

## Improved Performance of Multistage Lattice Vector Quantization with Hybrid Lattice Structure

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**Abstract:** Lattice vector quantization (LVQ) reduces computational load and design complexity due to its regular structure. In this letter, we introduce and analyze the performance of two hybrid combinations of two lattices i.e. the  $A_n A_n^*$  and  $A_n D_n^*$ . Experiment results show that multistage LVQ with lattice  $A_n A_n^*$  combination in four dimensional vector offers the least quantization errors with  $p = 0.0098$  as compared to other single stage lattices or combination of lattices.

**Key words:** LVQ, Quantization error, Hybrid

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### INTRODUCTION

Vector quantization technique has been employed in applications related to multimedia communications (Gersho, A. and R.M. Gray, 1992). There have been many research efforts done regarding lattice vector quantization (LVQ) as presented in (Gersho, A. and R.M. Gray, 1992; Conway, J.H. and N.J. Sloane, 1999; Agrell, E., 2002). The main reason for the choice of this technique is due to its lower computational load and design complexity.

Servetto *et al.* (1999) uses multistage LVQ where a sublattice of order one is formed within the main lattice. This produces two descriptions of source data for transmission in wireless channel. The multistage LVQ has been also successfully employed in wavelet based image compression scheme as presented in (Bai, H., 2007; Mukherjee, D. and S.K. Mitra, 2002). The work in (Mukherjee, D. and S.K. Mitra, 2002) presents a successive refinement uniform Voronoi lattice VQ (VLVQ) algorithm using two stages  $A_2$  lattice. Meanwhile in (Salleh, M.F.M. and J. Soraghan, 2007), the application of multistage LVQ (MLVQ) technique in video coding is presented.

A thorough performance analysis of single lattice has been well documented in (Conway, J.H. and N.J. Sloane, 1999) where Conway and Sloane have compared the performances of different lattices with several dimensions. Other works that compare the performance between lattices are presented in (Gao, Z., 1995; Postol, M.S., 2002). In Gao *et al.* (1995) compare the performance of specific lattices,  $Z$ ,  $E_8$ , and leech lattice for Quantization of the transformed image in the form of generalized Gaussian distribution. The work in (Postol, M.S., 2002) compare the performance of  $E_8$  and Leech lattice for motion estimation algorithm in video coding that reduce decoder computational complexity.

All of those lattice quantizer performance analysis in (Conway, J.H. and N.J. Sloane, 1999; Gao, Z., 1995; Postol, M.S., 2002) are based on single stage lattice. In this paper, the hybrid multistage LVQ (HMLVQ) technique that combines two types of lattices i.e. the  $(A_4$  and  $A_4^*)$  and  $(A_4$  and  $D_4^*)$  are proposed. Then, the performances of these quantizers are evaluated. Simulation results show that performance of HMLVQ with  $A_4 A_4^*$  combination outperforms others in term of Quantization errors ( $p$ ) for various dimensions ( $n = 2, 3, 4$  and  $E_8$ ).

#### **Multistage Lattice Vector Quantization (MLVQ):**

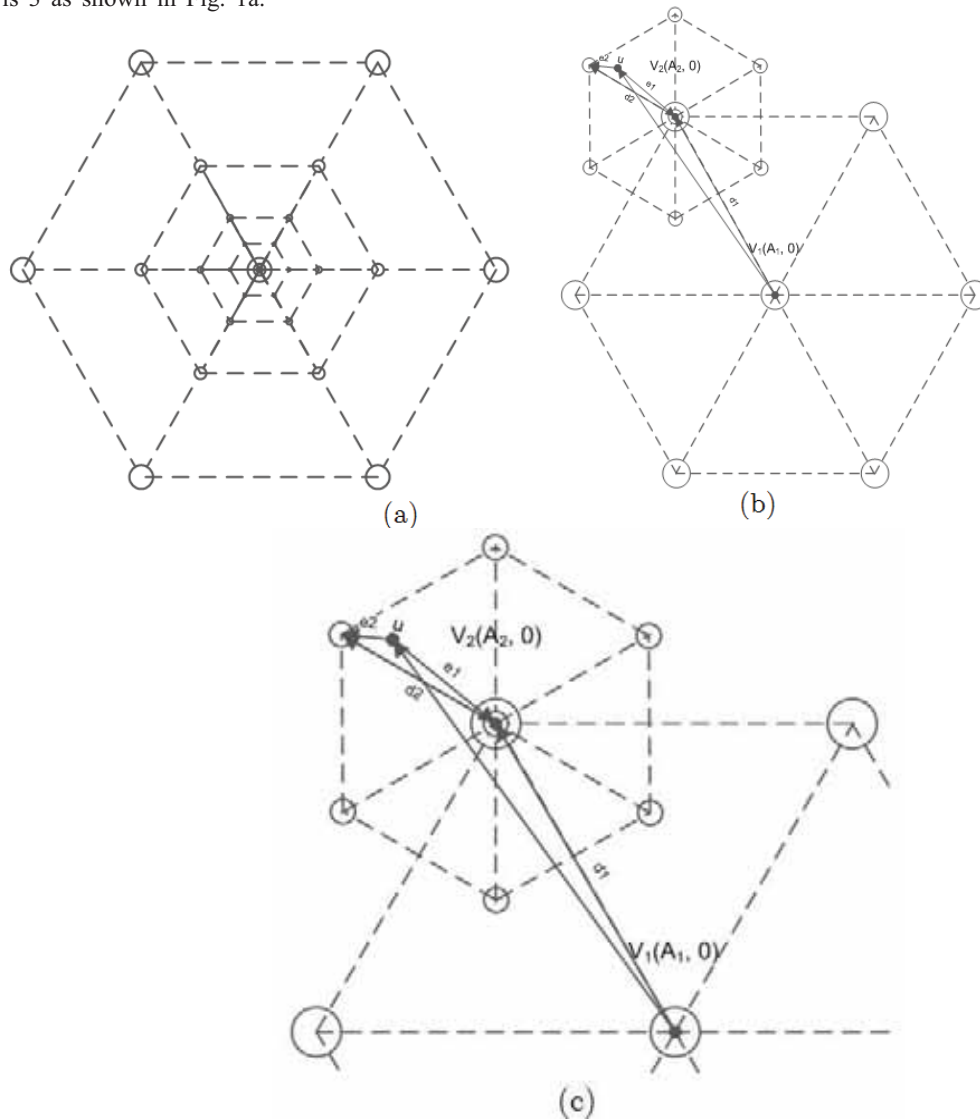
The MLVQ technique consists of several lattices, where the size of the inner lattice is reduced by a certain scaling factor. Fig. 1 depicts a simple example of multistage LVQ in 2 dimensions lattice  $A_n$ . As explained in (Gao, Z., 1995), the essence of multistage LVQ is to generate a series of decreasing scale of zero-centered Voronoi lattice regions with maximum of three sublattice  $A_n$  as shown in Fig. 1a. Unlike in (Gao, Z., 1995), in this work we generate an  $m$ -order or stages of sublattices. The technique introduces the veronoi regions of

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having  $A_n$  lattice as indicated by  $V_1(A_1,0)$ ,  $V_2(A_2,0)$ , ...,  $V_n(A_n,0)$  where  $V_1(A_1,0)$  as the main lattice. Each sublattice has a zero-centered voronoi region that has smaller scale from the main lattice.

The sublattices are arranged in such a way that no overlapping can be occurred as shown in Fig. 1. The scale down factor defines the size of sublattice given as  $1/2^m$  of the main lattice. The order of sublattice or stage is indicated by the value of  $m$ . For example, the scale down factor is  $1/8$  ( $r = 2^3 = 8$ ) and the order or stage is 3 as shown in Fig. 1a.



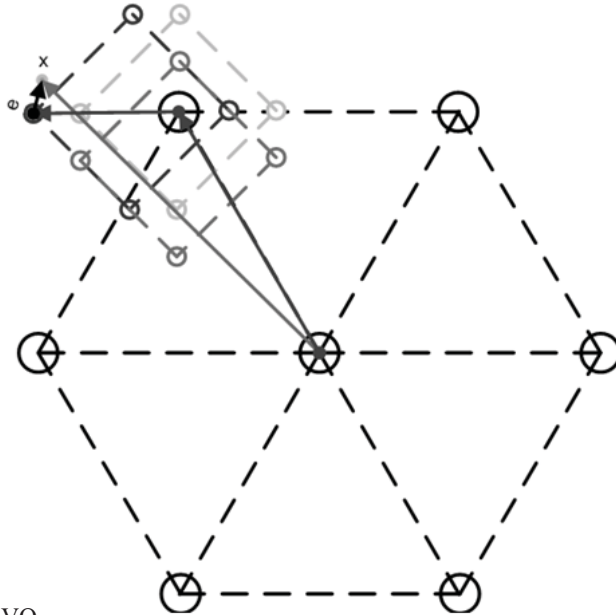
**Fig. 1:** 2-dimension Multistage Lattice Vector Quantization, a) 3<sup>th</sup> order of MLVQ scaling the points until 8 times, b) 1<sup>st</sup> order of MLVQ and vector operation c) Sublattice of 1<sup>st</sup> order of MLVQ.

The process of finding the closest point in multistage LVQ can be seen in Fig. 1b. First, the vector  $u$  is quantized by  $V_1(A_1,0)$  lattice with radius  $d_1$  (top left corner of lattice  $V_1$ ). The Quantization error  $e_1$  is given by  $e_1 = u - d_1$ . After that, the zero-centered lattice  $V_2(A_2,0)$  is constructed as shown in Fig.1c. The size of  $V_2(A_2,0)$  is reduced by  $1/2$  of lattice  $V_1$  and the center coordinate of  $V_2$  is subtracted by radius  $d_1$  and becomes the origin of  $V_2$  lattice.

Next, vector  $u$  is quantized using  $V_2(A_2,0)$  sublattice. This results vector  $u$  to be rounded to the point at the top left corner of sublattice  $V_2(A_2,0)$ . The Quantization error  $e_2$  is given by  $e_2 = d_2 - e_1$ . Thus, the quantization error  $e_2$  is smaller than  $e_1$  due to smaller size of sublattice as shown in Fig. 1c.

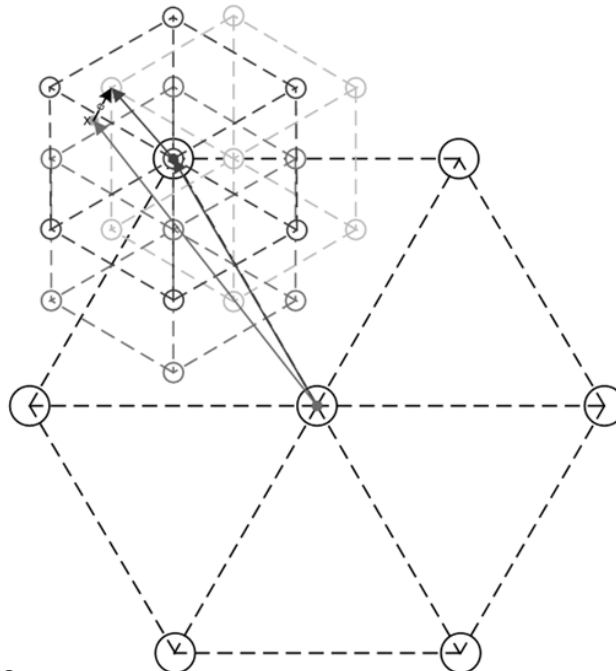
**Hybrid Multistage Lattice Vector Quantization:**

The idea of hybrid multistage lattice vector Quantization is to combine two different lattices to form a multistage process. The propose method uses the first lattice as  $A$ -type and the sublattice or the next stage lattice as different type with higher density.



**Fig. 2:** Hybrid  $A_2D_2^*$ MLVQ

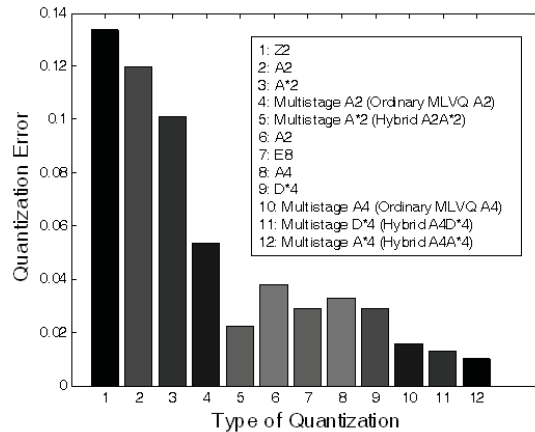
Fig. 2 shows an example of Hybrid MLVQ using  $A_2$  as the main lattice and  $D_2^*$  as the sublattice. First, the data or vector  $x$  is quantized by  $A_2$ . Then,  $x$  is quantized again using  $D_2^*$ . The process is exactly the same as the MLVQ process as explained in section 3. The difference is that the sublattice used here is denser than main lattice. The other combination is using  $A_2$  as the main lattice and  $A_2^*$  as the sublattice as shown in Fig. 3.



**Fig. 3:** Hybrid  $A_2A_2^*$ MLVQ

## RESULTS AND DISCUSSION

The Gaussian distribution source is used in simulation to obtain the performance of various lattice combinations. Fig. 4 shows the overall performances of several lattices with dimension 2, 3, 4 and 8 (for  $E$  type lattice). From that Figure, the performance of vector quantizer improves as the dimension of lattice increased. Secondly, comparing the performance of  $A_2A_2$  multistage LVQ with  $A_2A_2^*$  quantizer, the former offers better performance. This is because the density of the subsequent lattice is higher than first lattice. Comparing the performances of hybrid combinations using 4 dimensions lattice reveals that the combination of  $A_4A_4^*$  offers the best performance.



**Fig. 4:** Comparison of LVQ with several types of lattices.

### Conclusion:

This paper proposes various hybrid lattice combinations for multistage quantizer. The performance comparison of various single lattice quantizer as well as with multistage lattice quantizers has been conducted. The best performer is given by the hybrid combination of  $A_4A_4^*$ .

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