Analytical Analysis of Lock-on Range of Infrared Heat Seeker Missile

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Abstract: The lock-on range, R, of infrared (IR) heat seeker missile has been studied in details. An analytical expression form of R has been derived in term of target (assumed as aircraft hot metal tailpipe), atmosphere and infrared (IR) detector parameters. The R is represented in term of “Lambert W function” and as a function of target temperature, T, and atmosphere extinction coefficient, α, and also the noise equivalent irradiance (NEI) of the detector. Assuming clear sky background, the simulation results show that a high value of R can be achieved when the temperature of aircraft engine is high especially in takeoff case where the engine thrust is large. The R can be decreased by (1) increasing the attenuation coefficient of the propagation medium, i.e. by cooling the aircraft hot engine or any other obscurants that absorb or scatter the IR radiation (2) or by decreasing emissivity, e, of the target surface which depends on the applied coatings / paintings. However, the R of can be increased by reducing the NEI of the IR detector.

Key words: Heat seeker missile, infrared, lock-on range.

INTRODUCTION

Infrared (IR) homing is a passive system in which heat generated by the target is detected and homed on. Typically, it is used in the anti-aircraft role to track the heat of jet engines. It has also been used in the anti-vehicle role with some success. This means of guidance is also referred to as “heat seeking”. The IR heat seeker missiles have exploited techniques to acquire and intercept airborne targets, by passively detecting their IR energy. Developments in IR detection and tracking have led to the increasing effectiveness of IR guided missiles (fire and forget), which are now portable such as MANPADS (Man Portable Air Defense Systems) and easily available. IR heat seeker missiles have been responsible for the majority of aircraft losses since their introduction into service during the 1960s. Some statistics suggest that heat-seeking missiles have been responsible for more than 80% of all combat aircraft losses over the last 40 years (Titterton, D.H., 2006), see Fig. 1 for example. There are three main types (generations) of IR heat seeker missiles according to the type of reticle (optical modulator) (Kopp, C., 1982) and the detector (in missile head): spin-scan, conical-scan, and imaging. The latter is the most advance one because It cannot be easily fooled by counter-measure techniques.

The IR heat seekers detect the difference of IR energy between the target and background, i.e. contrast. Thus, IR stealth technology aims to, ideally, make this difference zero by reducing the target IR signature level (IRSL). Heat seeker missile uses atmosphere windows for detecting and tracking. Those windows are specified bands of wavelength in electromagnetic spectrum that characterized by low IR attenuation. In addition, those windows are interesting to the seeker designers because they include the IR radiation of hot turbojet aircrafts. In an effort to redress the balance between the aircraft and the missile, researches into systems that could be utilized to defeat seekers have been widespread. The development work has followed two main paths: defensive systems such as flares; and active systems such as jammers. Those countermeasures are designed to decreases the lock-on range, R, by increasing the attenuation of IR target. In this paper, we investigates the lock-on range, R, of IR heat seeker missile in details. A model is presented for lock-on range in term of target, which is considered as point source, atmosphere and detector parameters. We solve the R equation in term of Lambert
Fig. 1: Damage caused by a missile. The lucky part was that the missile hit the wing.

W function. The simulation results show that lock-on range, R, can be decreased by attenuating IR energy of the target or decreasing the target temperature. In addition, the lock on range can be decreased also by decreasing the emissivity of target surface. This can be achieved using special applied coatings/paintings (Mahulikar, S.P., et al., 2008).

**IR Radiation and Signature:**

The spectral radiant emittance (SRE) of blackbody* can be written as function of wavelength, \( \lambda \), and temperature, \( T \) (in Kelvin (K)), according to Planck's law as (Hudson, R.D., 1969)

\[
S_{\lambda} = \frac{c_1}{\lambda^5} \frac{1}{e^{\frac{c_2}{\lambda T} - 1}}
\]

where

- \( S_{\lambda} \) is the spectral radiant emittance (Wcm\(^{-2}\)\(\mu m\)\(^{-1}\))
- \( c_1 \) is first radiation constant, \( c_1 = 3.7415 \times 10^8 \) Wcm\(^{-2}\)\(\mu m\)\(^{-4}\)
- \( c_2 \) is the first radiation constant, \( c_2 = 1.43879 \times 10^4 \mu m K \).

The SRE of blackbody for different values of temperatures is shown in Fig. 2. More general representation of SRE of black body is shown in Fig. 3. It can be seen that SRE increases as temperature increases and the maximum SRE peaks lie over the range of short wavelengths. Principally, the range of temperatures (500K-900K) is very important to heat seeker designer because it includes the IR energy hot metal tailpipes of turbojet aircraft (Hudson, R.D., 1969). The peaks of SRE corresponding to above temperature range lie between wavelengths 3\(\mu m\) -5\(\mu m\) that most of heat seekers are designed to track and lock-on the target. Integrating the above equation with respect to the wavelength from zero to infinity gives us an expression for the radiant emittance, i.e. the flux radiated into hemisphere above blackbody 1 cm\(^2\) in area, which can be written as

\[
S = \sigma T^4
\]

where

- \( S \) is the radiant emittance (Wcm\(^{-2}\))
- \( \sigma \) is Stephan-Boltzmann constant, \( \sigma = 5.667 \times 10^{-8} \) W cm\(^{-2}\) K\(^{-4}\)
Fig. 2: Spectral radiant emittance of black body for different values of temperatures.

Fig. 3: (a) Spectral radiant emittance of black body vs. temperature and wavelength (b) the corresponding contour plot to show different optimum value curves of spectral radiant emittance.

The above law is known as Stephan-Boltzman law (Hudson, R.D., 1969). Stephan-Boltzman law provides a standard comparison; it describes an ideal radiator, i.e. blackbody which can be used to compare the radiation of any other sources. A factor can, be added, so it can be applied to sources that are non blackbodies. This factor is called emissivity, $\varepsilon$, (Hudson, R.D., 1969)

$$\varepsilon = \frac{S_{\text{source}}}{S_{\text{blackbody}}}$$

(3)

Therefore, SRE of any source can be written as

$$S_{\text{source}} = \varepsilon_{\text{source}} S_{\text{blackbody}}$$

(4)

where $\varepsilon_{\text{source}}$ is the emissivity of the source at specified temperature and wavelength. Fig. 4 shows the radiant emittance as a function of temperature, T, and for different values of emissivity, $\varepsilon$, where the curve of $\varepsilon = 1$ has the highest radiant emittance (black body). It can be shown that radiant emittance decreases as $\varepsilon$ decreases.
Fig. 4: Radiant emittance for different values of emissivities.

General representation of Stephan-Boltzman law is shown in Fig. 5 where it can be concluded that the target having high $\epsilon$ emits high radiant emittance which can be locked on by IR heat seeker.

Fig. 5: (a) General representation of radiant emittance and (b) the corresponding contour plot which show the curves of different optimum values of radiant emittance.

Researches in the process of developing an ‘electro chromic polymer’. These thin sheets cover the aircraft’s white skin and sense the hue, color and brightness of the surrounding sky and ground.

Atmosphere Attenuation:
IR energy is generally lost to the atmosphere mainly by absorption and scattering. Absorption is the energy loss due vibration and rotation of molecules (Barbara Stuart, 2004). Scattering is the energy loss due to redirection away from the detector. In general, the transmittance of atmosphere can be represented as (Hudson, R.D., 1969)

$$\tau = e^{-\alpha R}$$

where $R$ is the range between the IR source and the detector and $\alpha$ is extinction (or attenuation) coefficient which can be written as (Mahulikar, S.P., et al., 2008)
\[ \alpha = a + \gamma \]  

(6)

where \( a \) and \( \gamma \) are the absorption (due to water vapour) and scattering (due to fogs and clouds) coefficients respectively. The atmospheric windows are characterized by high \( \tau_a \), i.e. low \( \alpha \) (attenuation). The presence of high humidity, clouds, fog or smoke can dramatically influence IR absorption.

**IR Signature Level (IRSL) of the Target (Aircraft):**

For military systems, generally the targets are aircraft, missiles, ships, ground vehicles, tanks etc. The sources of radiation in military targets are mainly the hot engine parts, the exhaust plume and the high emissivity metal skin. The engine and exhaust plume produce a large amount of heat and makes the exhaust section as a main source of thermal radiation. The typical sources of radiation in jet aircraft, as shown in Fig. 6, are the hot-metal tailpipe, the exhaust gas plume, metallic skin and the aerodynamic heating which increases with the speed of aircraft.

![Fig. 6: Sources of IR radiation in an aircraft.](image)

The hot metal tailpipe and the stream of hot exhaust gases known as the plume. Exhaust gas temperature (EGT) is one of the most important criteria of engine performance. The temperature of the plume at the metal tailpipe is given as

\[ T_2 = 0.85 T_{\text{EGT}} \]  

(7)

where \( T_{\text{EGT}} \) is the EGT.

The tailpipe behaves typically as a graybody with total emissivity of about 0.9, with temperature equals to the EGT and an area equals to that of exhaust nozzle. The higher EGT of the aircraft is in takeoff case where the engine thrust is maximum. This high radiation of tailpipe can be easily detected by even less sensitive heat seeker missile. In this paper, we will assume that the thermal radiation is emanating from a single point, i.e. aircraft plume that radiates into hemisphere. According to that, the radiance of the target can be written as

\[ N_i = \frac{S_i}{\pi} = \frac{e_i \sigma \epsilon_i T_i^4}{\pi} \]  

(8)

where

- \( N_i \) is the radiance of the target (W cm\(^{-2}\) sr\(^{-1}\)), \( \pi \) is the sold angle.
- \( S_i \) is the radiance emittance of the target (Wcm\(^{-2}\))
- \( e_i \) is the emissivity of the target
- \( T_i \) is the temperature of the target which we will considered in °C. Note that the relationship between the radiance, \( N \), and the radiant emittance, \( S \), is \( N = S/\pi \) (Mahulikar, S.P., *et al.*, 2008).

In addition the radiant emittance of target background should be taken into account where the heat seeker detection depends on the difference of radiant emittance, i.e. contrast, between the attenuated IR radiant level of the target and its background. The radiance of target background can be modeled as a graybody with specified value of emissivity.
\[ N_b = \frac{S_b}{\pi} = \frac{\varepsilon_b \sigma T_b^4}{\pi} \]  
(9)

where

- \( N_b \) is the radiance of background (W cm\(^{-2}\) sr\(^{-1}\)).
- \( S_b \) is the radiance emittance of background (W cm\(^{-2}\)).
- \( \varepsilon_b \) is the emissivity of the background.
- \( T_b \) is the temperature of background (°C).

**Heat Seeker Missile:**

The tactical missile system used against aircraft includes three main sections: the guidance and control, the warhead and the propulsion sections as shown in Fig. 7. The guidance and control system is located in the front part of the missile and consists of the seeker, the guidance control unit and the rudders. Seeker head receives the IR radiation emitted from a heated source, typically the engine of aircraft, and converts this energy into an electric signal. The signal is processed in the guidance control unit that calculates control signals used for directing the missile via rudders and tailfins.

![Fig. 7: IR heat seeker missile.](image)

The block diagram of seeker head is shown in Fig. 8. It is comprised of the following major components: (1) IR dome for protection from the aerodynamic forces and weather, (2) optical system (mirrors) to focus the IR target energy onto detector, (3) reticule (or optical modulator) to provide directional information for track, (4) detector, to convert the IR energy to electrical signal Also optical filter may be put in front of the detector to pass only specified narrow wavelength band for locking-on and reject background noise.

![Fig. 8: Block diagram of seeker head.](image)
Lock-on Range:

The lock on range of heat seeker missile can be written as (Mahulikar, S.P., et al., 2008)

\[ R = \left( \left( N_t - N_b \right) \right) \mathcal{A}_t \tau_o \left( \frac{\pi}{2} D_o (NA) \tau_o \left( D^* \right) \left( \frac{1}{(\alpha \Delta f)^{1/2} \text{SNR}} \right) \right)^{1/2} \]  

(10)

where the definition of above parameters with their values used in calculation are:
- \( A_t \) is the target area, (3660 cm\(^2\) (aircraft nozzle))
- \( D_o \) is the diameter of optics, (3.8 cm)
- \( NA \) is the numerical aperture of the optics (0.25)
- \( \tau_o \) is the transmittance of the optics (0.81)
- \( D^* \) is the detectivity of detector, (5×10\(^{10}\) cm Hz\(^{1/2}\) W\(^{-1}\))
- \( \omega \) instance field of view, (1.9×10\(^{-5}\) sr)
- \( \Delta f \) is the electrical bandwidth, (200 Hz)
- \( \text{SNR} \) is the signal-to-noise ratio, (3)

Recalling that the transmittance of atmosphere, \( \tau_o \), is function of the range, \( R \), therefore it appears difficult to solve the above equation for \( R \). Previously the authors have used either numerical method such as Newton Raphson to solve this nonlinear equation. In this paper we present exact and full analytical solution.

Substituting equation (5), i.e. \( \tau_o = e^{-\alpha R} \), in equation (10) and re-arrange the terms, on obtains

\[ R(T) e^{-\frac{\alpha R(T)}{2}} = H^{1/2} \]  

(11a)

where

\[ H = \left( \left( N_t(T) - N_b(T) \right) \mathcal{A}_t \right) \left( \frac{\pi}{2} D_o (NA) \tau_o \left( D^* \right) \left( \frac{1}{(\alpha \Delta f)^{1/2} \text{SNR}} \right) \right) \]  

(11b)

where we consider the lock-on range, \( R \), is a function of temperature, \( T \), corresponding to the temperature of the contrast, \( N_t(T) - N_b(T) \), seen by the heat seeker.

The solution of equation (11a) can be written as

\[ R(T, \alpha) = \frac{2}{\alpha} W\left( \sqrt{H \alpha} \right) \]  

(12)

where \( W(.) \) is the Lambert’s W function.

The lock-on range, \( R \), is calculated as a function of temperature of target and for different values of extinction coefficient of the atmosphere as shown in Fig. 9. The target is assumed as graybody with \( \varepsilon = 0.9 \), i.e. aircraft tailpipe.

In addition the background is assumed as a graybody with \( T = 10^\circ C \) and \( \varepsilon = 0.98 \). According to equation (4) the value of background radiance, \( N_b \), is 0.011 W cm\(^{-2}\) sr\(^{-1}\). As shown in this figure, lock-on range, \( R \), increases as the temperature, \( T \), of target increases, therefore, hot parts of target for instance, the nozzle and tailpipe of jet aircraft, can be detected by the seeker from long distance. In addition, the \( R \) decreases as atmosphere extinction coefficient increases.

In addition, as shown in Fig. 10, the highest lock-on range is obtained at aircraft takeoff case because the temperature of aircraft tailpipe is high where exhaust gases temperature (EGT) reaches 635\(^\circ C \). While it is 515\(^\circ C \) and 485\(^\circ C \) in continuous and cruise cases respectively. According to above values of temperature, the lock-on range will be 37.167Km, 32.766Km and 31.6 Km respectively where the maximum lock-on range is obtained in aircraft takeoff case. From other hand, lock-on range decreases as extinction coefficient increases.
Fig. 9: Lock-on range of heat seeker against the temperature of the target (tailpipe of jet aircraft) for different values of atmosphere extinction coefficient.

Fig. 10: Lock-on range of heat seeker missile against extinction coefficient for different aircraft flying situations, i.e. different tailpipe temperature.

General calculation of lock-on range versus temperature and extinction coefficient is shown in Fig. 11. The decreasing of IR signature level of aircraft will decrease the lock-on range of heat seeker missile and provide longer time to the aircraft to manoeuvre, i.e. increase the probability of survivability.

Lock-on Range in Term of NEI:
In this section, we express the lock-on range, $R$, shown in equations (10) and (12) in term of noise equivalent power (NEP) and then noise equivalent irradiance (NEI) of the detector.

The NEP (W) is the radiant flux necessary to give an output signal equal to the detector noise while the NEI (W cm$^{-2}$) is the radiant flux density (irradiance) necessary to give an output signal equal to the detector noise. Generally NEP and NEI can be related to each other as
Fig. 11: (a) Lock-on range of IR heat seeker missile against extinction coefficient and aircraft tailpipe temperature. (b) The corresponding contour plot show different curves for different optimum values of lock-on range of heat seeker.

\[
\text{NEI} = \frac{\text{NEP}}{A_d}
\]  

(13)

where \( A_d \) is the detector area in cm\(^2\). When the detectors are compared, the best detector is the one with the lowest NEP and in turn the lowest NEI. Therefore, the lock-on range of heat seeker missile is expected to be increased as the NEI decreases (will be shown later).

The detectivity of detector can be expressed as (Mahulikar, S.P., et al., 2008)

\[
D^* = \frac{(A_d \Delta f)^{1/2}}{\text{NEP}}
\]  

(14a)

In addition, the detector area can be written in term of field of view, \( \omega \), and the equivalent focal length of the optics, \( f \), as

\[
A_d = \omega f^2
\]  

(14b)

Substitute (14b) in (14a), yields

\[
D^* = \frac{(\omega \Delta f)^{1/2} f}{\text{NEP}}
\]  

(15)

Furthermore, the numerical aperture, \( \text{NA} \), of the optics can be expressed in term of \( D_\phi \) and \( f \) as

\[
\text{NA} = \frac{D_\phi}{2f}
\]  

(16)

Substitute (15) and (16) in (11b) and after some simplification, one obtains

\[
H = \left( (N_i(T) - N_0(T)) A_f \right) (\pi D_\phi^2 \tau_\phi) \left( \frac{1}{4 \cdot \text{NEP} \cdot \text{SNR}} \right)
\]  

(17)
Substitute the above equation in (12), one obtains analytical expression of lock-on range, R, in term of NEP. In addition, by substituting (13) in (17), yields

$$H = \left( N_l(T) - N_b(T) \right) \frac{A_t}{A_d} \left( \pi D_o^2 \tau_o \right) \left( \frac{1}{4 \text{NEI} \cdot \text{SNR}} \right)$$

where it is in term of NEI and the ratio of target-to-detector areas.

From (18) and (16), it can be readily shown that as NEI decreases, R increases, therefore, the heat seeker missile having a detector with low NEI represents serious danger to the aircraft targets.

Figure 12 shows the effect of reduction of NEI on the lock-on range, R. As shown in this figure, as NEI decreases R increases for the specified value of atmosphere extinction coefficient. Developments in IR-detection technology are aiming to reduce the NEI, for instance, new generation of IR detectors are based on quantum well infrared detector to improve the detection ability of IR missile.

Fig. 12: Lock-on range of heat seeker missile against target temperature for different values of missile NEI detector.

More general details about the effect of NEI on R is shown in figure 13. This figure gives us a clear picture about the development efforts on the two sides. From one side, of the aircraft designers try to reduce the IR signature of military targets by reducing the emitted thermal radiation. From the other side heat seeker missile designers try to improve/develop the detection ability of IR seeker by reducing the NEI using advanced IR detectors.

Fig. 13: (a) Lock-on range of heat seeker missile against target temperature and NEI of the detector for $\alpha = 0.5 \times 10^{-6}$. (b) The corresponding contour plot show different curves for different optimum values of lock-on range of heat seeker.
One of the methods used in aircraft protection is the reduction of jet engine IR energy via cooling the exhaust gases plume. This can be done by injection water to the exhaust stream. This leads to decrease the transmittance parameter, $\tau_c$, due to increasing of attenuation coefficient, and then decrease IR signature level. This can be schematically represented by assuming that the exhaust gases plume will pass the cooling medium before the atmosphere one as shown in Fig. 12. Thus, the IR signature received by the seeker head will be attenuated by $e^{-at_c} \times e^{-aR} = e^{-acR}$ where $ac$ is the attenuation coefficient corresponding to the cooling medium (or any other attenuated medium such as smoke obscurants).

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