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Comparison of Homotopy-Perturbation Method and variational iteration Method to the Estimation of Electric Potential in 2D Plate With Infinite Length

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Abstract: Estimation of electric potential due to dependent of many electrical components to this parameter is one of the most significant issues in electrical engineering. In this work, two powerful analytical methods are investigated to analyze the electrical potential in 2D plate with infinite length. One is He's variational iteration method (VIM) and the other is He's homotopy-perturbation method (HPM). VIM is used to construct correction functional using general Lagrange multipliers identified optimally via by means of the variational theory, and the initial approximations can be freely arbitrarily chosen with unknown constants. HPM may be used to transform a difficult problem into a simple problem which can be easily solved. Comparison of this new method with finite element method (FEM) is applied to assure us about the accuracy of solution.

Key words: Electric potential, 2D plate, homotopy-perturbation method (HPM), He's variational iteration method (VIM) and finite element method (FEM).

INTRODUCTION

The solution of electromagnetic field problems in 2D plate with infinite length obtained by Partial differential equations which arise in real-world physical problems are often too complicated to be solved exactly And even if an exact solution is obtainable, will be required calculations that may be too complicated to be practical. Using Maxwell's equations electric potential and electric field are determined as solution of Poisson's equation in plate, while in that case Maxwell's equations are non-linear partial differential equations, which solution depends on the initial conditions and earlier media situation. These equations and existing boundary conditions on the separating surface electric scalar potential functions are determined as the solution of Laplace's equations in plate. In the numerical method, stability and convergence should be considered, to avoid divergent or inappropriate results. Therefore, approximate analytical solutions were introduced. Many different new methods have been presented recently. Among them the variational iteration method (VIM) and He's homotopyperturbation method (HPM) which were introduced by He (He, 1999; 1998a; 1998b; 2000; 2004; 2000; 1999) are the most effective and convenient ones for Laplace's equations. Developing the perturbation method for different usage is very difficult because this method has some limitations and based on the existence of a small parameter. Therefore, many different new methods have recently introduced some ways to eliminate the small parameter such as VIM. The methods have a useful feature in that it provides the solution in a rapid convergent power series with elegantly computable convergence of the solution. Figure (1) shows outlook of 2D plate with infinite length and its condition. Comparison of this new method to final element method (FEM) shows excellent agreement of these two methods. The rest of this paper is organized as follows: section 2 explains some relationship that governs stationary electricity and reach to necessity of powerful approach to estimate electric parameter. Section 3 and 4 describes in detail the proposed method. Sections 5 will illustrate and analyze method in various boundary conditions. Section 6shows the simulation results. Finally, conclusions are presented in Section 7.

2. Electric Potential in Differential Form:

Maxwell's equations are partial differential forms that are true at all space. One of these main equations is (2-1) that governs stationary electricity.

$$\nabla.D=\rho \tag{2-1}$$

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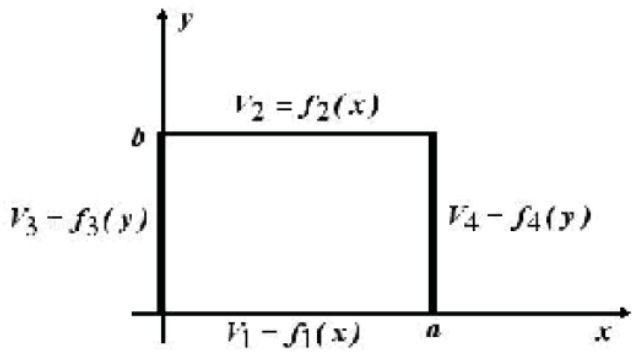


Fig. 1: Outlook of 2D plate with infinite length

$$\nabla \times E = 0 \tag{2-2}$$

Where ∇ is Del operator, D is electric field density, ρ is free volume charge density and E is electric field. Nature of non-eddy E as shown in (2-2) makes we have:

$$E = -\nabla V \tag{2-3}$$

At isotropic environment $D = \varepsilon E$, so (2-1) introduced as:

$$\nabla . \varepsilon E = \rho \tag{2-4}$$

Where$e$ is environment permittivity factor. Replacing (3-2) in (3-3):

$$\nabla . (\varepsilon \nabla V) = -\rho \tag{2-5}$$

In homogeneous environment ε is constant; so (2-5) change to:

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \tag{2-6}$$

Where∇^2 is laplacian operator; (2-6) is well known as Poisson's equation that is partial differential equation. In Cartesian coordinates:

$$\nabla^2 V = \nabla . \nabla = \frac{\partial^2 V}{\partial^2 x} + \frac{\partial^2 V}{\partial^2 y} + \frac{\partial^2 V}{\partial^2 z} \tag{2-7}$$

Finally replacing (2-7) with (2-6):

$$\frac{\partial^2 V}{\partial^2 x} + \frac{\partial^2 V}{\partial^2 x} + \frac{\partial^2 V}{\partial^2 x} = -\frac{\rho}{\varepsilon} \tag{2-8}$$

Solution of Poisson's equation usually is not easy. In environment with no free charge ρ =0, (2-8) can be simplified as:

$$\nabla^2 V = \frac{\partial^2 V}{\partial^2 x} + \frac{\partial^2 V}{\partial^2 y} + \frac{\partial^2 V}{\partial^2 z} = 0 \tag{2-9}$$

So Presence of powerful analytical approach to solve this problem seems to be critical.

Basic Idea of Variational Iteration Method:

To clarify the basic ideas of He's VIM, we consider the following differential equation:

$$L(U) + N(U) = g(x,t) \tag{3-1}$$

Where L is a linear operator, N a nonlinear operator and $g\ (t\ )$ an inhomogeneous term. According to VIM, we can write down a correction functional as follows:

$$U_{n+1}(x,t) = U_n(x,t) + \int_0^t \lambda(L(U_n)(\xi) + N(\tilde{U}_n)(\xi))d\xi, \quad n \geq 0 \tag{3-2}$$

Where λ is a general Lagrangian multiplier (Abdul-Majid, Mehdi Tatari, He, 1999; 1998; 2000)], which can be identified optimally via the variational theory (He, 2000). The subscript n indicates the nth approximation, and $\tilde{U}_n$ is considered as a restricted variation (He, 2000), i.e.

$\delta\tilde{U}_n$ % = 0. Therefore, we first determine the Lagrange multiplier l that will be identified optimally via integration by parts. The successive approximations U_{n+1} (x,t); $n \geq 0$ of the solution $U(x,t)$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function U_0. The zeroth approximation U_0 may be selected by any function that justifies at least two of the prescribed boundary conditions. With l determined, then several approximations U_n (x,t); $n \geq 0$ follow immediately. Consequently,

the exact solution may be obtained by using $U = \lim_{n\to\infty} U_n$

4. Basic Idea of Homotopy Perturbation Method:

To illustrate the basic ideas of the method, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, r \in \Omega \tag{4-1}$$

With the boundary conditions:

$$B\left(u, \frac{\partial u}{\partial u}\right) = 0, r \in \Gamma \tag{4-2}$$

Where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and G is the boundary of the domainΩ. A Can be divided into two parts which are L and$N$, where L is linear and N is nonlinear. Eq. *(4-1)* can therefore be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, r \in \Omega \tag{4-3}$$

If the nonlinear equation (3-1) has no "small parameters", we can construct the following homotopy:

$$H(v,p) = L(v) + pL(u_0) + p[N(v) - f(r)] = 0 \tag{4-4}$$

Where

$$v(r,p); \Omega \times [0.1] \to 1 \tag{4-5}$$

R and p is called the homotopy parameter. p ε [0, 1] is an embedding parameter and U_0 is the first approximation that satisfies the boundary conditions. According to HPM, the approximation solution of Eq. (4-4) can be expressed as a power series of p-terms:

$$v = v_0 + pv_1 + p^2 v_2 + ... \tag{4-6}$$

$$u = \lim_{P\to 1} = v_0 + v_1 + v_2 + ... \tag{4-7}$$

Substituting Eq. (4-4) into Eq. (4-1) results in Eq. (4-6) which is the approximate solution of Eq. (4-1). The Eq. (4-7) is convergent for most cases, and also the rate of convergent depends on $L(v)$ (He, 2000).

5. Applications:

In order to illustrate the method discussed above, we apply HPM and VIM to estimate potential in plate which is infinite in Z axis. It means that in Eq. (2-8):

$$\frac{\partial^2 V}{\partial^2 z} \to 0$$

There is no free charge in space, ρ !0. The Eq. (2-8) simplified as:

$$\frac{\partial^2 V}{\partial^2 x}+\frac{\partial^2 V}{\partial^2 y}=0 \tag{5-1}$$

V_1, V_2, V_3 and V_4 Are defined as initial boundary condition as it shown in figure (1).

Case study For HPM:

Boundary conditions of Laplace equation in this example are:

$$V(0,y)=0 \tag{5-2}$$
$$V(\pi,y)=\sinh\pi.\cos y$$
$$V(x,0)=\sinh x$$
$$V(x,\pi)=-\sin x$$

Substituting Eq. (5-1) into Eq. (4-4) in this case, we have:

$$H(x,y,p)=(1-P)[\frac{\partial^2 V(x,y)}{\partial x^2}]+P[\frac{\partial^2 V(x,y)}{\partial x^2}+[\frac{\partial^2 V(x,y)}{\partial y^2}]=0 \tag{5-3}$$

We Consider *V(x, y)* as:

$$V(x,y)=V_0(x,y)+PV_1(x,y)+P^2V_2(x,y)+..... \tag{5-4}$$

Substituting Eq. (5-4) into Eq. (5-3) and rearranging based on power of *p*-terms, we have:

$$P^0:\frac{\partial^2 V_0(x,y)}{\partial x^2}=0 \tag{5-5}$$

$$V_0(0,y)=0$$
$$V_0(\pi,y)=\sinh\pi.\cos y$$
$$V_0(x,y)=\sinh x$$
$$V_0(x,\pi)=-\sinh x$$

$$P^1:\frac{\partial^2 V_1(x,y)}{\partial x^2}+\frac{\partial^2 V_0(x,y)}{\partial x^2}=0 \tag{5-6}$$

$$V_1(0,y)=0$$
$$V_1(\pi,y)=0$$
$$V_1(x,y)=0$$
$$V_1(x,\pi)=0$$

$$P^2:\frac{\partial^2 V_2(x,y)}{\partial x^2}+\frac{\partial^2 V_1(x,y)}{\partial x^2}=0 \tag{5-7}$$

$$V_2(0,y)=0$$
$$V_2(\pi,y)=0$$
$$V_2(x,y)=0$$
$$V_2(x,\pi)=0$$

$$P^3:\frac{\partial^2 V_3(x,y)}{\partial x^2}+\frac{\partial^2 V_2(x,y)}{\partial x^2}=0 \tag{5-8}$$

$$V_3(0,y)=0$$
$$V_3(\pi,y)=0$$
$$V_3(x,y)=0$$
$$V_3(x,\pi)=0$$

Solving Eq. (5-5) with related boundary condition, we will obtain:

$$V_0(x,y)= x \cos y \tag{5-9}$$

The zeroth approximation $V_0\ (x,y)$ satisfies three boundary conditions when considering sinh x ≅ x. Solving Eqs. (5-6) – (5-8) we have:

$$V_1(x,y)=\frac{1}{3!}x^3\cos y \tag{5-10}$$

$$V_2(x,y)=\frac{1}{5!}x^5\cos y$$

$$V_3(x,y)=\frac{1}{7!}x^7\cos y$$

$$V(x,y)=\cos y(x+\frac{1}{3!}x^3+\frac{1}{5!}x^5+\frac{1}{7!}x^7+.....) \tag{5-11}$$

Therefore, the exact solution of $V(x,\ y)$ in closed form is:

$$V(x,y)= \sinh x.\cos y(5\text{-}12) \tag{5-12}$$

Case study For VIM:

Boundary conditions of Laplace equation in this example are:

$$V(0,y)=0 \qquad y<\pi \tag{5-13}$$
$$V(\pi,y)=\sinh\pi.\cos y$$
$$V(x,o)=\sinh x \quad x>0$$
$$V(x,\pi)=-\sinh x$$

The correction functional for this equation reads

$$V_{n-1}(x,y)= \tag{5-14}$$

$$V_n(x,y)+\int_0^x \lambda(\xi)\left(\frac{\partial^2 V_n(\xi y)}{\partial\xi^2}+\frac{\partial^2 V_n(\xi y)}{\partial\xi^2}\right)d\xi$$

This in turn gives

$$\lambda=\xi\text{-}x \tag{5-15}$$

Substituting this value of the Lagrange multiplier into the functional Eq. (5-14) gives the iteration formula:

$$V_{n-2}(x,y)= \tag{5-16}$$

$$V_n(x,y)+\int_0^x \lambda(\xi)\left(\frac{\partial^2 V_n(\xi y)}{\partial\xi^2}+\frac{\partial^2 V_n(\xi y)}{\partial\xi^2}\right)d\xi$$

Considering the given boundary conditions, it is clear that the solution contains cis y in addition to other functions that depend on *x*. Therefore, we can select $V_0(x,y)$ = xcos y approximation $V_0(x,y)$ satisfies three boundary conditions when considering sinh x ≈ x selection into Eq. (5-16) we obtain the following successive approximations

$$V_0(x,y) = x\cos y, \tag{5-17}$$

$$V_1(x,y) = \left(x + \frac{1}{3!}x^3\right)\cos y,$$

$$V_2(x,y) = \left(x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5\right)\cos y,$$

$$V_3(x,y) = \left(x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7\right)\cos y,$$

$$V_4(x,y) = \left(x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7 + \ldots\right)\cos y,$$

This gives the exact solution by

$$V(x,y) = \sinh x \cos y \tag{5-18}$$

Obtained upon using the Taylor expansion for sinh x . The Taylor expansion for sinh x.

6. Simulation by 2D FEM:

In order to analyze the electric potential, 2D Finite Element method was used for the purpose of the FEM to get the overall figure of the electric potential amounts in various points of the 2D plate with infinite length, to compare the electric potential obtained from FEM and analytic method (VIM). It should be mentioned that in this article, magneto-static analysis of PDE TOOLBOX software of MATLAB7 was used for programming and simulation of finite element method (FEM) and all limits were considered.

6.1 Finite Element Method:

The FEM is a numerical technique for obtaining approximation solution to boundary value problems of mathematical physics. Especially it has become a very important tool solve electrostatic problems because of its ability to model geometrically and compositionally complex problems.

The potentioal distribution which satisfies the differential equation, subject to proper boundary conditions, will also minimize the stored energy in the field and vice versa. Therefore one practical approach for solving the field problem is to approxmite and minimizes the stored energyin the field. To construct and approximate solution by finite element method, the complicated field region is discretized into a number of uniform or ununiform finite elements that are connected via nods. The potential within each element is approximated by an interpolation function. Thereafter the potential distribution in the variouse elements is interring element boundaries. The total energy is the sum of the individual element energies. Then, the total stored energy is minimized. The result of this minimization can be reformulated into a matrix equation:

$$\boldsymbol{S.A = N} \tag{6-1}$$

In this equation, S is the complex global matrix whose coefficients are functions of the geometry of region considered, material properties, boundary conditions and angular frequancies. N is the current vector. The nonlinear matrix equation can be interatively solved to get the potential distribution in the field. Using FEM to solve problems involves three stages. The consists of meshing the problem space into contiguous elements of the suitable geometry and assigning appropriate values of the material parameter – conductivity, permeability and permittivity – to each element. Secondary, the model has to be excited, so that the initial conditions are set up. Finally, the values of the potentials are suitably constrained at the limits of the problem space. The finite element method has the advantage of geometrical flexibility. It is possible to include a greater density of elements in regions where fields and geometry vary rapidly.

6.2 Simulation of Case study 2 using FEM:

Calculation of the electric potential distribution in the mentioned Case study is presented in this Section. The three-dimensional figure of the electric potential distribution for our defined models in Case study is obtained, is shown in bellow:

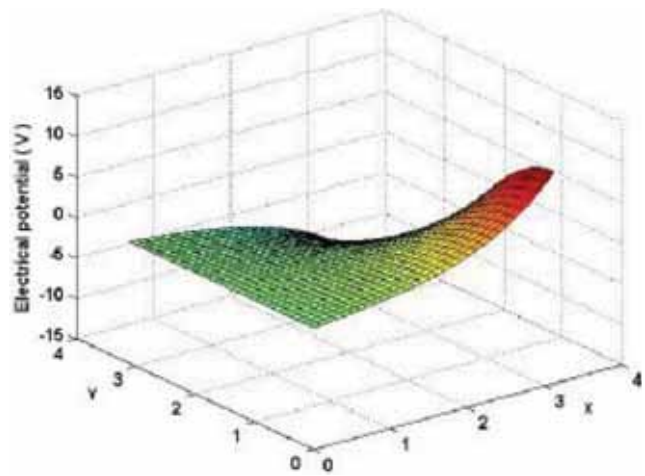


Fig. 2: Three-dimensional of the electric Potential in Case study using VIM.

Fig.2 shows 2D plot comparison between the FEM results and VIM analysis of electric potential distribution over x axis in $y = \frac{b}{8}$.

The HPM model dimensions of Case study are $a = b = \pi$. The three-dimensional figure of the electric Potential distribution for our defined model in Case study is obtained, as shown in Fig.3.

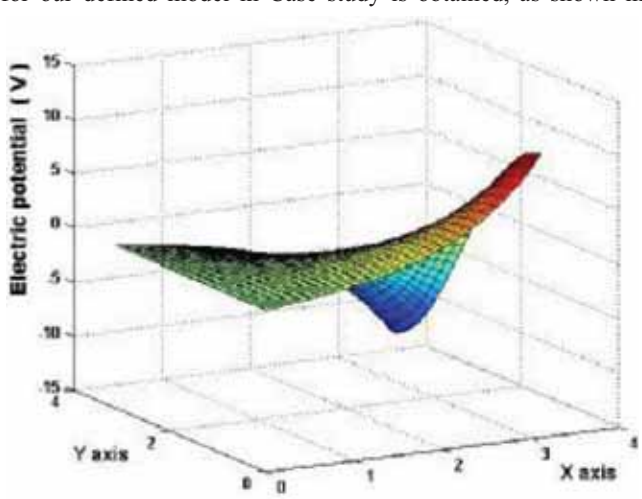
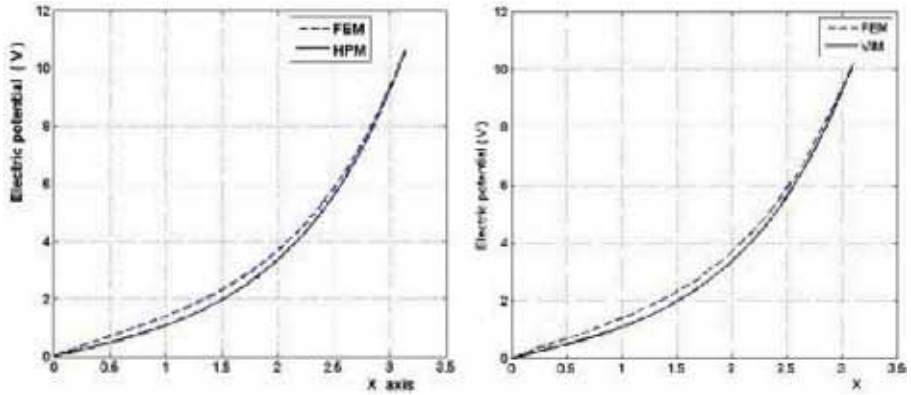


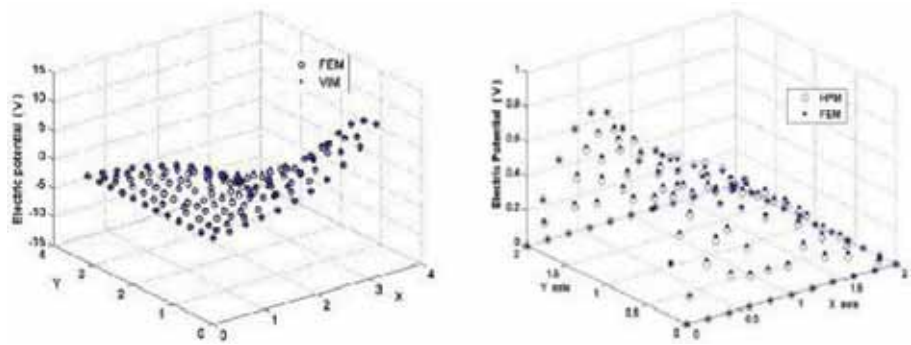
Fig. 3: Three-dimensional of the electric Potential in Case study using HPM.

The Fig.4 shows 2D plot comparison between the FEM results and HPM and VIM analysis of electric potential distribution over x axis in $y = \frac{b}{8}$. As it shows the results in this case study are same for both methods.



(a) (b)

Fig. 4: Comparison between the FEM with (a) HPM and (b) VIM results.



(a) (b)

Fig. 5: Comparison between the FEM and (a) VIM and (b) HPM in Case study.

Fig. 5:shows comparison three-dimensional of the electric potential distribution between the FEM and VIM over in Case study.

Conclusions:

In this paper, the authors have studied electric potential in 2D plate with infinite length through variational iteration method (VIM) and homotopy perturbation method (HPM) do not require small parameters, whereas the perturbation technique does. The results show that The VIM can give much better analytical approximations for electric potential equations than other methods. This is mainly because this technique is based on a general weighted residual method. The weighted factor or the general Lagrange multiplier l can be determined using the variational theory; the exacter the Lagrangian multiplier is derived, the more rapid the convergence to the exact solutions. The comparison of these methods reveals that the approximations obtained by the VIM converge to the exact solution faster than HPM. In VIM and HPM, the initial approximation can be arbitrarily chosen with unknown constants which can be defined through different methods. The calculations in VIM and HPM are simple and straightforward. In the two methods, the approximations are valid not only for small parameters but also for larger ones.

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