

Significance of Load Modeling Considering the Sources of Uncertainties in the Assessment of Transfer Capability for Interconnected Power Systems Using Interval Arithmetic

¹S.U. Prabha, ²C. Venkateshaiah, ³M. Senthil Arumugam

Faculty of Engineering and Technology Multimedia University Melaka Campus
75450 Melaka Malaysia

Abstract: The Total Transfer Capability (TTC) of a transmission system is a measure of unutilized capability of the system at a given time. A method for calculating TTC considering the load and line uncertainties are described in this paper. As the load representation has a significant impact on power system analysis results, in this work the buses were modeled using composite load modeling. This problem has not been discussed in the literature yet and is considered for calculating TTC in the present work. Interval Arithmetic (IA) method has been used to incorporate variations in load and line parameters. The IEEE 30 bus test system is used to illustrate the proposed methodology. The comparison has been done between conventional Optimal Power Flow (OPF) method and the proposed method. The quantification of load and line uncertainties and incorporation of composite load modeling lead to a better understanding of the underlying properties involved in TTC computation.

Key words: Total Transfer Capability, Power system planning, composite load modeling

INTRODUCTION

The electrical power systems are very large and complicated networks. Power system operators need to interpret and integrate multiple measured parameters. The Available Transfer Capability (ATC) indicates how much inter area power transfers can be increased without compromising system security. ATC is defined as Total Transfer Capability (TTC) minus the base case transfers together with adjustments to allow some margin of safety (NERC report, 1996). The Continuation Power Flow (Ajjarapu and Christy, 1992), Repeated Power Flow (Ejebe, *et al.* 1998; Prabha and Venkateshaiah, 2007), Security Constrained Optimal Power Flow (Yan.O.U and Chanan Singh, 2002) enable transfers by increasing the complex load with uniform power factor at every load bus in the sink area and increase the real power at generator buses in the source area in incremental steps until limits are incurred. For example, a line power flow could exceed its limit, a bus voltage magnitude could drop below the normal limit, or an operating point could disappear in voltage collapse. The real power transfer at the first encountered security violation is the TTC. These methods reported in the literature are appropriate and efficient in the management of transmission system. Due to the computational complexity of the problem, more work has been devoted for obtaining faster solutions and practically no attention has been paid to incorporate composite load modeling. Adequate representation of loads may result in considerable savings in system costs and increase the operation flexibility. The effects of modeling the load voltage dependence in PF and OPF has been considered (El-Hawary & Dias, 1987; Jawad Talaq, 1995; A.Chaturvedi, *et al.* 2006). Recently load modeling has been used for transient stability analysis (Y.Li, *et al.* 2007).

The uncertainties in load and line parameters are to be considered for accurate evaluation of ATC. In this paper, the quantitative uncertainty is represented by interval variables. The application of IA method for power flow analysis of transmission network has been proposed in literature (Zian Wang and Fernando.L.Alvarado, 1992; B.Das, 2002; A.Chaturvedi, 2006). In this paper, the probabilistic nature of the system input parameters and the composite load modeling are represented in terms of interval variables and has been considered in the calculation of TTC. The problem is formulated as an optimization problem, where the objective function is to maximize the power transfer between one area to another area without violating any of the system constraints.

Corresponding Author: S.U.Prabha, Faculty of Engineering and Technology Multimedia University Melaka Campus
75450 Melaka Malaysia
Telephone : 006-06-2523954; Handphone : 006-016-2934054; Facsimile : 006-06-2316552
E-mail: prabha.umapathy@mmu.edu.my

MATERIALS AND METHODS

Problem Formulation:

The objective function for the OPF reflects the maximum power transfer from one bus/area to another bus/area.

Objective Function

$$f(x) = \text{Max}(\sum_{i \in \Omega} \Delta P_{Di}) \tag{1}$$

Equality Constraints

$$P_{Gi} - P_{Di} - \sum_{j=1}^n |V_i| |V_j| (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) = 0$$

$$Q_{Gi} - Q_{Di} - \sum_{j=1}^n |V_i| |V_j| (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) = 0 \tag{2}$$

Inequality Constraints

The generators maximum and minimum output real and reactive powers are as follows.

$$|P_{Gi}|_{\min} \leq |P_{Gi}| \leq |P_{Gi}|_{\max}$$

$$|Q_{Gi}|_{\min} \leq |Q_{Gi}| \leq |Q_{Gi}|_{\max} \tag{3}$$

For the maintenance of system security, the transmission line MVA ratings are taken into account.

$$|S_{ij}| \leq |S_{ij}|_{\max} \tag{4}$$

To maintain the quality of electrical service and system security, bus voltage limits are considered.

$$|V_i|_{\min} \leq |V_i| \leq |V_i|_{\max} \tag{5}$$

where,

- ΔP_{Di} active power increment of load bus
- P_{Gi}, Q_{Gi} real and reactive power generation at bus i
- P_{Di}, Q_{Di} real and reactive load demand at bus i
- n bus number of the system
- V_i, V_j voltage magnitude at bus i, j
- S_{ij} line MVA limit

Interval Arithmetic Method:

Interval Arithmetic is a powerful tool to determine the effects of uncertain data. It can deal with numbers that vary within a range. The basic concepts of interval arithmetic are discussed as follows.

An interval number $X = [x_1, x_2]$ is the set of real numbers x such that $x_1 \leq x \leq x_2$. Here x_1 and x_2 are known as the lower limit and upper limit of the interval number, respectively. Let $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ be the two interval numbers. The basic arithmetic operations addition, subtraction, multiplication and division of these two interval numbers are defined in Eqns. (6)-(9) as below (Alefeld.G and Herzberger.J, 1983).

$$X + Y = [x_1 + y_1, x_2 + y_2] \tag{6}$$

$$X - Y = [x_1 - y_1, x_2 - y_2] \tag{7}$$

$$X * Y = [\min(x_1 * y_1, x_1 * y_2, x_2 * y_1, x_2 * y_2), \max(x_1 * y_1, x_1 * y_2, x_2 * y_1, x_2 * y_2)] \tag{8}$$

$$X \div Y = X * Y^{-1} \tag{9}$$

However, for the purpose of power flow analysis, calculations are based on complex number rather than real numbers. The basic relations involving complex interval numbers are described as follows. In general, any complex number $Z = X + iY$, where i is the complex operator, is said to be a complex interval number if both its real and imaginary part, X and Y respectively are interval numbers. Therefore X can be represented as $X = [x_1, x_2]$ and Y can be represented as

Uncertainties in Load and Line Parameters:

The system is assumed to operate under normal condition but line and load parameters vary with in a certain range. In this paper, $\pm 10\%$ variation in load parameters and $\pm 3\%$ for line parameters are considered from their rated nominal value.

$$P(k)_l = 0.90 P(k) \text{ and } P(k)_u = 1.10 P(k)$$

$$Q(k)_l = 0.90 Q(k) \text{ and } Q(k)_u = 1.10 Q(k)$$

$$R(jj)_l = 0.97 R(jj) \text{ and } R(jj)_u = 1.03 R(jj)$$

$$X(jj)_l = 0.97 X(jj) \text{ and } X(jj)_u = 1.03 X(jj)$$

Various case studies have been conducted and results of the following are presented.

- system under normal operating condition
- uncertainties due to load parameters only
- uncertainties due to line parameters only

System Load Modeling:

It is very well acknowledged that load representation is an important element in load flow studies, as it affects the over all system performance during operation and planning. More realistic and accurate load models are likely to improve the accuracy of the results.

Generally, loads can be modeled as constant power, constant current, constant impedance and exponential types. For constant power load, the power demand is constant regardless of voltage. For constant current load, the power demand is proportional to supply voltage and for constant impedance load its power demand is proportional to square of the supply voltage.

Mathematically, we can express these loads as

$$P = P_0 V^{k1} \tag{10}$$

$$Q = Q_0 V^{k2} \tag{11}$$

Where, for differential load models, the coefficient $k1$ and $k2$ are as follows: (James.A.Momoh, 2001)

- $k1 = k2 = 0$ for constant power loads
- $k1 = k2 = 1$ for constant current loads
- $k1 = k2 = 0$ for constant power loads
- $k1 = 1.38$ and $k2 = 3.22$ for exponential loads

P_0 and Q_0 represent the specified active and reactive powers at nominal voltage and V is the actual voltage magnitude in per unit.

The practical load can be combination of these four types of loads. Specific proportional combination of these loads forms a composite load and it is modeled as follows.

$$P_L^0(i) = P_L(i) [\alpha_0 + \alpha_1 V + \alpha_2 V^2 + \alpha_3 V^{e1}] \tag{12}$$

$$Q_L^0(i) = Q_L(i) [\beta_0 + \beta_1 V + \beta_2 V^2 + \beta_3 V^{e1}] \tag{13}$$

In this present work, the load demand is composed as follows. It is considered as 20% constant power, 30% constant current, 25% constant impedance and 25% exponential loads. Various case studies have been conducted and the results and the discussions are presented in the next section.

RESULTS

The IEEE 30 bus system shown in Fig.1, is adopted to illustrate the proposed method. The system is divided into 3 areas. The system has six generators, with two generators in each area. We assume each area as a utility. The utility in a certain area wants to import power from another area. Hence TTC is evaluated between areas.

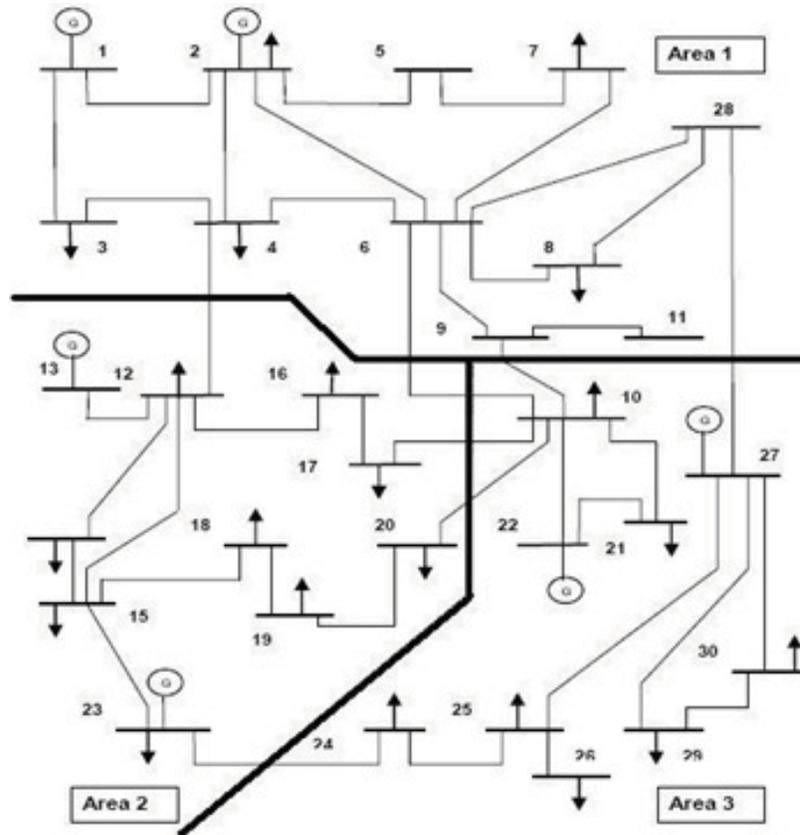


Fig. 1: IEEE 30 bus test system

The transfer capability is computed from a base case constructed from system information available at a given time. When the load demands in the system vary within some intervals, the bus voltage and other quantities such as line flows and line losses also varies within certain intervals. The interval of variation of bus voltages for the system under consideration have been calculated for all the uncertainties and are presented in Table 1,2 and 3. In Table 1 the voltage magnitude obtained for constant and composite power load models with fixed inputs (no uncertainties in load and line parameters) are compared. In Table 2 the voltage magnitude obtained due to uncertainties of load parameters for constant and composite power loads are given. Table 3 presents results of voltage magnitude when uncertainties in line parameters are considered. In these tables, the

symbols V_{ll} and V_{ul} denote the lower limit and upper limit of the voltage respectively. From the results, one can observe that, the voltage magnitude is lower in the case of constant power load model compared to the voltage magnitude obtained from composite load model. Thus by considering the constant power load will certainly lead to over-designing of the power system. Also voltage collapse might occur much later than the predicted maximum load, if one considers the constant power load model.

Case Study for TTC:

Several cases were studied and results for power transfer from area 2 to area 1 is presented and discussed below. Using the proposed interval arithmetic method, TTC is evaluated for both fixed input parameters and for various uncertainties. The loads are modeled as composite loads. The active loading of area 1,(in MW) before and after the transaction is shown in Table 4.

Table 1: Voltage Magnitudes for constant and composite power load models with fixed inputs

| Bus # | Fixed line and load parameters | |
|-------|--------------------------------|---------------------------------|
| | Constant power load $V_i(p.u)$ | Composite power load $V_i(p.u)$ |
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 0.983138 | 0.984319 |
| 4 | 0.980093 | 0.981475 |
| 5 | 0.982406 | 0.983612 |
| 6 | 0.973184 | 0.975015 |
| 7 | 0.967355 | 0.969414 |
| 8 | 0.960624 | 0.963251 |
| 9 | 0.980506 | 0.981543 |
| 10 | 0.984404 | 0.985012 |
| 11 | 0.980506 | 0.981543 |
| 12 | 0.985468 | 0.9861 |
| 13 | 1 | 1 |
| 14 | 0.976677 | 0.977546 |
| 15 | 0.980229 | 0.98101 |
| 16 | 0.977396 | 0.978295 |
| 17 | 0.976865 | 0.977791 |
| 18 | 0.96844 | 0.969765 |
| 19 | 0.965287 | 0.966771 |
| 20 | 0.969166 | 0.970469 |
| 21 | 0.993383 | 0.993548 |
| 22 | 1 | 1 |
| 23 | 1 | 1 |
| 24 | 0.988566 | 0.988803 |
| 25 | 0.990215 | 0.990523 |
| 26 | 0.972194 | 0.973196 |
| 27 | 1 | 1 |
| 28 | 0.974715 | 0.976593 |
| 29 | 0.979597 | 0.980358 |
| 30 | 0.967883 | 0.969174 |

Table 2: Voltage Magnitudes for constant and composite power load models with uncertainties due to load parameters only

| Bus # | Constant Power Load | | Composite Power Load | |
|-------|---------------------|---------------|----------------------|-----------|
| | $V(i) ll$ p.u | $V(i) ul$ p.u | $V(i) ll$ | $V(i) ul$ |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.981306 | 0.986537 | 0.982428 | 0.987622 |
| 4 | 0.9773 | 0.983761 | 0.978658 | 0.985123 |
| 5 | 0.979675 | 0.984989 | 0.980961 | 0.986158 |
| 6 | 0.96903 | 0.977216 | 0.970887 | 0.979069 |
| 7 | 0.963214 | 0.972314 | 0.965346 | 0.974312 |
| 8 | 0.95401 | 0.965198 | 0.956845 | 0.967843 |
| 9 | 0.977627 | 0.982225 | 0.978717 | 0.983251 |
| 10 | 0.982294 | 0.984897 | 0.98298 | 0.985472 |
| 11 | 0.977627 | 0.982225 | 0.978723 | 0.983825 |
| 12 | 0.983673 | 0.986949 | 0.984475 | 0.987409 |
| 13 | 1 | 1 | 1 | 1 |
| 14 | 0.974063 | 0.978931 | 0.97513 | 0.97959 |
| 15 | 0.978056 | 0.982241 | 0.978964 | 0.982886 |

Table 2: Continue

| | | | | |
|----|----------|----------|----------|----------|
| 16 | 0.974298 | 0.97883 | 0.975336 | 0.97962 |
| 17 | 0.973539 | 0.977869 | 0.974591 | 0.978736 |
| 18 | 0.965117 | 0.971416 | 0.9666 | 0.97258 |
| 19 | 0.961235 | 0.968089 | 0.962899 | 0.969423 |
| 20 | 0.965531 | 0.971485 | 0.966989 | 0.972663 |
| 21 | 0.992167 | 0.992787 | 0.992014 | 0.993311 |
| 22 | 1 | 1 | 1 | 1 |
| 23 | 1 | 1 | 1 | 1 |
| 24 | 0.987247 | 0.989803 | 0.987522 | 0.989993 |
| 25 | 0.989156 | 0.991279 | 0.989502 | 0.991544 |
| 26 | 0.969092 | 0.974968 | 0.970209 | 0.97586 |
| 27 | 1 | 1 | 1 | 1 |
| 28 | 0.970329 | 0.978686 | 0.97224 | 0.980578 |
| 29 | 0.977483 | 0.981787 | 0.978294 | 0.982461 |
| 30 | 0.964467 | 0.97125 | 0.96586 | 0.972397 |

Table 3: Voltage Magnitudes for constant and composite power load models with uncertainties due to line parameters only

| Bus # | Constant Power Load | | Composite Power Load | |
|-------|---------------------|-------------|----------------------|----------|
| | V(i) ll p.u | V(i) ul p.u | V(i) ll | V(i) ul |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.983488 | 0.984535 | 0.984608 | 0.98558 |
| 4 | 0.979993 | 0.981257 | 0.981371 | 0.982544 |
| 5 | 0.981824 | 0.982977 | 0.983071 | 0.98414 |
| 6 | 0.972369 | 0.974102 | 0.974242 | 0.975852 |
| 7 | 0.966832 | 0.9689 | 0.968928 | 0.970859 |
| 8 | 0.958474 | 0.958474 | 0.961222 | 0.963625 |
| 9 | 0.979353 | 0.980634 | 0.980411 | 0.981622 |
| 10 | 0.983113 | 0.984147 | 0.983727 | 0.984721 |
| 11 | 0.97953 | 0.980634 | 0.980411 | 0.981622 |
| 12 | 0.984867 | 0.985837 | 0.985504 | 0.986432 |
| 13 | 1 | 1 | 1 | 1 |
| 14 | 0.977289 | 0.977289 | 0.976667 | 0.978118 |
| 15 | 0.979542 | 0.980819 | 0.980337 | 0.981562 |
| 16 | 0.975853 | 0.977355 | 0.976745 | 0.978189 |
| 17 | 0.974976 | 0.976508 | 0.975868 | 0.977343 |
| 18 | 0.967297 | 0.969315 | 0.968653 | 0.970585 |
| 19 | 0.963586 | 0.965822 | 0.965116 | 0.967253 |
| 20 | 0.967554 | 0.969544 | 0.968889 | 0.970794 |
| 21 | 0.992253 | 0.992718 | 0.992437 | 0.99289 |
| 22 | 1 | 1 | 1 | 1 |
| 23 | 1 | 1 | 1 | 1 |
| 24 | 0.988173 | 0.988881 | 0.988413 | 0.989107 |
| 25 | 0.989914 | 0.990527 | 0.99023 | 0.990823 |
| 26 | 0.97117 | 0.972908 | 0.972206 | 0.97388 |
| 27 | 1 | 1 | 1 | 1 |
| 28 | 0.973798 | 0.975448 | 0.975716 | 0.977241 |
| 29 | 0.979001 | 0.980292 | 0.979785 | 0.981025 |
| 30 | 0.966859 | 0.968894 | 0.968194 | 0.970143 |

Table 4: Active loading of Area 1

| Bus No. | 2 | 3 | 4 | 7 | 8 |
|---------|-------|------|------|-------|-------|
| Before | 21.7 | 2.35 | 7.43 | 21.95 | 28.67 |
| After | 23.92 | 2.59 | 8.19 | 24.19 | 31.6 |

The load at area 1 increases from 82.1 to 90.49 MW and the limiting condition was the overloading of line 6-8. The sum of the loads at the sink area is taken as the TTC. Table 5 gives the total load of area 1 and total real and reactive losses in interval parameters for base case.

Table 5: Base case results

| Varying Parameters | Real Load P_{ll} (MW) | Real Load P_{ul} (MW) | Reactive Load Q_{ll} (MVAr) | Reactive Load Q_{ul} (MVAr) | Total real loss P_{ll} (MW) | Total real loss P_{ul} (MW) | Total reactive loss Q_{ll} (MVAr) | Total reactive loss Q_{ul} (MVAr) |
|---------------------|-------------------------|-------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------------|-------------------------------------|
| Fixed line and load | 82.1 | 82.1 | 53.93 | 53.93 | 2.217 | 2.217 | 8.28 | 8.28 |
| Load Uncertainty | 73.9 | 90.32 | 48.53 | 59.31 | 1.702 | 2.988 | 6.78 | 10.73 |
| Line Uncertainty | 82.1 | 82.1 | 53.93 | 53.93 | 2.163 | 2.308 | 8.04 | 8.57 |

Table 6: Interval values for the operating parameters for constant power load

| Varying Parameters | Total Transfer Reactive Load Capability | | Total Real Loss | | Total Reactive Loss | | Total Loss | | | |
|---------------------|---|-------------|-----------------|----------|---------------------|----------|------------|----------|----------|----------|
| | Lower limit | Upper limit | Q_{ll} | Q_{ul} | P_{ll} | P_{ul} | Q_{ll} | Q_{ul} | S_{ll} | S_{ul} |
| | (MW) | (MW) | (MVAr) | (MVAr) | (MW) | (MW) | (MVAr) | (MVAr) | (MVA) | (MVA) |
| Fixed line and load | 98 | 98 | 63.37 | 63.37 | 2.68 | 2.68 | 11.52 | 11.52 | 11.83 | 11.83 |
| Load Uncertainty | 88.89 | 108.65 | 59.45 | 72.64 | 2.22 | 3.874 | 8.78 | 14 | 9.06 | 14.53 |
| Line Uncertainty | 90.49 | 104.23 | 59.9 | 73.4 | 2.808 | 2.99 | 10.27 | 13.44 | 10.65 | 13.77 |

Table 7: Interval values for the operating parameters for the composite power load

| Varying parameters | Total Transfer Reactive Load Capability | | Total Real Loss | | Total Reactive Loss | | Total Loss | | | |
|---------------------|---|-------------|-----------------|----------|---------------------|----------|------------|----------|----------|----------|
| | Lower limit | Upper limit | Q_{ll} | Q_{ul} | P_{ll} | P_{ul} | Q_{ll} | Q_{ul} | S_{ll} | S_{ul} |
| | (MW) | (MW) | (MW) | (MVAr) | (MW) | (MW) | (MVAr) | (MVAr) | (MVA) | (MVA) |
| Fixed line and load | 90.49 | 90.49 | 59.43 | 59.43 | 2.435 | 2.435 | 9.35 | 9.35 | 9.66 | 9.66 |
| Load Uncertainty | 86.38 | 105.58 | 56.74 | 69.33 | 1.962 | 3.579 | 8.06 | 13.03 | 8.30 | 13.51 |
| Line Uncertainty | 88.09 | 101.23 | 57.44 | 72.58 | 2.696 | 2.878 | 9.17 | 11.47 | 9.56 | 11.83 |

Table 6 and 7 presents the values of different operating parameters after the transaction has been carried out, considering the constant power load model and composite load model. The TTC values are obtained and presented in interval form for all categories of uncertainties. It is observed the application of IA method suggests a wider range of intervals. Also, one may observe that after transaction, the loss obtained in constant power load is comparatively more than composite load model.

Conclusion:

The proposed technique has been successfully implemented on IEEE 30 bus test system and the results obtained were found to be satisfactory. The contributions can be summarized as follows:

- a scheme has been implemented to incorporate the uncertainties in load and line parameters for the calculation of TTC
- a method based on composite load modeling is proposed in the evaluation of TTC and compared with constant load modeling and the analysis of the results are reported
- both of the above are implemented by using interval arithmetic method which has been proved as a useful tool in literature

The results obtained from the proposed method are found to be more informative in qualitative terms about the system analysis when compared to the conventional deterministic approach. It is proposed that this method can be a effectively used by the utility engineers in planning and operation of power systems and particularly for the issues concerning to transfer capability.

REFERENCES

Ajjarapu, V. and C. Christy, 1992. The continuation power flow: A tool for steady state voltage stability analysis. *IEEE Transactions on Power Systems*, 7: 416-423.

Alefeld, G. and J. Herzberger, 1983. *Introduction to interval computations*. New York Academic Press.

Biswarup Das, 2002. Radial Distribution system power flow using interval arithmetic. *Electrical power and energy Systems*, 827-836.

Chaturvedi, A., K. Prasad and R. Ranjan, 2006. Use of Interval Arithmetic to incorporate the uncertainty of load demand for radial distribution system analysis. *IEEE Transactions on Power delivery*, 21: 1019-1021.

Chaturvedi, A., R.R. Anjan, K. Prasad and N.C. Sahoo, 2006. Probabilistic distribution based interval arithmetic load flow algorithm for radial distribution system. *International Journal of Power and Energy Systems*.

Ejebe, G.C., J. Tong, J.G. Waight, J.G. Frame, X. Wang and W.F. Tinney, 1998. Available transfer capability calculations. *IEEE Transactions on Power Systems*, 13: 1521-1527.

El-Hawary, M.E and L.G. Dias, 1987. Incorporation of Load models in load flow studies. *IEE Proceedings*, 134: 27-30

Jawad Talaq, 1995. Modeling and Elimination of Load Buses in power flow solutions. *IEEE Transactions on Power Systems*, 10(3): 1154-1158.

James, A., Momoh, 2001. *Electric Power System Applications of Optimization*. Marcel Dekker, Inc.

Li, Y., H.D. Chiang, B. K. Choi, Y.T. Chen, D.H. Huang and M.G. Lauby, 2007. Representative static models for transient stability analysis: development and examination. *IET Generation, Transmission and Distribution*, 1(3): 422-431.

NERC reference document, 1996. Available transfer capability definitions and determination.

Prabha Umapathy and C.Venkateshaiah, 2007. A novel approach for the assessment of available transfer capability in power systems. *College science in India*, 1(3): 1-4.

Yan, O.U. and Chanan Singh, 2002. Assessment of available transfer capability and margins. *IEEE Transactions on Power Systems*, 17: 463-468.

Zian Wang and Fernando, L. Alvarado, 1992. Interval Arithmetic in power flow analysis. *Transactions on power Systems*, 7: 1341-1349.