two decades, the traditional statistical methods assumed that the concentration of chemical elements follows a normal or log-normal distribution. A geochemical anomaly is defined as a region where the concentration of the element of interest is greater than a certain threshold value.

Various quantitative methods have been developed in this field, and among them, statistical methods have played an important role. Conventional statistical methods used for geochemical anomaly separation such as moving-averages, Kriging, probability graphs, univariate and multivariate analysis methods (Sinclair, 1974, 1976, 1991; Govett et al., 1975; Miesch, 1981; Stanley, 1988; Garrett, 1989; Stanley and Sinclair, 1989; Cheng et al., 1996) are primarily concerned with the frequency distributions of element concentration values and correlation coefficients among multiple variables. Spatial statistical methods such as moving average techniques, Kriging and spatial factor analysis (Grunsky, 1997; Agierberg, 1995) can take into account spatial correlation and variability within neighboring samples in addition to concentration value frequency distributions and correlation coefficients. Spatial methods usually provide better results in geochemical pattern recognition. In addition to frequency distributions and spatial correlation and variability of concentration values, geometrical
properties and scale-independent characteristics of anomalies can be considered in fractal, multifractal methods (Cheng et al., 1994, 1996, 1999, 2000). Ideally, the geochemical patterns have to be analyzed and classified according to the following four aspects: value frequency distribution; spatial correlation and variability; geometrical properties (shape and orientation) of anomalies; and scale independence of patterns.

During most of the history of science, fractals and chaos were systematically ignored. This is no longer possible, especially in geosciences, as clearly demonstrated by numerous applications. In recent years, applications of fractals and multi-fractals have been increasing in the earth sciences.

Fractals is a natural consequence for geological phenomena that display self-similarity resulting from scale-independence properties, such as the frequency-size distributions of sediments and porosity, faults, earthquakes, volcanic eruptions, mineral deposits and oil fields. Fractals are more or less important in nearly all geoscientic disciplines. Scientists interested in fractal applications come from widely different backgrounds both inside and outside the earth sciences.

The fractal filter is defined in Fourier space by applying the fractal concentration-area model (Cheng, Agteberger and Ballantyne, 1994) to the power spectrum of the processed geochemical field. They are irregularly shaped filters due to the anisotropy and complex inner structure of the geochemical field. Each filter represents the regions where the power-spectrum shows the same or similar scaling properties. The inverse Fourier transform applied to the filtered signals will yield distinct geochemical patterns in the spatial domain with the corresponding scaling properties in the frequency domain. The patterns obtained using the fractal filters may or may not relate to the mineralization that we are interested in. In some complicate geological environments, the anomalies divided by the scaling properties may correspond to multiple processes, some of which may not be related to specific mineralization processes. Therefore, the interpretation or identification of the “meaningful anomalies” or “real anomalies” related to the mineralization is needed.

This paper will introduce the recently developed concentration-area (C-A) and power spectrum-area (S-A) fractal methods for geochemical anomaly separation. Both the C-A and S-A are developed for pattern recognition on the basis of self-similarity and self-affinity. The former is applied in the space domain whereas the later in the frequency domain. The implementations of these two methods involve plotting the values of contours and the areas enclosed by the contours in log-log paper. Straight-line segments are fitted to the values to construct power-law relations to represent the different self-similarities. The cutoff values obtained by intersecting these straight-line segments can be applied to separate the contours into groups each has the similar shapes. In the application of the C-A method these contours separated in the space domain often correspond to the patterns reflecting distinct underlying processes. In the application of the S-A method, filters can be constructed on the basis of these contours separated in the Fourier frequency domain. The patterns converted back to the space domain from the Fourier Transformed signals in the frequency domain with the filters applied will provide the decomposed patterns reflecting distinct underlying processes. Both the C-A and S-A will be used in the current paper for geochemical anomaly separation. In this study we will first introduce the theory of the C-A and S-A methods, and then gives a case study of application to the geochemical data from stream sediments in the western Iran, Kurdistan, Shurab Haji.

For this study, 107 stream sediments samples were analyzed for As, Sb, Bi, Hg, Ti, Ag, B, Ba, Co, Cr, Cu, Mn, Ni, Pb, Sn, W, Mo, Au, Zn, Be. The map created from the Cu content values will be employed in this paper to illustrate the application of the C-A and S-A methods.

MATERIAL AND METHODS

2-1-Study Area:
Shorab Haji 1:50,000 sheet is located at Ghorvah 1:100,000 sheet. This sheet is located at northwestern of Ghorvah. It is located in Kurdistan province in the northwestern of Iran. It is bounded on the north by latitude 35° 30’, on the south by latitude 35° 15’, on the east by longitude 47° 45’, and on the west by longitude 47° 30’.The area is roughly rectangular in shape with a width of approximately 22 Km and length of 27 Km(Fig.1).

The most populous city in this region is Ghorvah, inhabited by a population of 90,000. The study area is surrounded by Hamadan-Sanandaj road. Totally, with regard to accessible roads, the region has a good situation. The study area covers about 600 km².

Dominant winds in the area are north-westerly. The climate of the study area is characterized by drought periods usually extending from May to September. Rainfall is varied in this area. It is dry and hot in summers, cold in winters and there are big differences in temperature between night and days, summer and winter. It is the hottest in July (36°C) and the lowest temperature measured was -20°C.
The geology of the area mostly consists of sedimentary complex with acid and basic from Pleistocene-Holocene age. Lithology consist mostly of sedimentary and volcanic rocks, spanning from Mio-Plio to the recent age. The geological setting is characterized by Quaternary and Neogene Formations that underwent several complex phases of deformation. The sedimentary rock include; stratigraphic units, made up of limestone, dolostone, and terrigenous sediments (clay, siltstone, sandstone, conglomerate). The Mio-Plio units, made up mostly of trachyandesite, latite, dacite, altered of trachyandesite, latite and dacite (quartz, feldspar, cristobalite, kaolinite); Quaternary sediments, which occur mostly in plains are made up of low level terraces, alluvial sediments.

The region consists of plains and the topography is very low. According to scientific studies, ghorvah, in which the study area Shorab Haji is situated, is located in Sanandaj-Sirjan zone. At north there are volcanic activities, and consequently intermediate to basic volcanic rocks are present. Composition of rocks is mostly latite to andesite. Their texture is Porphyritic-glomeruporphyritic, and consists of phenocryst of Plagioclase, Biotite. Some of these rocks have been altered. These rocks (MP) consist of feldspar, cristobalite, and Kaolinite. These rocks are cream-coloured. Epidotization is present and because of this event, Amphibole changed to Epidote. Italian geologists evaluated the age of these volcanic activities 3.7 to 7.8 Million years ago.

Regarding the economic geology, study area is comprised of dimension stones. Dimension stone can be defined as a natural rock that fulfils certain qualitative criteria and is therefore extracted and processed to definite dimensions for use in the building, construction, and monument industries. Dimension stone deposits are found in two types of rock in the study area:

a) Marble: In some part of the study area, there are marbles with the color of white and light grey. Their tonnage is high, but their quality is low. Although there are some places that, the quality of the stones is good.

b) Intrusive rocks: There are valuable reserves of these rocks that had not been noted before, however more work has been done recently in this region. In Ghorvah 1:100,000 sheet reports indicate that, there are other important and valuable materials including pozzolan, kaolin, iron and titanium.

2-2-Sampling and Analysis:

Media such as stream sediments and lake sediments are sampled and analyzed by geochemists to detect geochemical patterns reflecting the underlying geological structures. Different schemes of geochemical sampling are required in different stages of mineral exploration; for example, in the early stage of a geological survey, geochemical reconnaissance may be the dominant activity involving sampling secondary median such as stream or lake sediments, with each sample covering a large area of influence. A small number of samples can be representative of a relatively large area.

Stream sediment sampling was carried out in February 2001. A total of 107 samples were collected from stream sediment in Shorab Haji 1:50,000 Sheet. The sampling density was about 1 per 6Km². Figure 3 shows the location of the sampling points. The coordinate of the sample points were detected with GPS, and plotted...
Fig. 2: Simplified geological map of Shorab Haji 1:50000 sheet.

on topographic map with the scale of 1:50,000. Systematically collected samples were stored in 4 Kg plastic bags. Samples were dried with iron at 105°C for 24h. Dried samples were passed through a 2mm plastic sieve and separated from pebbles. Finally dried samples were passed through an 80 mesh sieve and 100gr was chosen for chemical analysis. Detection of the elements Ti, Ag, B, Ba, Co, Cr, Cu, Mn, Ni, Pb and Sn were performed with X-ray fluorescence spectrometry and by Atomic Absorption Spectrometry (AAS) for As, Sb, Bi and Hg, and by Polarography for W, Mo. The analytical methods and detection limits of elements tested in the present paper are shown in table 1.

2-3- Method:

Cheng et al. (1994) suggested that the geochemical background and anomalies, respectively, correspond to different power-law distributions, thus creating a multifractal appearance. Based on this, they proposed an element concentration–area (C–A) model. The Power Spectrum-Area (S-A) method is derived from Concentration-Area method by replacing power spectrum with concentration, which in turn follows the power-law of fractals. Thus, first we discuss the Fractal Concentration-area (C-A) method.
Fig. 3: Location of the sampling points in stream sediments.

2-3-1- Fractal Concentration-area Model (C-A):

Cheng (1994) developed a fractal method to separate geochemical anomalies from the background. It has been applied to analysis various types of geochemical data such as lake sediment, rock samples and humus (Sim, Agterberg and Beaudry, 1999). In this paper we used this method for stream sediment samples. It involves a concentration-area plot on log-log paper showing power-law relations between the area $A(\alpha)$ with an element concentration value higher than $\alpha$ and the concentration value $\alpha$;

$$A(\alpha) = C\alpha^\alpha$$

Where $C$ is a constant and $\alpha$ is the exponent that may have several values for different ranges of geochemical concentration values $\rho$. $A(\rho)$ denotes the area with concentration values greater than the contour value $\rho$. $A(\rho)$; values are obtained by box-counting of original element concentration values. By box-counting, one superimposes a grid with cells on a study region. The area $A(\rho)$ for a given $\rho$ is equal to the number of cells (multiplied by cell area) with concentration values greater than $\rho$. Stream sediments data from 107 samples in the Shorab Haji region (about 600 km$^2$ in area), Kurdistan, Iran, were analyzed using the above model. Figure 4, 5, 6 and 7 shows Zn, Ni, Cu and Au stream sediments (107 samples) geochemical image maps, respectively. The C–A model of these elements also shown in Fig 8, 9, 10 and 11.

On the log-log paper, the values of $A(\rho)$ against the $\rho$ may be fitted by a number of straight lines. The break(s) of the straight lines and the corresponding values $\rho$ can be picked up and used as the cutoff(s) to separate geochemical values into different components such as anomalies and background.
Table 1: List of elements, analytical methods, and detection limits tested in the present paper.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Analytical methods</th>
<th>Detection (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti</td>
<td>XRF</td>
<td>500</td>
</tr>
<tr>
<td>Ag</td>
<td>XRF</td>
<td>0.05</td>
</tr>
<tr>
<td>B</td>
<td>XRF</td>
<td>10</td>
</tr>
<tr>
<td>Co</td>
<td>XRF</td>
<td>5</td>
</tr>
<tr>
<td>Cu</td>
<td>XRF</td>
<td>5</td>
</tr>
<tr>
<td>Mn</td>
<td>XRF</td>
<td>100</td>
</tr>
<tr>
<td>Ba</td>
<td>XRF</td>
<td>50</td>
</tr>
<tr>
<td>Pb</td>
<td>XRF</td>
<td>2</td>
</tr>
<tr>
<td>Sn</td>
<td>XRF</td>
<td>2</td>
</tr>
<tr>
<td>Cr</td>
<td>XRF</td>
<td>20</td>
</tr>
<tr>
<td>Ni</td>
<td>XRF</td>
<td>5</td>
</tr>
<tr>
<td>As</td>
<td>AAS</td>
<td>1</td>
</tr>
<tr>
<td>Bi</td>
<td>AAS</td>
<td>0.1</td>
</tr>
<tr>
<td>Sb</td>
<td>AAS</td>
<td>0.2</td>
</tr>
<tr>
<td>Hg</td>
<td>AAS</td>
<td>0.05</td>
</tr>
<tr>
<td>Au</td>
<td>AAS</td>
<td>0.0003</td>
</tr>
<tr>
<td>W</td>
<td>P</td>
<td>1</td>
</tr>
<tr>
<td>Mo</td>
<td>P</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 4: Stream sediments map showing Zn distribution. Fig. 5: Stream sediments map showing Ni distribution.

**AAS:** Atomic Absorption; **XRF:** X-ray Fluorescence Spectrometry; **P:** Polarography

The method has been used to decompose anomalies and background of geochemical data from soil samples (Cheng, Xu and Grunsky, 1999, 2000) and till samples (Cheng, Li and Xu, 2000). In the current paper, this method was applied to process stream sediment geochemical maps from the Shorab Haji, Kurdistan, Iran, for mineral exploration. Figure 12, 13, 14 and 15 shows Cu, Au, Ni and Zn anomalies obtained using the C-A method.

**2-3-1- Fractal Power Spectrum-area Model (S-A):**

The S-A method, based on a Fourier spectral analysis, is a fractal filtering technique used to separate the anomalies of an element from its original values. It also uses both frequency and spatial information for geochemical map and image processing. The basic geological assumption for the S–A method is that a geochemical field or image generated by specific geological processes may be discriminated in terms of its fractal properties. The scale invariant property of most geological processes (e.g. erosion processes, mineralizing events, magnetic field of the earth’s crust, distribution of earthquakes and volcanic eruption) often show “self-similarity” or “self-affinity”. These properties can be measured in both the frequency and the spatial domain (Turcotte, 1997). In the frequency domain, such properties can be represented by means of power spectra.
Fig. 6: Stream sediments map showing Cu distribution.

Fig. 7: Stream sediments map showing Au distribution.

Fig. 8: Fractal Concentration-Area plot for Zn data.

Fig. 9: Fractal Concentration-Area plot for Ni data.
The fractal filter to be used is defined on the basis of the power-law properties of a power spectrum in the frequency domain (Fig. 12). The purpose of this is to divide the power spectrum into components characterized by similar scaling properties. It is an irregularly shaped filter, due to the anisotropic and usually complex intrinsic structure of the geochemical data.

Fig. 12: Zn anomalies obtained using the C-A method.

Fig. 13: Ni anomalies obtained using the C-A method.
The filter can be used to identify anomalies from the background, and to extract other meaningful patterns from the original map (see Xu and Cheng, 2001 for more details).

The S–A method involves the use of the following relationship:

$$A(S) \propto CS^a$$

$A(S)$ denotes the area with power spectrum value greater than the contour value; $C$ is a constant and $a$ is the exponent that may have several values for different ranges of power spectrum values $S$. $A(S)$ value is obtained by box counting of original element concentration values. By box counting, one superimposes a grid with cells on a study region. The area $A(S)$ is equal to the number of cells (multiplied by cell area) with power spectrum values greater than $S$. 

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Fig. 14: Cu anomalies obtained using the C-A method.

Fig. 15: Au anomalies obtained using the C-A method.

Fig. 16: The powers spectrum distribution of Zn.

Fig. 17: The powers spectrum distribution of Ni.
Fig. 18: The powers spectrum distribution of Cu.

Fig. 19: The powers spectrum distribution of Au.

Fig. 20: Fractal Spectrum-Area plot for Zn data.

Fig. 21: Fractal Spectrum-Area plot for Ni data.

Fig. 22: Fractal Spectrum-Area plot for Cu data.

Fig. 23: Fractal Spectrum-Area plot for Au data.
Stream sediment data from 107 samples in the Shorab Haji region (about 600 km² in area), Kurdistan, Iran, were analyzed using the above model. The S–A model of Zn, Ni, Cu and Pb shown in Fig.20,21,22 and 23, respectively. In order to apply the S-A method to the values of elements content in stream sediments samples, the original map has to be converted into the frequency domain by means of 2-D Fourier Transformation. The powers spectrum distribution of Zn, Ni, Cu and Pb shown in Fig.16,17,18 and 19, respectively.

A log–log plot shows the relationship between the area and the power spectrum values on the Fourier transformed map of the power spectrum. The values on the log–log plot were modeled by fitting straight lines using least-squares. Distinct classes can be generated, such as lower, intermediate, and high power spectrum values, approximately corresponding to original values, anomalies and noise of geochemical values in the spatial domain. An irregular filter was applied on these distinct patterns to remove the anomalies and noise related to intermediate and high power-spectrum values. The image, converted back to a spatial domain with the filter applied, shows patterns that, indicate an area that represents anomaly geochemical patterns (Fig. 24,25,26,27).

RESULTS AND DISCUSSIONS

Results of separation of geochemical anomalies by S-A for elements in stream sediment samples from Shorab haji sheet (about 600 km²), Kurdistan Province, Iran, are presented in the following:

**As:**
The As anomaly appears at northeastern (samples 540 and 541) and southeastern (Samples 712, 713 and 714) of the region. These areas are located at travertine and carbonate rocks. Because travertine and carbonate rocks related to hydrothermal system and post magmatic process, the presence of As at these locations are natural. Another As anomaly is located at northwestern (Samples 677, 678, 679, 680, 681, 682 and 683) of the region, where the host rocks are trachyandesite, latite, dacite, tuff, breccia and other volcanic and metavolcanic rocks related to volcanic activities of Sheyda volcano.

**Au:**
Au was an effective and critical element for separation anomaly from background in the study area. The anomaly map of Au gives some area as important, but the most important area is located at northwestern of the region and corresponds to Sheyda volcano. Another less important area is located at southeastern of the region (Samples 540 and 541). This anomaly corresponds to As anomaly and the host rocks are travertine and carbonates.

**Zn:**
The Zn anomaly is located at northwestern (Samples 677, 678, 679, 680, 681, 682 and 683) of the region, where the volcanic and metavolcanic rocks related to volcanic activities of Sheyda volcano are present.

**W:**
The W anomalies are located at northeastern (samples 540 and 541) and northwestern (Samples 677, 678, 679, 680, 681, 682 and 683) of the region. This element is trace in the area and has no relationship with mineralization, itself.

**Cu:**
Cu has dispersion anomaly (fig.27), but the most important anomaly was corresponded at As and Co anomaly located at tuff and breccia. These rocks are the production of metamorphic and alteration process. Probably the concentrations of these elements are controlled by metamorphic factors.

**Mo:**
The Mo Anomaly appears at northwestern (Samples 677, 678, 679, 680, 681, 682 and 683) of the region, where the host rocks are volcanic and metavolcanic rocks that related to volcanic activities of Sheyda volcano.

**Co:**
The Co anomaly located at east (sample 540 and 541) and southeastern (sample 712,713 and 714), and northwestern (sample 700) of the study area. They are related to hydrothermal system and post magmatic process, and also volcanic activities of Sheyda volcano.
Fig. 24: Zn anomalies obtained using the S-A method.  
Fig. 25: Ni anomalies obtained using the S-A method.  
Fig. 26: Cu anomalies obtained using the S-A method.  
Fig 27: Au anomalies obtained using the S-A method.

Sn:  
Careful investigations on the region show that the Sn anomalies have no relationship with mineralization. Geological studies confirm this.

Mn and Ni:  
The Mn and Ni anomalies are located at southeastern of the study area. These anomalies are less important and these no relationship between mineralization and the anomalies. The host rocks are alluvial and travertine.

4-Conclusions:  
Fractal filtering technique by decomposing an image or map is able to separate meaningful and mineralized area from other areas. This technique has been recently developed and the principle of this method is based
on the frequency information and processing in frequency domain. This technique is able to neutralize regional effects. Fractals filter is defined at forier space in which power spectrum is separated based on the self-similarity.

Probabilistic analyses are useful techniques for processing geochemical data, but when the geochemical background is high, these methods probably fails to delineate meaningful and anomaly area. In comparison with the filters constructed in the concentration-area (C-A) method, the filters generated by power spectrum area (S-A) method can suggest anomaly regions with less area. As a result, the costs of project for more explorations at this case will be less.

Integrating the resultant maps with the various geological features often provides an insight into the multi-scale and multi-degree mineralization controlling factors. The case study has shown that the areas in northwestern region near the Shyda volcano may reflect the significant ore-controlling factors which should be given more attentions in the further exploration for the metamorphic-hosted Au, Zn, Mo deposits in the study area.

REFERENCES


