Sliding Mode Control with PID Sliding Surface of an Electro-hydraulic Servo System for Position Tracking Control

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Abstract: This paper presents the position tracking performance of an electro-hydraulic hydraulic servo (EHS) system using sliding mode control (SMC) with proportional-integral-derivative (PID) sliding surface. In modelling process, a mathematical model of the EHS system is developed by considering its nonlinearities as represented by a Lu-Gre friction model. The control strategy is derived from the developed dynamics equation and stability of the control system is theoretically proven by Lyapunov theorem. Simulation results show that the proposed controller has a better tracking performance compared to conventional PID controller.

Key words: sliding mode control; electro-hydraulic servo system; position tracking control; PID sliding surface

INTRODUCTION

Electro-hydraulic servo (EHS) system has attracted a great attention as an actuator in heavy engineering applications due to its capability in providing high forces with compact design (Merrit, 1967). By utilizing the advantageous of EHS system, different applications such as aircrafts (Karpenko and Sepehri, 2009), manufacturing machines (Pluta, 2008), fatigue testing (Ruan et al., 2006) and automotive application (Sam et al., 2004) established that the actuator system can be more well-known and crucial nowadays. The sophisticated design of EHS system with the versatility of electronic and hydraulic components offers a massive improvement and performance for those applications.

However, it is well-known that an EHS system is typically a complicated system suffers from nonlinearities, uncertainties and disturbances. These inconveniences may lead to degradation of control performance in force, pressure or position tracking of EHS system. Nonlinear flow/pressure characteristics, fluid compressibility, actuator friction and internal/external leakages are identified as the major sources of nonlinearity exist in the EHS system (Jelali and Kroll, 2003). Recently, Kalyoncu and Haydim (2009) discussed the mathematical model of the EHS system by including the effect of compressibility, external leakage in actuator and internal leakage in the servo valve. Most of proportional flow valves suffer from hard nonlinearities such as dead-zone due to asymmetric overlap in the spool valve design (Knohl and Unbehauen, 2000; Bessa et al., 2010). Besides, the friction phenomena are also affect the tracking performance and often considered as a nonlinear model in developing the dynamics equation of EHS system (Mihajlov et al., 2002).

Numerous control strategies have been proposed and reported in the literature to encounter the difficulties in controlling the EHS system. The increasing numbers of works dealing with EHS system over the past decades involved a linear control, nonlinear control and intelligent control approaches such as neural network (NN) (Knohl and Unbehauen, 2000), self-tuning Fuzzy-PID (Zulfatman and Rahmat, 2009; Cetin and Akkaya, 2010), model reference adaptive control (MRAC) (Ziaei and Sepehri, 2001; Kirecci et al., 2003) and generalized predictive control (GPC) (Sepehri and Wu, 1998). It is found that, the nonlinear control strategy has been an efficient tool and extensively used for controlling the nonlinear system. In recent years, sliding mode control (SMC) method is gaining research interest in controlling the EHS system (Liu and Handroos, 1999; Chuang and Shiu, 2004; Chin et al., 2005; Bessa et al., 2010).

SMC is recognized as a one of the most potential approach in nonlinear control field and has been proved to the problem of maintaining the stability for controlling many classes of model that are subjected to...
parameter variations and external disturbances (Edwards and Spurgeon, 1998). SMC has widely implemented in many engineering applications including pneumatic systems (Surgenor and Vaughan, 1997), active suspension systems (Sam and Osman, 2005) and active magnetic bearing systems (Husain et al., 2008). Current study has been published by Eker (2010) utilizing the second-order SMC for electromechanical plant. The experimental application is an extended work presented by Eker (2006) using the proportional-integral-derivative (PID) sliding surface for second order system. In works published by Eker and Akmal (2008), the sliding surface is augmented with an integral action to improve the tracking performance compared with the conventional design.

In this paper, the position tracking control performance of EHS system is evaluated using SMC with PID sliding surface. The nonlinear dynamics model consists of servo valve and hydraulic actuator incorporating with friction model is derived. Based on the EHS model, the SMC control law is developed from the third-order system and the stability is analyzed in the sense of Lyapunov theorem. Afterward, position tracking control is performed and comparison with the conventional PID controller is presented.

**Mathematical Modeling of EHS System:**

Dynamics equation of EHS system consist of servo valve and hydraulic actuator as illustrated in Figure 1. The hydraulic actuator motion is controlled by modulating the hydraulic oil flow from the cylinder chambers using a servo valve. The mass is attached with a spring and damper that generates the counter force against the actuator.

![Electro-hydraulic servo system configuration](image)

**Fig. 1:** Electro-hydraulic servo system configuration

In the electronic design, the spool valve position in servo valve is driven by the torque motor. The electrical signal, $V$ is used to drive the current in coil as represented in (1).

$$ V = \frac{dI}{dt} L_c + R_c I $$

(1)

where $L_c$ and $R_c$ are the coil inductance and resistance respectively.

The servo valve dynamics in (2) can be considered as second-order system relates the current drive from the torque motor and the spool valve position, $x_v$.

$$ \frac{d^2 x_v}{dt^2} + 2\zeta\omega_n \frac{dx_v}{dt} + \omega_n^2 = I \omega_n^2 $$

(2)

where $\zeta$ is the servo valve damping ratio and $\omega$ is the natural frequency.

In mechanical design of servo valve, critical-centered is considered where the spool valve is unexposed from flow leakages and dead-zone problems for each port. The servo valve control the flows $Q$ in each chambers in the actuator can be models from the orifice equations relates the spool valve position $x_v$ and pressure difference $P_c$. For the ideal orifice equation:
The flow relations for each chambers are given in (4) and (5) by neglecting the internal leakages effects in servo valve (Sohl and Bobrow, 1999).

\[ Q_A = \begin{cases} K_A x_v \sqrt{P_S - P_A} & x_v \geq 0 \\ K_A x_v \sqrt{P_A - P_R} & x_v < 0 \end{cases} \]  
(4)

\[ Q_B = \begin{cases} -K_B x_v \sqrt{P_B - P_R} & x_v \geq 0 \\ -K_B x_v \sqrt{P_R - P_B} & x_v < 0 \end{cases} \]  
(5)

where the coefficient gain is assumed to be \( K = K_A = K_B \) for a symmetrical valve.

The hydraulic actuator is modeled from the dynamics of volume of each chamber as follows:

\[ V_A = V_{line} + A_p(x_s + x_p) \]  
(6)

\[ V_B = V_{line} + A_p(x_s - x_p) \]  
(7)

where \( V_{line} \) is the volume of the pipeline and the zero position is located at the center of the cylinder. From the flows and volume equations, pressure from each chamber can be determined by defining the relation between flow rate, bulk modulus, external leakages and volume rate.

\[ P_A = \frac{\beta}{V_{line} + A_p(x_s + x_p)} \int \left( Q_A - q_{wa} - q_a - \frac{dV_A}{dt} \right) dt \]  
(8)

\[ P_B = \frac{\beta}{V_{line} + A_p(x_s - x_p)} \int \left( \frac{dV_B}{dt} - q_{wb} - q_b - Q_B \right) dt \]  
(9)

The total force generated from the actuator can be expressed in (10) from the overall dynamics equation of spring, damper, moving mass and friction.

\[ F_p = A_p(P_A - P_B) = M_p \frac{d^2 x_p}{dt^2} + B_s \frac{dx_p}{dt} + K_s x_p + F_f \]  
(10)

Lu-Gre friction model is preferred for representing the friction in EHS system since it captures most of the friction behavior (Kalyoncu and Haydim, 2010; Mihajlov et al., 2002). The combination of pre-sliding characteristics of the Dahl model with the steady state friction behavior like Coulomb friction, viscous friction and the Stribeck effect in the Lu-Gre friction model can be stated as follows:

\[ F_f = \sigma_o z + \sigma_1 \frac{dz}{dt} + \alpha_2 x_p \]  
(11)

\[ \frac{dz}{dt} \] is the average of bristle deflection which can be represented as:

\[ \frac{dz}{dt} = \dot{x}_p - \frac{\sigma_0}{g(\dot{x}_p)} \]  
(12)

The Stribeck function in (13) is a function where expressing the steady-state friction behavior at a constant velocity.
The supply pressure is generated by the pumps and drive to the servo valve. Generally, EHS system is equipped with a pressure regulator to regulate the maximum pressure operating in that system. The continuity dynamics between the pump and servo valve can be written as:

$$P_S = \frac{\beta}{V_s} \int (Q_{pump} - Q_s) dt$$  \hspace{1cm} (14)

With this dynamics system model, some parameters may impossible to gather and the controller design is more complicated to design. Then, the simplified model can be used in controller design where the EHS system can be represented using a third-order perturbed linear model incorporating uncertainties and disturbances as in (15).

$$\ddot{x}_p(t) = -(A_n \pm \Delta A)\dot{x}_p(t) - (B_n \pm \Delta B)x_p(t) + (C_n \pm \Delta C)u(t) + d(t)$$  \hspace{1cm} (15)

where $d(t)$ consist of external load disturbance, nonlinear friction and leakage. $A_n$, $B_n$ and $C_n$ are the nominal system parameters. The bounded uncertainties $A$, $B$, $C$ are the uncertainties exist from the unmodeled dynamics. The third-order of the EHS system can be rearranged as:

$$\ddot{x}_p(t) = -A_n\dot{x}_p(t) - B_n\dot{x}_p(t) + C_nu(t) + L(t)$$  \hspace{1cm} (16)

where the lumped uncertainty is defined by:

$$L(t) = \pm \Delta A\dot{x}_p(t) \pm \Delta B\dot{x}_p(t) \pm \Delta C u(t) + d(t)$$  \hspace{1cm} (17)

and the upper bound of the lumped uncertainty $L_{\text{max}}$ is

$$L_{\text{max}}(t) = \Delta A|\dot{x}_p(t)| + \Delta B|\dot{x}_p(t)| + \Delta C|u(t)| + d(t)$$  \hspace{1cm} (18)

**Sliding Mode Control Design:**

SMC is a type of variable structure control (VSC) developed in the early of 60’s in Russia (Edwards and Spurgeon, 1998). It is established that the most crucial step in designing the SMC is the construction of the sliding surface which is expected to respond the desired or control specifications and performances. The states trajectories are forced to be reached and stayed on the sliding surface as depicted in Figure 2.

![Fig. 2: Phase portrait of a sliding motion in sliding mode control](image)

The PID sliding surface in the SMC design in Figure 3 can be expressed in following equation where $k_p$, $k_i$ and $k_d$ are the PID parameters (Eker, 2010). For a third-order system, the sliding surface can be defined as in (19).
The tracking error can be determined as a difference between the trajectory and actual position of the actuator.

\[ e(t) = x_r(t) - x_p(t) \]  \hspace{1cm} (20)

Since the linear model is a third-order model, the third derivative of the error is:

\[ \dddot{e}(t) = \dddot{x}_r(t) - \dddot{x}_p(t) \]  \hspace{1cm} (21)

The control signal in SMC design consists of equivalent control and switching control where the control action is corresponding with the sliding phase and reaching phase. The equivalent control is determined when \( s(t) = 0 \), while the switching control is described when \( s(t) \neq 0 \). The common control structure of SMC can be represented as:

\[ u_{\text{smc}}(t) = u_{\text{eq}}(t) + u_{\text{sw}}(t) \]  \hspace{1cm} (22)

In sliding mode phase, the tracking error will converge to equilibrium point and the sliding surface is supposed to be \( s(t) = \dot{s}(t) = \ddot{s}(t) = 0 \) where in this situation, the tracking error is trapped in the sliding surface.

The equivalent control is obtained when \( \dddot{s}(t) = 0 \) and the second derivative of the sliding surface,

\[ \dddot{s}(t) = k_p\dddot{e}(t) + k_d\dot{e}(t) + k_d\dot{e}(t) \]  \hspace{1cm} (23)

By substituting (21) into (23) and let \( \dddot{s}(t) = 0 \) without lumped uncertainty (\( L = 0 \)), the equivalent control can be defined as:

\[ u_{\text{eq}}(t) = (k_d C_n)^{-1} \left( k_p\dddot{e}(t) + k_d\dot{e}(t) + k_d\dot{e}(t) (\dddot{x}_r + A_n\dddot{x}_p + B_n\dot{x}_p) \right) \]  \hspace{1cm} (24)

The switching control can be chosen as a sign function of sliding surface (Eker, 2010),

\[ u_{\text{sw}}(t) = \lambda s(t) + k_s \text{sign}(\dot{s}(t)) \]  \hspace{1cm} (25)

where \( \lambda, k_s \in \mathbb{R}^+ \) and \( \text{sign}(\dot{s}(t)) \) denotes signum function as:

\[ \text{sign}(\dot{s}(t)) = \begin{cases} 
1, & \dot{s}(t) > 0 \\
0, & \dot{s}(t) = 0 \\
-1, & \dot{s}(t) < 0 
\end{cases} \]  \hspace{1cm} (26)

To verify the stable convergence behavior of nonlinear controller, Lyapunov theorem as a well-known approach is used in stability analysis (Mihajlov et al., 2002; Liu and Handroos, 1999; Chin et al., 2005). Lyapunov function can be chosen to prove the stability as:

\[ V(t) = \frac{1}{2} s^2(t) + \frac{1}{2} \ddot{s}^2(t) \]  \hspace{1cm} (27)

with \( V(0) = 0 \) and \( V(t) > 0 \) for \( s(t) \neq 0 \). To guarantee the trajectory move from reaching phase to sliding phase and ensure the stability, it is necessary to follow the reaching condition: \( \dot{V}(t) < 0 \), for
By substituting (21), (22) and (23) into (28),

\[
\dot{V}(t) = s(t)\dot{s}(t) + \dot{s}(t)\ddot{s}(t)
\]

\[
= s(t)\dot{s}(t) + \dot{s}(t)[k_p\ddot{x}(t) + k_i\dot{x}(t) + k_d(x(t) + A_1\dot{x}(t) + B_2\dot{x}(t) - C_1(u_{eq}(t) + u_{sw}(t)) + L(t))]
\]

\[
= s(t)\dot{s}(t) + \dot{s}(t)[-k_dC_1\lambda\dot{s}(t) - k_dC_1k_s\text{sign}(\dot{s}(t)) - k_dL(t))]
\]

\[
= s(t)\dot{s}(t) - k_dC_1\lambda\dot{s}(t) - k_dC_1k_s\text{sign}(\dot{s}(t)) - k_dL(t))\dot{s}(t) \leq [s(t)[s(t) - k_dC_1\lambda\dot{s}(t) - k_dC_1k_s\text{sign}(\dot{s}(t)) - k_dL(t))]
\]

\[
\leq [s(t)[s(t) - k_dC_1\lambda\dot{s}(t) - k_dC_1k_s\text{sign}(\dot{s}(t)) - k_dL(t))]
\]

\[
\leq -[s(t)[s(t)(k_dC_1\lambda - 1) + k_dC_1k_s - k_dL_{\text{max}}(t)) < 0
\]

When the system is in the reaching phase where \(s(t) \neq 0\), \(\dot{s}(t) \neq 0\) the requirement for SMC parameters to guarantee the stability is \(k_d > \frac{L_{\text{max}}}{C_1}\), \(\lambda > \frac{1}{k_dC_1}\) and \(\dot{V}(t)\) is negative definite.

The discontinuous function in (25) caused a chattering in the control signal. To avoid the chattering effect that can be harm to the system, the hyperbolic tangent function with boundary layer \(\varphi\) can be proposed (Eker, 2010).

\[
\dot{u}_{sw}(t) = \lambda s(t) + k_s \tanh(\frac{\dot{s}}{\varphi})
\]

Fig. 3: Sliding mode control with PID sliding surface

RESULTS AND DISCUSSION

In simulation study, the parameters used in nonlinear model of EHS system can be seen in Table 1. In the early design of SMC, the nominal values of EHS system can be identified using linearization of the EHS model or any identification techniques (Knohl and Unbehauen, 2000). To reduce the chattering effect, the \(\varphi\) parameter is set by sensitivity method until the controller achieves the adequate performance. The trade-off between the chattering and high-speed performance should made in tuning this parameter.
Table 1: Parameters of EHS System (Kalyoncu and Haydim, 2009)

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass</td>
<td>( M_p )</td>
<td>9 kg</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>( B_s )</td>
<td>2000 Ns/m</td>
</tr>
<tr>
<td>Spring stiffness</td>
<td>( K_s )</td>
<td>10 N/m</td>
</tr>
<tr>
<td>Total actuator displacement</td>
<td>( X_s )</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Piston area</td>
<td>( A_p )</td>
<td>645x10^6 m^2</td>
</tr>
<tr>
<td>Servo valve damping ratio</td>
<td>( \zeta )</td>
<td>0.48</td>
</tr>
<tr>
<td>Servo valve natural frequency</td>
<td>( \omega )</td>
<td>534 rad/s</td>
</tr>
<tr>
<td>Servo valve coil resistance</td>
<td>( R_c )</td>
<td>100 Q</td>
</tr>
<tr>
<td>Servo valve coil inductance</td>
<td>( L_c )</td>
<td>0.02 A</td>
</tr>
<tr>
<td>Servo valve gain</td>
<td>( K )</td>
<td>2.38x10^-5 m^3/2/kg^1/2</td>
</tr>
<tr>
<td>Bulk modulus of hydraulic fluid</td>
<td>( \beta )</td>
<td>1.4x10^9 N/m^2</td>
</tr>
<tr>
<td>Pump pressure</td>
<td>( P_s )</td>
<td>2.1x10^7 Pa</td>
</tr>
<tr>
<td>Return pressure</td>
<td>( P_r )</td>
<td>0 Pa</td>
</tr>
</tbody>
</table>

To simulate the friction model, the nonlinear Lu-Gre friction is implemented in the dynamics of the EHS system where parameters are tabulated in Table 2.

Table 2: Friction Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stribeck velocity</td>
<td>( v_s )</td>
<td>0.032 m/s</td>
</tr>
<tr>
<td>Coulomb friction</td>
<td>( a_c )</td>
<td>370 N</td>
</tr>
<tr>
<td>Stribeck friction</td>
<td>( a_s )</td>
<td>217 N</td>
</tr>
<tr>
<td>Viscous friction</td>
<td>( a_v )</td>
<td>2318 N/m/s</td>
</tr>
<tr>
<td>Bristles stiffness</td>
<td>( c_b )</td>
<td>5.77x10^6 N/m</td>
</tr>
<tr>
<td>Bristles damping</td>
<td>( c_l )</td>
<td>2.28x10^4 N/m/s</td>
</tr>
</tbody>
</table>

P, PD and PID controllers are used for performance comparison in the position tracking control. The parameters of each controller in Table 3 are taken from Kalyoncu and Haydim (2009) which have been optimized by genetic algorithm.

Table 3: P, PD and PID Controller Parameters

<table>
<thead>
<tr>
<th>Controller</th>
<th>Kp</th>
<th>Ki</th>
<th>Kd</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>59.7117</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PD</td>
<td>206.1246</td>
<td>0</td>
<td>1.1490</td>
</tr>
<tr>
<td>PID</td>
<td>221.0415</td>
<td>3.5260</td>
<td>1.2573</td>
</tr>
</tbody>
</table>

The results of position tracking performance are present concurrently for the SMC, P, PD and PID controller in Figure 4 and the control effort produced by SMC is shown in Figure 5. From the step input trajectory of \( x_r = 0.05 \) m, the results obtained with SMC controller is better compared to P, PD and PID controllers where the proposed controller produced more fast responses to reach the desired trajectory. In sliding surface design, the parameters are set to \( \lambda = 1 \), \( k_s = 10 \) and \( \phi = 15 \). The phase plot of SMC as can be seen in Figure 6 shows that the system reach the sliding phase and stay on that surface until meet the equilibrium point where the error and derivative of error approaching to zero. It is also can be seen that no chattering occur in sliding surface and its control effort during the position tracking control.

Comparative analysis from the controller performance based on steady state error and mean-square-error (MSE) for each controllers are tabulated in Table 4. SMC resulted better accuracy in position tracking which resulted the lowest value in MSE.

Table 4: Error Analysis

<table>
<thead>
<tr>
<th>Control method</th>
<th>SMC</th>
<th>P-controller</th>
<th>PD-controller</th>
<th>PID-controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state error</td>
<td>(-1.8917x10^3)</td>
<td>1.664x10^4</td>
<td>9.9614x10^3</td>
<td>(-2.666x10^4)</td>
</tr>
<tr>
<td>Mean-square-error</td>
<td>8.2324x10^4</td>
<td>2.7210x10^4</td>
<td>1.1717x10^4</td>
<td>1.0709x10^4</td>
</tr>
</tbody>
</table>

The tracking performance is evaluated with different trajectories to ensure that the proposed controller can achieved same performance with the step response performance in Figure 4. In Figure 7, the tracking performance is illustrated. Meanwhile, Figure 8 shows the control effort produced using SMC. Based on the error analyses, control effort and observation on the tracking performance, the SMC provides more convenient
and better performance in position tracking control and ensured that the control system in under stable condition. Is it found that equivalent control $u_{eq}$ can be neglected in this simulation study. The assumption where the position tracking is not one of reaching sliding surface, but one of near or already on sliding surface can be made. In applications where stability in not an issue, the term $u_{eq}$ often has a minimal effect in system response, so the equivalent control can be omitted (Surgenor and Vaughan, 1997). Consequently, a simple control method without much control effort can be made with this design instead of using conventional PID controller.

Fig. 4: Tracking performance of different controller for position control

Fig. 5: Control effort of SMC
Fig. 6: The phase plot of the SMC

Fig. 7: Tracking performance with different trajectories

Fig. 8: Control effort of SMC for different trajectories
Conclusion:
In this study, the performance of SMC, P, PD and PID controllers are evaluated for position tracking control. Theoretical analysis is developed in SMC and to ensure that the system is under stable condition, the Lyapunov approach has been utilized. The numerical simulations show that the proposed controller provides better performance in tracking accuracy and response. The control effort produced from SMC without the chattering effect also is practical to be used in real application. In conclusion, a simple control method without much control effort and better performance can be made with the SMC design based on PID sliding surface as an alternative of using conventional PID controller.

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