Numerical and Experimental Investigation on Laminar Hot-gas Flow with Injected Water Spray

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Abstract: One-dimensional space-marching technique is acquired to set up computational fluid dynamics (CFD) code to describe the characteristics of the high-temperature gas flow with injected water spray. General equations of conservation have been proposed to describe the flow characteristics. Binary diffusion, developed by Chapman and Enskog, is used to calculate the evaporation rate of water droplets. The model of drag on immersed body is used to describe the action of momentum transfer between gas and liquid phases. The flow is treated as laminar flow and the gas is treated as an ideal gas in order to achieve the computational results. To provide the comparative data for the CFD, high-temperature laminar gas flow through a 9.8-cm inside diameter pipe is arranged in adiabatic environment. Spray of water with the average of 0.07-0.08 mm droplet diameter is injected downstream into the flow at the rate of 7.2 ml/s. Momentum and heat transfers occur between hot gas and water droplets due to the difference of velocity and temperature. Droplets evaporate and are accelerated during traveling with the flow, causing the flow characteristics change of the gas phase. Both numerical and experimental results share similarity of the characteristics of the flow, including temperature, density and velocity of the flow.

Key words: laminar, spray, evaporation

INTRODUCTION

Injecting water spray into hot-gas stream is a convenient method to lower temperature of the gas. Heat of the gas is transferred to the water droplets as the droplet temperature increases and the droplets evaporate into steam. The process is useful for reconditioning hot gas to a preferred temperature. Steam can also be produced with this process with proper settings. The process has been used in applications, such as gas turbine and steam turbine.

Injecting water spray into hot-gas stream creates two-phase flow. Water droplets act as particles floating in the hot-gas stream. Mass, momentum and heat are transferred between gas and liquid due to the differences of velocity and temperature. Maher M. Abou Al-Sood and Madjid Birouk (2008) created three-dimensional numerical model to investigate the effect of turbulence on heat and mass transfer rates of a droplet exposed to a hot-air stream. The study indicated that the effect of radiation is less significant at temperature lower than 1,000°C. The experiment for this study is performed at hot-gas temperature of 300-350°C and the radiation effect is neglected.

In this study, evaporation is the major process which causes change to the flow. Evaporation process increases volume and mass of the gas phase. At the same time, latent heat of evaporation decreases the temperature of the hot gas. The droplet size also decreases during evaporation. Parts of liquid droplets transform into steam and mix with hot gas, increasing the humidity degree of the flow. A certain increasing
rate of the flow humidity can decrease the evaporation rate of droplets, as experimented by Yoshida and Hyodo (1970). The influence on droplet evaporation rate from surrounding condition was also studied by S.S. Sazhin, I.N. Shishkova, A.P. Kryukov, V.Yu. Levashov and M.R. Heikal (2007).

The objective of this study is to investigate characteristics of the hot-gas flow with injected water spray. After the numerical model is set up, its outcomes will be compared with experimental results. The comparison will determine whether the model is able to explain the characteristics of the flow, the product of the process, and the required system to achieve those results. Numerical and experimental models are built for inviscid laminar flow under adiabatic environment. Assumptions and theory of the flow are described in the next section.

2. Numerical Model and Theory:

Circular pipe flow with constant diameter is used for setting up numerical model. Product of LPG-air combustion is used for a hot-gas stream in both numerical and experimental settings. The numerical model is one-dimensional laminar flow with the following assumptions:
1. Flow is compressible.
2. Flow is inviscid.
3. No heat transfer through a pipe wall.

Water is sprayed at the beginning of the flow. Droplet diameter and droplet density are assumed to be consistent over the cross-section area. The flow system is shown in Fig. 1.

![Fig. 1: Model of hot-gas flow carrying water droplets through a circular pipe.](image)

2.1 LPG-air Combustion:

\[3.076 \text{C}_3\text{H}_8 + \text{C}_4\text{H}_{10} + a\text{(O}_2+3.76\text{N}_2) \rightleftharpoons 13.228\text{CO}_2 + 17.304\text{H}_2\text{O} + (a-15.266) \text{O}_2 + 3.76an\text{H}_2\text{O}\]

Assumed that LPG with weight proportion of 70% propane (C\textsubscript{3}H\textsubscript{8}) and 30% butane (C\textsubscript{4}H\textsubscript{10}) is used in combustion process to produce hot gas in numerical and experimental settings, gas mixture of the combustion product is determined from chemical reaction in Equation (1). Mole of air, a, in combustion process is a function of air-fuel ratio, which is measured from experiment. The composition of gas mixture determines thermal, chemical and physical properties of the gas in calculation.

2.2 Ideal Gas and Equation of State:

The numerical model is set up for low pressure and high temperature gas flow. The absolute pressure of the flow is approximately at atmospheric pressure by allowing gas to expand freely when its properties change due to effect from evaporation and drag. Gas under these conditions behaves as an ideal gas. Charles and Gay-Lussac’s equation of state is applied to describe behavior of the hot gas (Yunas, A. Cengel and Micheal, A. Boles, 2002).

\[p = \rho \frac{R_u}{M} T\]

The numerical scheme uses equation of state to monitor change of gas properties in the flow. The molar mass term represents the composition of the gas which continuously changes as flow proceeds.

2.3 Assumptions for Droplets:

The following assumptions are used to describe condition of droplets during the evaporation process.
1. The evaporation is quasi-steady
2. Droplet temperature is uniform and assumed to be a fixed value below the boiling point of liquid.
3. Mass fraction of vapor at droplet surface is determined by liquid vapor equilibrium at droplet temperature.

Fig. 2 demonstrates model of mass fraction species A (water) around droplet which is used in the calculation. Mass fraction of species A in the flow is assumed to be uniform everywhere as, in this study, the droplet diameter is relatively small and droplet density is high.

Fig. 2: Mass fraction of droplet in quiescent environment (Turns, 2000)

Droplets are treated as the source of mass as they float in the flow field and evaporate (Maher, M. Abou Al-Sood and Madjid Birouk, 2008; Liu, W. and C.J. Rutland,). The rate of droplet flowing through each increment space is constant. In numerical model, the droplet distribution and velocity are assumed to be uniform within single increment space. However, the density of droplets is an inverse function of droplet velocity. Model of droplet distribution is shown in Fig. 3. The droplet density decreases while droplet velocity increases as flow proceeds.

Fig. 3: Droplet distributions in each increment space

2.4 Evaporation:

Equation for evaporation is based on Fick’s law, describing the rate at which species A diffuses through species B. Diffusion coefficient of two species (\(D_{ab}\)) is a main property to determine evaporation rate of the droplets while \(D_{ab} \propto T^{3/2}/p\). Droplets, as liquid phase, are treated as incompressible objects and their properties are determined separately from gas phase which is compressible (Stephen, R. Turns, 2000).

Chapman and Enskog derived a working equation for diffusion coefficient from solving Boltzmann equation (Reid, Prausnitz and Poling, 1986). Applying the ideal gas principle, the equation can be written as:
Transfer number is defined as:

\[ B_Y = \frac{Y_{A,Y} - Y_{A,Y_{\infty}}}{1 - Y_{A,Y}} \]  

(4)

Transfer number \( B_Y \) represents surrounding condition of droplets. In this case, \( B_Y \) as a function of mass fraction of water in hot gas, indicates the degree of humidity where evaporation occurs. Yoshida and Hyodo (1970), and later, Schwartze and Brocker (2000) concluded that degree of humidity in atmosphere influenced evaporation rate of liquid. Injecting a certain number of droplets can cause humidity change in the flow; therefore, \( B_Y \) is treated carefully in this calculation.

Equation for evaporation rate of single droplet can be written as:

\[ \dot{m}(r) = 4\pi r_D^2 D_{AB} \ln \left[ 1 + B_Y \right] \]  

(5)

Vapor mass is assumed to mix with the main flow immediately and homogeneously after evaporates. This assumption is reasonable when droplet density is high. The effect of mass and momentum changes of the gas phase are included in mass and momentum conservation equations; further description is in the following section 2.6.

The energy term for evaporation is described in latent heat term in energy conservation equation, (section 2.6). Amount of latent heat is determined, based on evaporation rate, for each increment space.

2.5 Drag on Immersed Body:

Droplet shape is assumed to be spherical throughout the process. The effect from droplet deformation is neglected in this study. Drag, \( F_D \) for droplet immersed in stream of hot gas is described by:

\[ F_D = \frac{1}{2} \rho C_D A_d \left| u_i - u_d \right| (u_i - u_d) \]  

(6)

Drag coefficient is calculated by (Jaber Almedeij, 2008).

\[ C_D = \frac{24}{R_e} (1 + 0.15 R_e^{-0.687}) ; \quad 2 < R_e < 800 \]  

(7)

2.6 Gas Phase Conservation Equations:

The system is set as a one-dimensional steady-state adiabatic laminar flow. The flow is two phases of fluid combined. Gas phase is the one of interest. Droplets are treated as particles in the flow and interact with the flow with hydrodynamics force in the form of drag. Mass conservation equations are set up to describe total gas flow and vapor fraction in the flow to monitor the increasing of water vapor in the flow.

Mass conservation equation of total gas

\[ \nabla \cdot (\rho \vec{u}) = \dot{m}_{vapour} \]  

(8)

Mass conservation equation of vapor

\[ \nabla \cdot (Y_{A,Y} \vec{u}) = \dot{m}_{vapour} \]  

(9)

Momentum conservation equation

\[ \nabla \cdot (\rho u \vec{u}) = -\frac{\partial p}{\partial x} + \rho \cdot \vec{F}_D \]  

(10)
Energy conservation equation

\[
\nabla \cdot \left( \rho (\varepsilon + \frac{u^2}{2}) \right) = \frac{\partial (\varepsilon u_p)}{\partial x} + \rho D_v \cdot \ddot{u} - \dot{m}_{\text{vapour}} \frac{h_f}{\Delta T_{\text{vapour}}} - \dot{m}_{\text{vapour}} C_D (T_w - T_{\text{boil}})
\]

2.6 Numerical Approach:

FORTRAN computer code is written to perform the calculation for one-dimensional steady-state laminar flow. Space-marching algorithm is used to predict the forwarding flow as it is time independent. First set of calculation is calculated using first order forward difference and then second order central forward difference is applied to the rest of marching steps. Terms of evaporation rate and drag force are calculated separately for an individual droplet on each marching step. Droplet velocity and distribution are also calculated separately for each increment space. Small increment steps are used in order to limit discretization error and maintain stability of the calculation. Flow chart of the algorithm is shown in Fig. 4.

![Flow chart of computer code](image)

3. Experiment:

There are several models of experiment on two-phase flow and droplet evaporation. Experiment of two-phase flow together with evaporation in a tube was done by G. Sun and G.F. Hewitt, G. Castanet, C. Maqua, M. Orain, F. Grisch and F. Lemoine (2007) investigated the physical change of droplet stream using laser-induced fluorescence techniques. This experiment focuses on flow properties under influence of droplet evaporation within the flow.

Initial setting of the experiment

Experiments are performed at four different initial conditions to verify the results from the computer code. The set-up model for experiment is shown in Fig. 5. The experiment is set at the following conditions:

1. Hot gas stream is a product gas of LPG-air combustion at inlet temperature between 307-322°C with flow velocities between 8.1-14.2 m/s.
2. Hot gas stream is reconditioned to reach laminar condition before entering investigation zone. The Reynolds’s number of the flow is in the range of 17,600-33,900.
3. The flow path is 9.8-cm diameter circular pipe.
4. Water spray is injected into the flow at the beginning of the investigation zone at flow rate of 7.1 ml/s.
5. Initial droplet size entering the investigation zone is 0.07-0.08 mm.
6. Initial droplet velocity is 1.5 m/s.
7. Investigation zone is insulated to minimize heat transfer through the pipe wall. Temperature, pressure and velocity of the flow are measured through series of tap holes on the pipe wall. Each hole is 20 cm apart.

![Model for Experiment](Image)

**Fig. 5: Model for Experiment**

### 3.2 Uncertainty Analysis:

Uncertainty consideration of the experiment data is based on U95 uncertainty model. Random error is calculated from raw data. Systematic error is calculated from sensitivity of measuring instruments on the following assumptions:

**Analog instrument**
- ± 10% of smallest major division for laboratory quality instruments.
- ± 30% of the smallest major division for industrial quality instruments.

**Digital instrument**
- ± 3 units of the last digit displayed for laboratory quality instruments.
- ±1% of full scale for industrial quality instruments.

### RESULTS AND DISCUSSION

#### 4.1 Numerical and Experimental Results:

Four experiments with different initial conditions are performed to verify the numerical results. The initial conditions of those experiments are as follows:

- **Exp. a)** initial temperature = 307°C, initial droplet diameter = 0.08 mm and initial flow velocity = 8.1 m/s
- **Exp. b)** initial temperature = 310°C, initial droplet diameter = 0.07 mm and initial flow velocity = 10.0 m/s
- **Exp. c)** Initial temperature = 317°C, initial droplet diameter = 0.08 mm and initial flow velocity = 10.0 m/s
- **Exp. d)** Initial temperature = 322°C, initial droplet diameter = 0.07 mm and initial flow velocity = 14.2 m/s

Fig. 6a to 6c demonstrates the numerical and experimental results of Exp. a). Temperature, density and velocity of the flow are presented in comparison in each graph. The numerical and experimental results show a good similarity on the characteristics of the flow with maximum temperature, density and velocity errors of 1.28%, 0.94%, and 1.14% respectively.

Under adiabatic environment, heat transfer process occurs only between hot gas and droplets. Heat is used for increasing droplet surface temperature to reach evaporating status, for evaporating, and for increasing vapor temperature to reach the temperature balance with the flow. In computer code, the evaporated mass of droplets is assumed to be the only part of droplets that goes through heat transfer process. Not only influenced by flow temperature, evaporation rate also depends on droplet size. The numerical results of droplet diameter and evaporation rate are shown in Fig. 7.

Considering that the water vapor density from droplet evaporation is lower than the initial hot gas density, the flow density should be decreasing as accumulated water vapor in the flow is getting larger as flow proceeds. Therefore, the flow increasing of density indicates that the decreasing temperature dominates such effect and causes increasing of the flow density. The experimental results also show flow velocity decrease along the pipe. While the expansion of evaporating vapor should cause the flow velocity to increase, drag force on droplets, which causes droplet velocity decrease, is the factor that leads to the contrary effect. Therefore, the experimental flow velocity decrease indicates that the effect of drag force on droplets dominates the effect of the evaporating vapor expansion on the overall flow velocity. However, reverse effects can be expected in different settings.
The results of other three experiments agree with the previous experiment with maximum temperature, density and velocity differences of 1.97%, 1.14% and 1.60% respectively, comparing to the numerical results.
4.2 Numerical Error Investigation on Droplet Deformation:

In numerical calculation, a droplet is assumed to maintain spherical shape at all times. However, in actual flow, a droplet continuously deforms under drag force. In case of laminar flow, drag force acts on a droplet in one direction and forces spherical droplet to deform into spheroid. Droplet deformation under drag force occurs at the beginning zone of the flow where velocity difference between droplet and flow presents. R. J. Haywood, M. Renksizbulut and G.D. Raithby (1994) developed a numerical model of droplet shape during transient deformation stage and suggested the droplet shape at velocity higher than $0.128U_y$ to be prolate. Fig. 8 demonstrates droplet shape in actual flow at 30 cm downstream, displaying prolate spheroid shape of the droplet of 1:2 axis proportion. The deformation affects both drag and evaporation rate as projection area and surface area are different from sphere for equal volume.

![Droplet shape in actual flow](image)

Fig. 8: Droplet shape in actual flow

The actual shape of droplet is applied in the computer code from the start of calculation to investigate the maximum limit of calculation error. Input for drag and equation for evaporation rate are modified for spheroid-shape droplet. The equation used for evaporation rate for spheroid-shape droplet is:

$$m(r_c) = \frac{(9\pi + 4\pi^2)}{18} r_c \rho D_{ab} \ln \left( \frac{1 - Y_{A,e}}{1 - Y_{A,s}} \right)$$

(12)

Error limit for Exp. d) due to the droplet deformation on temperature, density and velocity are 1.13%, 0.54% and 0.46% respectively. Other experiments share the same level of error limit as the droplet sizes are proximately the same.

4.3 Influence of Droplet Surface Temperature on Gas Properties:

Maqua, Castanet, Grisch and Lemoine (2007) investigated the temperature change inside droplets during evaporation in air flow. The investigation indicated that droplet temperature is not stable under the process of evaporation and heat transfer due to temperature difference between droplets and their surroundings. The phenomenon should also cause the droplet surface temperature to change during traveling with the flow.

Droplet surface temperature is one of the initial variables for the computer code. The variable is used to determine mole fraction of water vapor at droplet surface and has high influence on the evaporation rate. Droplet surface temperature is practically difficult to measure because droplet size is usually very small and floats along the gas stream at relatively same speed. In this experiment the water temperature is measured before it is injected from spray nozzle and is used as reference for estimating droplet surface temperature. The values of droplet surface temperature in the calculation are obtained by trial and error comparing to the final temperature of the flow in each experiment. Fig. 8a to 8c shows the effect of varying droplet temperature in the computer code. The deviation of flow temperature is getting larger as flow proceeds further, indicating that the selection of droplet temperature is a major influence to the calculation accuracy.

4.4 Influence of Gas Humidity on Evaporation:

Yoshida and Hyodo (1970) experimented on wetted wall column and proved that evaporation rate of water into air varied by the degree of air humidity. Schwartz and Brocker (2000) confirmed the phenomenon theoretically by applying film theory to heat and mass transfer to a liquid surface exposed to a gas flow. When the spray nozzle injects enough amounts of droplets into the hot gas flow, degree of humidity in the hot gas increases as droplets evaporate and steam is accumulated as the flow proceeds.
The effect of increasing humidity is accounted in the computational process as surrounding’s mass fraction of steam, $Y_{AB}$, in the conservation of mass and evaporation equation. Fig. 9 compares rate change of droplet diameter in form of $D^2$. The results agree with those of Yoshida and Hyodo’s (1970) and the diameter decreasing rate does not follow $D^2$ law of evaporation.

The increasing humidity of the flow also affects other flow temperature, density and velocity as shown in Fig. 10a to 10c. The results indicate increasing error as the flow proceeds due to accumulated moisture in the flow.
5. Summary and Conclusion:

Numerical method is used to describe the characteristics of the hot-gas stream with injected water spray. Evaporation and drag force of droplets are major effects to the flow properties. The numerical algorithm is a second-order accurate CFD, associated with drag and binary diffusion. Set of experiments is done to verify the algorithm. Initial conditions are set to be same in both numerical and experimental. The numerical model, written in FORTRAN, provides outcome with good agreement to the experimental results. Maximum temperature, density and velocity errors are 1.97%, 1.14% and 1.60% respectively. Flow characteristics agree with earlier research of Yoshida and Hyodo (1970). The increasing humidity of the flow has significant effect to the flow properties when droplet flow rate is sufficiently high. Another important factor that affects the calculation result is droplet surface temperature. This factor has not been included in the algorithm but the surface temperature is set to be constant throughout the process. The study shows that the result’s deviation from surface temperature input grows larger as flow proceeds and affects with the whole flow properties.

The properties at any positions in the flow are predictable by numerical scheme. Deviation of numerical results from droplet deformation and surface temperature selection are presented. However, some physical errors; such as droplet break-up, droplet size non-uniformity, droplet collision and radiation effect, etc., have not been investigated. Some errors can cause significant effect to the result in calculation in some specific situations and should be included in the calculation in the next version.

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8. Nomenclature:

- \( p \) = pressure
- \( \rho \) = density
- \( R_u \) = universal gas constant
- \( M \) = molar mass
- \( D_{AB} \) = diffusion coefficient, \( \text{m}^2/\text{s} \)
- \( T \) = temperature, \( \text{K} \)
- \( p \) = pressure, \( \text{Pa} \)
- \( \sigma_{AB} \) = characteristic length, \( \text{A} \)
- \( \Omega_p \) = diffusion collision integral
- \( Y_{A,s} \) = mass fraction of vapor at droplet surface
- \( Y_{A,¥} \) = mass fraction vapor of surrounding
- \( C_D \) = drag coefficient
- \( A_d \) = projected area in the stream direction
- \( u_i \) = stream velocity
- \( u_d \) = droplet velocity
- \( u \) = flow velocity
- \( m_v \) = source term of evaporating droplet within increment space
- \( r_e \) = radius of the longest axis of spheroid