Portfolio Selection Using Fuzzy Mean-Variance Approach

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Abstract: Portfolio selection issue continuously gaining an interest among scholars. The portfolio’s ability to maximize the diversification benefit for investors becomes main focus of portfolio management. Portfolio selection model ability to solve uncertainty issue in investing is a key in deriving a new and robust model. Previous studies show that there are many factors that influence the portfolio’s diversification benefit. In this paper, we utilized a fuzzy approach to innovate the expected asset’s return variable in the mean-variance model. The model was derived by adopting the idea in Markowitz[15] (mean-variance model) and Vercher, Bermudez & Segura [24] (VBS fuzzy model). The fuzzy mean-variance model was used to construct 139 efficient portfolios based on the large and small market capitalization. The portfolios performances were examined using efficient frontier index (EFI) in the whole period of study, in rising, sideway and falling stock market trends. As a result, empirical evidence of the model revealed that the new model able to maximize portfolio’s diversification benefit especially in the falling market trend outperformed the conventional mean-variance and the VBS fuzzy model.

Key words: Portfolio, fuzzy approach, mean-variance model, efficient frontier index.

INTRODUCTION

Fluctuation in stock market is unpredictable and it is random in nature. Due to this behavior investors need to be very cautious in monitoring stock market movement. The past Asean economic crisis in 1997-1998 and the recent sub-prime problem in the USA and Europe in 2008-2009 have caused great loss to the public investors. One of the strategies to overcome the uncertainty in investment is by investing in form of portfolio. By having a right combination of asset and correct asset allocation, investors can diversify away the element of unsystematic risk in the investment. Therefore, unit trust investments become one of good alternatives for investment due to satisfied diversification offered.

Unfortunately, many of previous studies have shown that the unit trust performance is not as good as expected. Many of them are unable to outperform market benchmarks. Fauziah & Mansor(2007) in their study in 1998-2006 found that, generally the Malaysian unit trust performance is underperformed the market benchmark. Studies in other countries also show the same trends. Investigation by Sharpe, (1966) found that in the USA market, only 32% of the mutual funds outperformed the DJIA, he also concluded that the past performance of the funds was not the best predictor of future performance. Other finding by Jensen [10] has strengthened the result on the funds’ performance over time when he concluded that after taken into consideration the operating expenses of a mutual fund, on average the mutual funds could not beat a buy-and-hold strategy. As a result, the portfolio selection strategy and model should be improved further. Therefore, fund managers and public investors are really needed for a robust model that is able to overcome the uncertainty in investing and maximize the portfolio’s diversification benefit.

Previous literatures show that the fuzzy approach is an alternative tool to model the uncertainty data. The approach has been widely applied in engineering, computing, biology engineering and management sciences. Studies by Zhang et. al. (2003), Wang et al. (2005), Bilbao-Terol et al. (2006), Vercher et al. (2007), Lin & Liu, (2008) and Li & Xu, (2009) show that the fuzzy approach also applicable in portfolio selection.

Researchers had introduced and discussed various types of portfolio selection models and approach, each of them has its own advantages and weaknesses. Therefore, the paper’s objective is to examine the portfolio diversification benefit when the asset’s expected return was measured by using a fuzzy approach. The finding was supported by empirical evidence.
Literature Review:

Portfolio selection issue continuously gaining an interest among scholars. Markowitz\cite{15} has initiated significant contribution to the finance body of knowledge when he introduced the mean-variance model which is become foundation to the modern portfolio theory (MPT). Markowitz idea on the mean-variance approach then being expended by Sharpe, (1966), Mossin, (1966) and Lintner (1965). The modern portfolio theory then evolved to Capital Asset Pricing Theory when risk free rate asset was included into the portfolio and then evolved to Arbitrage Pricing Theory.

General objectives of portfolio management are to diversify away the investment diversifiable risk and to maximize the return. By having the right combination of assets, these objectives can be achieved. Markowitz’s mean-variance model has incorporated the asset variance and co-variance factors as main contributors to the portfolio risk. Variance measures the volatility of asset return form the average of rate of return for both negative and positive return. By using Markowitz’s model, the portfolio variance can be minimized by having weak or negative assets correlation in the portfolio. Since then, the model is well accepted by investors and fund managers that aimed to construct an efficient portfolio with the highest diversification benefit.

Portfolio diversification is influenced by many factors that govern the portfolio selection criteria such as the firm sizes, financial ratios, stock markets and investor’s judgment. Reinganum, (1981) has conducted a study on abnormal return in small firm portfolio in the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX). He had ranked the firm’s market value and divided it into 10 equally weighted portfolios. The risk-adjusted returns for extended periods of 10 to 15 years have indicated that the small firms consistently superior than the larger firm. He has claimed that the firm size is more dominant than PE ratio in influencing the portfolio performance as reported by Basu, (1977). Subsequently, Basu, (1977) reexamined Reinganum’s works for different study period and different portfolio construction methods and found that the small and low PE ratio portfolios have highest risk-adjusted returns. In Malaysian case, Sazali et al., (2004) evidently showed that for long term, the Malaysian domestic-small firm’s portfolio provided the highest diversification benefit compared to other portfolio classification such as domestic-large firms, international-developed and developing countries portfolio. The results suggested that in the long term, there are smaller stocks on the Bursa Malaysia which are correlated at the low values with each other as compared to assets of international portfolios or a portfolio of larger stocks on the exchange.

Besides the assessment of portfolio’s efficiency, diversification also can be achieved by having appropriate number of asset. According to Tang, (2004) portfolio diversification also can be achieved by having sufficient number of assets in the portfolio. Previous studies show that the numbers of required asset are varied. It ranged from 10 to 40 assets. Statman, (1987) and Evans & Archer, (1968) have proposed that the appropriate numbers of assets in a portfolio are between 10 to 15 and less than 40 respectively. Additionally, finding by Solnik, (2007) showed that the asset number is around 20 assets for the US stocks and international portfolios. In Malaysian stock market, Zulkifli et al. (2008) revealed that 15 stocks are sufficient to diversify away the diversifiable risk in the stock market.

Solnik\cite{21} noted that international diversification is more dominant than inter-industry diversification. To encounter this view, Cavaglia, (2000) had investigated the importance of industry diversification beside of inter country diversification. With data from 21 developed equity markets and various industries covered the period December 1985 through November 1999, they presented evidence that industry factors have been growing in relative importance and may now dominate country factors. Furthermore, their evidence suggests that, diversification across global industries has provided greater risk reduction than diversification by countries. They concluded that industry allocation is an increasingly important consideration for active managers of global equity portfolios and those investors may wish to reconsider home-biased equity allocation policies.

In the context of globalization, international markets have become more open, leading to a common perception that global capital markets have become more integrated. This integration resulting higher correlation would imply about the diversification potential across countries. Therefore, international diversification becomes more common to investors. Previous studies show that there are diversification benefits in international markets as well as in domestic market. Solnik, (1974), Santis & Gerard, (1997), Lewis, (2006), Driessen & Laeven, (2007) confirmed this matter.

In conclusion, asset selection and asset allocation are very important in constructing a portfolio. Regardless either the portfolio is having different asset criteria, market or industry. Therefore a robust model is needed to construct a robust portfolio which is able to maximize the portfolio diversification benefit in all market situations.
Methods:
Fuzzy portfolio model derivation can be based on so many factors depends on the scope of the study. For example, Vercher et al. (2007) has defined asset return data as a fuzzy number, otherwise Bilbao[4] have defined asset beta value as a fuzzy number and Fatma & Mehmet, (2005) had used asset financial ratios data as a fuzzy number that being used in analysis. All the approaches have it own strength and weaknesses.

The paper focuses on the portfolio selections based on three different models that are mean-variance model by Markowitz, (1952), the fuzzy mean-semi-variance model by Vercher et al., (2007) and the newly innovated fuzzy mean-variance model.

1. Mean-Variance (MV) Model:
Initial model on portfolio selection was introduced by Markowitz, (1952). In his model, Markowitz has incorporated the covariance into the model which explained that the pairs of asset correlation are very important in determining the portfolio risk. The Markowitz MV model assumes that the asset return is normally distributed and investors are trying to maximize their return and minimize the risk as they are risk averse. The MV model is presented as below.

\[
\text{Minimize} : \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \text{Cov}_{i,j} \\
\text{Subject to} : R_p \geq \sum_{i=1}^{n} w_i r_i \\
\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0
\]

where \( R_p \) is a portfolio return. \\
\( \sigma_p \) is a portfolio risk. \\
\( w_i \) is investment weighted in each asset \( i \).
\( r_i \) is asset \( i \) rate of return. \\
\( \text{Cov}_{i,j} \) is a co-variance between asset \( i \) and \( j \). \\
\( n \) is a number of asset in the portfolio.

2. Fuzzy Semi-variance Model:
Vercher, (2007) (VBS fuzzy model) has introduced a new version of the portfolio selection model when they replace the variance with semi-variance as a risk measure. The asset return is defined as a fuzzy number since it changes is unpredictable and uncertain in nature. The VBS fuzzy model then was transformed into fuzzy number formation. This model based on semi-variance risk measure and has fulfilled all the fuzzy properties, lemma and propositions. The utilization of semi-variance as a risk measure in fuzzy number environment has created a new dimension for investor since it is able to represent investor’s real situation in investing. The VBS fuzzy model is as below.

\[
\text{Min} \quad \sigma_{FP}^2 = \sum_{i=1}^{n} \left( a_{u,i} - a_{l,i} + \frac{1}{2} \left( c_i - d_i \right) \right) w_i \\
\text{Subject to} : \text{Max} \quad R_{FP} = \sum_{i=1}^{n} \left[ \left( \frac{1}{2} \right) \left( a_{u,i} + a_{l,i} \right) + \left( \frac{1}{4} \right) \left( d_i - c_i \right) \right] w_i \\
\sum_{i=1}^{n} w_i = 1, \quad \forall w_i \geq 0.
\]
where \( a_{i,60} \) is asset \( i \)th return at 60th percentile.
\( a_{i,40} \) is asset \( i \)th return at 40th percentile.
\( c_i \) is asset \( i \)th return spread between 40th percentile and 5th percentile.
\( d_i \) is asset \( i \)th return spread between 95th percentile and 60th percentile.
\( w_i \) is an investment weight imposed in asset \( i \)th.
\( \sigma_p \) is a fuzzy portfolio risk.
\( R_{fp} \) is a fuzzy portfolio return.
\( n \) is number of asset in the fuzzy portfolio.

In this model, asset return is defined as a fuzzy number. Therefore, the expected return variable is derived based on the percentile of the asset return in the data distribution. Adopting this approach, the expected asset return is determined based on the actual data distribution and able to solve part of the normality problem in the conventional mean-variance model.

3. Fuzzy Mean-Variance Model:
Mean-variance (MV) model is a well-accepted method and has been applied in industries due to its simplicity and practicality. In our model, we adopted the expected fuzzy return in the Vercher et al.’s model into mean-variance model. This implementation enables us to solve part of the normality problem in the mean-variance model. In this model, the importance of asset co-variance in portfolio selection is maintained. The innovated MV model is given as

\[
\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \text{Cov} \rho_{i,j}
\]

Subject to: \( R_p \geq \sum_{i=1}^{n} w_i \left[ \frac{1}{2} \left( a_{i,60} - a_{i,40} \right) + \frac{1}{4} \left( d_i - c_i \right) \right] \)
\[
\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0
\]

where \( R_p \) is a portfolio return.
\( \sigma_p \) is a portfolio risk.
\( w_i \) is investment weighted in each asset \( i \).
\( r_i \) is asset \( i \) rate of return.
\( \text{Cov} \rho_{i,j} \) is a co-variance between asset \( i \) and \( j \).
\( n \) is a number of asset in the portfolio.
\( a_{i,60} \) is asset \( i \)th return at 60th percentile.
\( a_{i,40} \) is asset \( i \)th return at 40th percentile.
\( c_i \) is asset \( i \)th return spread between 40th percentile and 5th percentile.
\( d_i \) is asset \( i \)th return spread between 95th percentile and 60th percentile.

4. Diversification Benefit Measures:
There is several diversification benefit measures can be applied in assessing portfolio performances. Among others are Sharpe Index, Treynors Index, risk adjusted return, abnormal return and efficient frontier curve. But the most dominant measurement is the efficient frontier due to its rigorousness in providing risk and return information in portfolio analysis study. Efficient frontier is a set of feasible portfolio return and risk level that offers the highest profit at any level of risk or lowest risk at any level of return. The area under the efficient frontier curve is a potential feasible portfolio event not at the highest preferences compared to the one on the efficient frontier curve.

5. Efficient Frontier Index (EFI):
In order to identify the superiority of the model, Sazali et al. (2004) noted that the efficient frontier index (EFI) can be used as a good indicator. The EFI index will identify the superiority of the efficient frontier in maximizing a portfolio diversification benefit. The highest index value shows that the portfolio is having highest return for every unit of risk. Therefore, the higher EFI is preferable. The EFI formula is as below.
\[ EFI = \left( \sum_{i=1}^{n} \frac{R_i}{\sigma_i} \right) \left( \sum_{i=1}^{n} \frac{R_i - R_{\text{Lowest}}}{\sigma_i - \sigma_{\text{Lowest}}} \right) \] (4)

where \( EFI \) is the Efficient Frontier Index
\( R_i \) is the portfolio \( i \) return on efficient frontier
\( \sigma_i \) is the portfolio \( i \) variance on efficient frontier
\( R_{\text{Lowest}} \) is the portfolio \( i \) lowest return on efficient frontier
\( \sigma_{\text{Lowest}} \) is the portfolio \( i \) lowest variance on efficient frontier

EFI is efficient in order to identify the most ‘north-west’ portfolio which indicates the highest return at every level of risk.

6. Portfolio Optimization:

Portfolio selection problems can be solved by using quadratic, linear programming, genetic algorithm, fuzzy mathematical programming or neural network programming approaches. Previous studies have one similarity that is to construct efficient portfolios that are able to maximize portfolio return and minimize portfolio risk. They are different in optimization tools and several minor aspects.

Linear programming approach is chosen as a tool to solve the optimization problem in the entire portfolio selection models. Portfolios selection is obtained by using the Solver function in MS Excel. In the function, the entire model must be set into the system correctly one by one. The Solver function will run a simulation to seek for the solution. Once the system coalesce the objective and the constraints, it will give the appropriate asset selection, asset allocation, portfolio risk and return level. The process is repeated several times until efficient frontiers are obtained for all the situations. The portfolio performances are then presented in the form of efficient frontier curves and index (EFI).

RESULTS AND DISCUSSIONS

The effectiveness of the MV model, the VBS fuzzy model and the fuzzy MV model are tested in the selection of asset for large size and small size portfolio. The portfolio sample is chosen from the listed companies in Bursa Malaysia for the period starting from January 1998 to June 2009 based on its sizes. The large size portfolio consist of the 30 largest market capitalization companies, while the small size portfolio consist of the 30 smallest market capitalization companies in Bursa Malaysia in the period of study. Based on literature, portfolio number of asset is set at 30. The portfolios’ performance is then tested in the whole period of study, in the rising, sideways and in the falling stock market trend. The sub-periods of market trend are based on the Bursa Malaysia Composite Index historical data and is given in Table 1.

<table>
<thead>
<tr>
<th>Period</th>
<th>BM Composite index point</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>End</td>
<td>Trend</td>
</tr>
<tr>
<td>9/1/1998</td>
<td>4/1/2000</td>
<td>Rising</td>
</tr>
<tr>
<td>1/6/2004</td>
<td>1/1/2006</td>
<td>Sideway</td>
</tr>
<tr>
<td>1/11/2007</td>
<td>1/6/2009</td>
<td>Falling</td>
</tr>
</tbody>
</table>

We constructed efficient portfolios based on large market capitalization and small market capitalization. The portfolio’s efficient frontier is presented in the Figure 1 below.

![Efficient Frontier of Small Market Capitalization Portfolios via VBS Fuzzy and MV Models while Market in Rising Trend](image-url)
The curves show the efficient frontier of a small market capitalization portfolio constructed using the mean-variance model and the VBS fuzzy model. The superiority of the portfolio can be seen clearly in the curve, where the more ‘North-West’ portfolio is better because it offers higher portfolio return for every unit of portfolio risk. It obviously shows that mean-variance portfolio is more superior due to it is more ‘North-West’ than the VBS fuzzy portfolio. In many cases, the efficient frontier is overlapping each other and quite difficult to visually differentiate the superior portfolio. In this scenario, efficient frontier index (EFI) introduced by Sazali et al. (2004) can be used to identify the portfolio superiority. The higher value of EFI shows that the portfolio is more superior.

In Table 2 below, we have calculated an EFI for the portfolios in the above example. The EFI value of the small market capitalization portfolio via mean-variance model is 89.58. While the EFI for the small market capitalization portfolio via VBS fuzzy model is only 51.51. Therefore, we can conclude that the portfolio via mean-variance model is more superior.

Table 2: EFI of Small Market Value Portfolios via VBS Fuzzy and MV Models While Market in Rising Trend

<table>
<thead>
<tr>
<th>SD</th>
<th>Ri</th>
<th>A=Ri/SD</th>
<th>B=(Ri-Rl) / (Sd1-SD)</th>
<th>EFI=A.B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>MV</td>
<td>Fuzzy</td>
<td>MV</td>
<td>Fuzzy</td>
</tr>
<tr>
<td>18.39%</td>
<td>8.46%</td>
<td>0.460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.31%</td>
<td>9.00%</td>
<td>0.466</td>
<td>0.594</td>
<td></td>
</tr>
<tr>
<td>19.63%</td>
<td>10.00%</td>
<td>0.509</td>
<td>1.249</td>
<td></td>
</tr>
<tr>
<td>20.80%</td>
<td>11.00%</td>
<td>0.516</td>
<td>0.864</td>
<td></td>
</tr>
<tr>
<td>23.08%</td>
<td>12.00%</td>
<td>0.520</td>
<td>0.756</td>
<td></td>
</tr>
<tr>
<td>27.11%</td>
<td>14.00%</td>
<td>0.516</td>
<td>0.636</td>
<td></td>
</tr>
<tr>
<td>27.89%</td>
<td>13.00%</td>
<td>0.466</td>
<td>0.479</td>
<td></td>
</tr>
<tr>
<td>28.68%</td>
<td>15.00%</td>
<td>0.523</td>
<td>0.636</td>
<td></td>
</tr>
<tr>
<td>32.26%</td>
<td>15.23%</td>
<td>7.95%</td>
<td>0.472</td>
<td>0.247</td>
</tr>
<tr>
<td>32.41%</td>
<td>15.29%</td>
<td>8.50%</td>
<td>0.472</td>
<td>0.262</td>
</tr>
<tr>
<td>33.94%</td>
<td>16.00%</td>
<td>9.31%</td>
<td>0.471</td>
<td>0.274</td>
</tr>
<tr>
<td>34.31%</td>
<td>16.08%</td>
<td>9.50%</td>
<td>0.469</td>
<td>0.277</td>
</tr>
<tr>
<td>35.65%</td>
<td>16.35%</td>
<td>10.50%</td>
<td>0.459</td>
<td>0.295</td>
</tr>
<tr>
<td>38.13%</td>
<td>16.87%</td>
<td>11.00%</td>
<td>0.442</td>
<td>0.289</td>
</tr>
<tr>
<td>38.77%</td>
<td>17.00%</td>
<td>11.41%</td>
<td>0.438</td>
<td>0.294</td>
</tr>
<tr>
<td>39.69%</td>
<td>17.29%</td>
<td>12.00%</td>
<td>0.436</td>
<td>0.302</td>
</tr>
<tr>
<td>41.98%</td>
<td>18.00%</td>
<td>12.85%</td>
<td>0.429</td>
<td>0.306</td>
</tr>
<tr>
<td>42.39%</td>
<td>18.10%</td>
<td>13.00%</td>
<td>0.427</td>
<td>0.307</td>
</tr>
<tr>
<td>45.96%</td>
<td>19.00%</td>
<td>13.83%</td>
<td>0.413</td>
<td>0.301</td>
</tr>
<tr>
<td>50.97%</td>
<td>15.00%</td>
<td></td>
<td>0.294</td>
<td>0.377</td>
</tr>
<tr>
<td>54.86%</td>
<td>17.00%</td>
<td></td>
<td>0.310</td>
<td>0.400</td>
</tr>
<tr>
<td>61.73%</td>
<td>19.00%</td>
<td></td>
<td>0.308</td>
<td>0.375</td>
</tr>
<tr>
<td>65.62%</td>
<td>20.00%</td>
<td></td>
<td>0.305</td>
<td>0.361</td>
</tr>
<tr>
<td>68.54%</td>
<td>20.90%</td>
<td></td>
<td>0.305</td>
<td>0.357</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.90</td>
<td>4.68</td>
<td>10.06</td>
<td>11.02</td>
<td>89.58</td>
</tr>
</tbody>
</table>

The EFI value is very helpful especially in evaluating many and more complicated efficient frontiers. We fully utilized it’s in the following section.

We also have examined the performance of the mean-variance model, VBS fuzzy model and the fuzzy mean-variance portfolio selection model in constructing a well diversified portfolio. The models were tested in the large and small market capitalization portfolio and in different stock market trends. Figure 2 and Figure 3 below show the efficient frontier of the portfolios via the mean-variance and the VBS fuzzy model.

Fig. 2: Efficient Frontier of Large Market Value Portfolio Using MV model and VBS Fuzzy Model in Different Periods
Due to the complexity of the efficient frontier curves, efficient frontier index (EFI) for all portfolios are presented in Table 3 below. Portfolio construction for the large market capitalization sample shows that the fuzzy mean-variance model is able to derive the highest EFI for all period of analysis. In the whole period analysis, the EFI is 7.67, followed in the rising trend, the EFI is 281.79, in the sideway trend the EFI is 111.54 and in the falling trend the EFI is 0.93. This shows that the portfolio construction via fuzzy MV model is able to maximize portfolio diversification benefit for the large market capitalization sample. While the portfolio construction via mean-variance and the VBS fuzzy models give the mixed results.

Table 3: EFI of mean-variance, VBS fuzzy and fuzzy MV model in the Whole Trends

<table>
<thead>
<tr>
<th>Type of Whole Period</th>
<th>Rising Trend</th>
<th>Sideway Trend</th>
<th>Falling Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>VBS Fuzzy</td>
<td>MV</td>
<td>VBS Fuzzy</td>
</tr>
<tr>
<td>Large MV</td>
<td>1.26</td>
<td>1.76</td>
<td>229.72</td>
</tr>
<tr>
<td>Small MV</td>
<td>1.99</td>
<td>0.87</td>
<td>7.67</td>
</tr>
</tbody>
</table>

Portfolio construction for the small market capitalization sample shows that the fuzzy mean-variance model is able to derive the highest EFI for the whole period and in the falling period only. In the rising and sideway trends, it seems that the mean-variance model gives a better result. This shows that, for the small market capitalization sample, the portfolio construction via fuzzy MV model is able to maximize portfolio diversification benefit in the long term and in the falling market trend.

Conclusion:

In this paper we have examined the performance of the mean-variance model, the VBS fuzzy model and the fuzzy mean-variance model in constructing a portfolio in different market trends. The models were tested on the selection of large and small market capitalization companies sample taken from Bursa Malaysia. In deriving the newly innovated fuzzy mean-variance model, we have defined asset return as a fuzzy number due to its uncertainty, vague and volatility. The fuzzy return modeling has taken into consideration the skewness element of return data. The proposed model is able to solve part of the normality problem in the mean-variance model and at the same time is able to maximize the portfolio diversification benefit in the long term and in the falling market trends outperformed the mean-variance and the VBS fuzzy model. The result is significant because the critical investment period is during falling market trend and investors can invest in the rising and sideway market trend because most investment can perform well.

In model derivation, the fuzzification is made only to the element of the asset return. It would be interesting to investigate the effect of fuzzification to other parts of the model such as in asset covariance or in asset correlation. We expect interesting finding is waiting in deriving a robust model that is efficient to maximize portfolio diversification benefit.

REFERENCES


