Performance of DS-CDMA System using Low-Rate Turbo-Hadamard Codes

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Abstract: The aim of this paper is to analyze the performance of low-rate codes in direct sequence code division multiple access (DS-CDMA) systems. While the DS-CDMA systems have been observed to afford capacity gains as compared to TDMA systems, additional benefits in capacity can be obtained through the use of advanced coding techniques before the information data is spread employing the pseudo-noise (PN) sequences. An important approach is to use low-rate codes that can provide both the effective channel coding and spectrum spreading. Low-rate codes could be employed in DS-CDMA networks to accomplish data spreading and provide coding gain as well. One class of such low-rate codes is the Turbo-Hadamard codes which are formed from Hadamard code arrays. In conventional DS-CDMA system, channel coding and spreading is performed separately but in modified system, it is performed simultaneously using low-rate codes. We have used Turbo-Hadamard code as a low-rate code in this system. The performance of the two systems has been evaluated on the basis of SNR VS BER. The simulation result show that modified system performs approximately 0.7dB better than the conventional system.

Key words: Low-Rate codes, Turbo-Hadamard Codes, Conventional CDMA system, Modified CDMA system

INTRODUCTION

The DS-CDMA (direct sequence code division multiple access) based techniques have increasingly been employed in digital cellular mobile networks because of their capability to offer higher system capacity over traditional multiple access techniques (Pickholtz, R. L. et al., 1991; Milstein L. B. et al., 1992). DS-CDMA systems are characterized by a number of distinctive features as compared with the FDMA and TDMA systems. These include high-traffic capacity, macro-diversity and soft-handoff, soft capacity, high-spectrum efficiency, graceful degradation, operation at low power and suitability for variable data rates and multiple services with diverse QoS requirements. CDMA has the ability to maximize the number of simultaneous users in given cell by an accurate power control and further does not require any frequency planning. Moreover, CDMA has have gained increased attention in the backdrop of growing needs for integration of diverse services such as voice, video and data with varying quality constraints in wireless cellular networks (Ericsson, 1991).

CDMA systems are (Andrew J. Viterbi, 1995) are characterized by the fact that their bandwidth is much greater than the information rate. The large redundancy inherent in spread spectrum signals is required to overcome severe level of interference that are encountered in the transmission of digital information over the channel. Coded information (John G. Proakis et al., 2003) is also characterized by bandwidth expansion factor greater than unity and it is an efficient method of reducing redundancy, hence powerful codes play an important role in CDMA systems. This not only gives the processing and coding gain but more users can be accommodated in the system. That is why it is viable method for providing digital cellular telephone service to mobile users (Panayiotis et al., 2001). The CDMA capacity is limited primarily by the interference level in the system and therefore not fixed, whereas for FDMA and TDMA systems, the capacity is mostly limited by the bandwidth.

The CDMA system capacity can be directly increased by reducing the interference in the system (Gilhousen, K. L. et al., 1991). The main kind of interference in DS-CDMA systems is the Multiple Access Interference (MAI) that results when the cross-correlation among various spreading code sequences is not zero. Although, using orthogonal spreading sequences can help reduce the MAI to some extent; however, advanced techniques are needed to minimize MAI and improve capacity. Additional benefits in capacity can be obtained...
through the use of advanced coding techniques before the information data is spread employing the pseudo-noise (PN) sequences (Andrew J. Viterbi, 1995). One of the key techniques in this context is channel coding, also referred to as forward error correction (FEC). Forward Error Correction (FEC) coding when combined with interleaving can be viewed as a type of time diversity technique. CDMA systems have been shown to provide optimal performance when effective FEC techniques are combined with spreading. An important approach is to use low-rate codes that can provide effective channel coding and spectrum spreading simultaneously. Low-rate codes can be utilized in DS-CDMA systems to attain the data spreading as well as afford coding gain (A. J. Viterbi, 1990).

The emergence of Turbo codes (Claude Berrou et al., 1996) initiated a fresh interest in the development of low SNR codes. Excellent weight distribution and tractable (sub-optimal) decoder complexity are the main features that characterize Turbo codes. For code-spread communication systems to operate in the regime of low signal-to-noise ratio (SNR), the role of low-rate codes is highly critical. Hadamard codes and super-orthogonal Convolutional codes are more conventional low-rate coding techniques; however, they provide small coding gain as the performance of these codes is not optimal in terms of approaching the Shannon limit. The availability of high performance low-rate codes with rates approaching zero have helped realizing the ultimate capacity in AWGN (additive white Gaussian noise) channels, referred to as the Shannon Limit, i.e., $Eb/No = 1.59$ dB. One such code is the Turbo-Hadamard that is constituted by parallel concatenation of Convolutional-Hadamard codes (Li. Ping et al., 2003). The focus of this work is on the performance assessment of the Turbo-Hadamard codes in a DS-CDMA system.

The rest of the paper is organized as: Section I describes the conventional DS-CDMA system whereas the Section II presents the modified DS-CDMA system under consideration in this work. A brief overview of Turbo-Hadamard codes is provided in Section III. The simulation analysis of both the conventional and modified DS-CDMA systems is discussed in Section IV whereas the Section V concludes the work.

I. Conventional DS-CDMA System:

A typical DS-CDMA system as considered in (Hasan, Abu-ul-Ala., 2003) is shown in the Figure 1. The inputs to this system are information bits from the users who desire to utilize the channel. The information bits $\{i_1, i_2, ..., i_k\}$, from each user are input to the system. These information sequences will be processed individually and then added together so that can be transmitted over the channel. As mentioned earlier, all users share the same channel and data from other users appears as a noise to an individual user, hence it is necessary to use error correction mechanism in the system. For this purpose, turbo code (L. R. Bahl et al., 1974) is used as an outer code and repetition code is used to spread the sequence. A spreading sequence of length $N$ chips is used. In this system, the same spreading sequence is used for all users. Therefore we interpret spreading as a repetition code. The spreading factor can also be given as

$$N = \text{Spreading Factor} = \frac{\text{Chip Rate}}{\text{Data Rate}} \quad 1.1$$

When the chip rate (John G. Proakis et al., 2003) is in chips per second and data rate is in code bits per second. The overall code rate is defined as

$$R = \frac{R_o}{N} \quad 1.2$$

where $R_o$ is the rate of outer code.

These chips are BPSK modulated. BPSK symbols are $\{+1, -1\}$, where zero is mapped to $+1$ and $1$ to $-1$ respectively. Each user has to be assigned a unique code or sequence. To achieve this Pseudo-Noise sequence is generated for each user. These PN-sequences are different from each other, and we represent it as $\{PN_1, PN_2, \ldots, PN_k\}$. Scrambling is done using these PN sequences. A PN sequence is multiplied by user chips as shown in figure 1.1. The scrambled chips represented as $\{s_1(t), s_2(t), \ldots, s_k(t)\}$ are added and transmitted over the channel, where $t$ is discrete time. This can be expressed as

$$s(t) = s_1(t) + s_2(t) + \ldots + s_k(t) \quad 1.3$$

From the channel, AWGN is added to this signal. Hence at the receiver, the sum of all user chips, and the noise is received. If AWGN is represented by $n(t)$, then the received signal $r(t)$ can be expressed as
At the receiver, signals from different users will be separated by autocorrelation of the received signal with each of the possible user PN-sequence. The next step is to decode these code bits to get estimated information bits. The decoded bits are represented as \{i_1, i_2^, \ldots, i_k^\}. We have seen that the data of each user, which was added before transmission is separated again.

In the DS-CDMA system presented in Figure 1, all users have the same transmit time. Nevertheless, this can be regarded as a model of an asynchronous system, because the correlation properties of the PN sequence (Simon Haykin, 2004) remain the same when different times are introduced in the model. Since each user transmits a pseudo random signal of bandwidth W and average power P, the capacity of the system depends on the number of users. If every user is treated by its own, the other user’s signal acts as interference at the receiver of each user. Each user’s signal can be treated as Gaussian random variable. Therefore, each user’s signal is corrupted by Gaussian interference of power \((K-1)P\) and additive Gaussian noise of power \(WN_o\).

Therefore, capacity per user as presented in (John G. Proakis et al., 2003) will be

\[
C_k = W\log_2[1 + P/(WN_o + (K-1)P)]
\]

where \(K\) is the number of users.

If \(E_b\) is the bit energy and \(N_o\) is the noise density (single-sided PSD of AWGN corrupting information bits), and assuming the bit energy \(E_b\) to be unity and \(\sigma_n^2\) is variance of the white Gaussian Noise, then

\[
N_o = 2\left(\frac{\sigma_n^2}{2}\right)(R_o/N)
\]

\[
E_b/N_o = N/(R_o\sigma_n^2)
\]

\[
\sigma_n^2 = N/(R_o(E_b/N_o))
\]

**Fig. 1:** Conventional CDMA system

**Outer Code (Turbo Code):**
Turbo code is used as an outer code in the conventional DS-CDMA system. It consists of recursive and systematic Convolutional encoders concatenated in parallel separated by an interleaver as shown in Figure 2. A Convolutional encoder of rate 1/2 and constraint length 2 is used as component code in it. By convention the systematic bits of top encoder are transmitted while the second encoder systematic bits are not transmitted. However, parity outputs of both encoders are transmitted. The maximum likelihood method is used in the decoding process which determines prior and posterior probabilities at each decoding step (Claude Berrou et al., 1996).

**PN Sequence:**
CDMA systems completely rely on the availability of pseudo noise (PN) sequence. The PN sequence is usually generated by means of linear feedback shift register. A linear feedback shift register is constituted of simple shift register with a binary weighted modulo-2 sum of the taps fed back to the input as shown in Figure 3 (John G. Proakis et al., 2003; Simon Haykin, 2004).
Feedback connections can be specified in short hand form with a feedback polynomial. As an example, a polynomial representation is

$$G(x) = X^3 + X^1 + 1 = \sum_{i=0}^{r} c_i X^i$$

The coefficients of this polynomial are the feedback coefficients $c_0, c_1, \ldots, c_r$. At each clock cycle, the linear shift register contents are shifted right. The sequence in the shift register, $a_n$ moves through each term and is determined from the previous $r$ terms according to the formula

$$a_n = c_0 a_{n-1} + c_1 a_{n-2} + \ldots + c_r a_{n-r} = \sum_{i=0}^{r} c_i a_{n-i}$$

where $c_1$ to $c_r$ are connection variables (1 for connection and 0 for no connection). However considering non-zero index and with $D$ as delay operator, we get a generating function of the sequence

$$G(D) = a_0 + a_1 D + a_2 D + \ldots = \sum_{n=0}^{\infty} a_n D^n$$

And the number of units of delay for each term corresponds to the power of $D$ of that term of this polynomial. Every linear shift register is periodic. If a primitive polynomial is used for feedback, the period is

$$P = 2^r - 1$$

We restrict ourselves to uplink of a mobile communication system. We assumes that user’s transmit times are asynchronous. Therefore user’s signals interfere with each other in the receiver. An example of this kind of system is UMTS uplink. The downlink is not considered here because the inter-user interference is much less and the user’s signals are approximately orthogonal.
II. Modified CDMA System:
In conventional CDMA system, channel coding (M. Bossert, 1999) and spreading is performed separately. In modified CDMA system (Panayiotis et al., 2001), spreading is not done by repetition code but achieved by coding the information bits. For this purpose Low-Rate codes as in (Li. Ping et al., 2003) are used to achieve this objective because Low-Rate codes perform spreading and error correction simultaneously. Low-Rate codes are powerful codes which provide strong immunity against noise but also give coding gain which does not provide the repetition code. The modified CDMA system is shown in Figure 4.

The overall code rate is determined by

$$R = R_o$$  \hspace{1cm} \text{(2.1)}

Now $R_o$ is the code rate of the Low-Rate code.

In case of modified CDMA system, the variance of the white Gaussian noise would be

$$\sigma_n^2 = 1/(R_o(E_b/N_o))$$  \hspace{1cm} \text{(2.2)}

III. Low-Rate Code (Turbo-Hadmand Code):
The Convolutional codes are characterized by the fact that they are highly structured. On one hand, this characteristic allows the implementation of encoders and decoders with reasonable complexity. While on the other hand, this same property causes the performance of Convolutional codes to be considerably lower than the random coding bounds predicted by Shannon (Claude Berrou et al., 1996). Hadamard codes have the property that they are orthogonal to each other and this property makes them useful in channel coding schemes. Li Ping in (Li. Ping et al., 2003) proposed a new Turbo like scheme called Turbo-Hadamard codes. These codes can be used in a CDMA system because these codes performs channel coding and spreading simultaneously. These codes possess some random noise like properties. As a consequence, the performance of Turbo-Hadamard codes can approach the Shannon bound.

A general block diagram Turbo-Hadamard encoder is shown in Figure 5 (Li. Ping et al., 2003). To form a Turbo-Hadamard code, several convolutional Hadamard codes are concatenated in parallel. A triplet { $D^{(m)}$, $P^{(m)}$, $q^{(m)}$ }
\((q(m), P(m))\) is considered to be the \(m\)th component codeword with code rate = \(r/2\) where \((q(m), P(m))\) are the respective redundancy outputs and \(D(m)\) is the \(m\)th interleaved version of \(D\) for \(m = 1, 2, ..., M\). The code rate is reduced to \(r/M\) once \(M\) such component codes are concatenated. It is to be noted that all \(M\) component codes repeat the information bits. The overall codeword can be redefined as \(\{D, q(1), P(1), ..., q(M), P(M)\}\) to improve the transmission efficiency as depicted in Figure 5 (Li. Ping et al., 2003). If \(r\) is taken as the order of the Hadamard code, the effective rate of the code would be as given by Eq. 3.1 below,

\[
R = \frac{r}{[r+M(2^r-r)]} 
\]

Assume that there are \(M\) independent and identically distributed i.i.d random variables that constitute the right hand side of Eq. 3.2 which for a Turbo-Hadamard code provides the \(W\), the overall parity weight,

\[
W = W_1 + W_2 + ... + W_M 
\]

where \(W_n\) is the parity weight for the component code, \(n\). If \(M\) is sufficiently large, \(W\) approaches Gaussian according to the central limit theorem. Therefore, a Turbo-Hadamard code resembles very closely to a random code (with Gaussian distribution). A random code is generally considered to be a “good code”, specifically as the code length approaches infinity. Due to the assumption that \(M\) component codes are independent and identically distributed i.i.d random variables, it is mandatory that these component codes have independent parity weights. This independence can be ensured by reducing the dependency among the input weights and output weights of the \(M\) component codes. For recursive component codes, the correlation among the input weights and output weights is comparatively lower that that for the non-recursive component code (Li. Ping et al., 2003).

**The Component Code:**

The Turbo-Hadamard code is constructed from hybrid concatenation of Hadamard codes and Convolutional codes. Let ‘\(C\)’ represent the Convolutional-Hadamard code that is employed as component in the given scheme. Figure 6 shows the way ‘\(C\)’ is generated. The incoming information bit stream is divided in blocks referred to as \(d_k\). There are \(r\) information bits in each of the \(d_k\) blocks. Assume ‘\(q\)’ to be the parity of the block \(d_k\). Utilizing a rate-1/2 Convolutional code \(C^*\), encode ‘\(q\)’ = ‘\(q_k\)’, and thus a parity sequence \(q = \{q_k\}\) is generated. Then a Hadamard code is used to encode \(\{d_k, q_k\}\) resulting in \(c_k = \{d_k, q_k, p_k\} \in \{±h\}\). The final output is a code sequence \(c = c_k\) that represents a codeword in ‘\(C\)’ (Li. Ping et al., 2003).

**Fig. 6:** A Convolutional-Hadamard Encoder

**Fig. 7:** Codeword Length of Convolutional-Hadamard Encoder

Figure 7 illustrates codeword in ‘\(C\)’. With \(c_k\) its \(k\)th row, the code is organized in an array. The codeword is decomposed as \((D, q, P)\), where \(p_k\) is the \(k\)th row of \(P\) and \(d_k\) is the \(k\)th row of \(D\). The final codeword does not contain the intermediate vector \(q\). The trellis diagram \(T^*\) of \(C\) is shown in Figure 8 (Li. Ping et al., 2003).
Fig. 8: Trellis of Convolutional-Hadamard Encoder

The set of states defined for the trellis T are the same as those in T'. There is one to one correspondence among the indexes for the states used in T and T'. For the for state transition $s_k$ to $s_{k+1}$ in T, assume that $(q'_k, q_k)$ are the coded bits. There are $2^{r-1}$ possible $d_k$ that correspond to a certain $q_k'$ as $q_k'$ is the parity check of $d_k$. Therefore, $2^{r-1}$ combinations of $(d_k, q_k)$ are obtained and a unique codeword is provided by the each combination. These $2^{r-1}$ code words are denoted by $h(q_k', q_k)$. By arranging the $2^{r-1}$ corresponding branches between the states $s_k$ and $s_{k+1}$, the trellis diagram T can thus be constructed. The development of the decoding algorithm and computation of the BER bounds for the code is greatly facilitated by the trellis representation. A Convolutional code having a generator polynomial, $1/(1+x)$ along with a Hadamard code of length-8($r = 3$) form the basis of this Convolutional-Hadamard code. Assume that the constraint length of $C'$ is $v'$. The trellis for the resulting Turbo-Hadamard code can be terminated to state-0 by appending 'v' extra rows to 'D'. All the information bits except one bit are set to zero in these rows. The trellis can always be terminated into state-0 by choosing a proper value of that one bit. The decoding is performed recursively as used in Turbo Codes (Li. Ping et al., 1998; Li. Ping et al., 2001).

IV. Performance of Conventional and Modified DS-CDMA Systems:

Conventional CDMA system is analyzed here using simulations that involve varying number of users and several other parameters. The number of interfering users, I is the total number of users, K minus the user itself. Or we can write it as

$$I = K - 1$$

In the presence of multiple users K, the total number of users will always be greater than or equal to 2, and I, the interference from other users will be greater than or equal to 1. Or simply

$$K \geq 2, I \geq 1$$

The modified system is also observed using different parameters. A recursive systematic Convolutional encoder of rate 1/2 and constraint length 2 is used as a component code in the Turbo-Hadamard encoder. The recursive systematic Convolutional (RSC) is further concatenated with a Hadamard code to make it a Low-Rate code as shown in Figure 6 (R. Johannesson et al., 1999). Hadamard codes have the property that they are orthogonal, linear and systematic codes. The interleaver used is semi random. The performance of random interleaver is in iterative decoding is beneficial because it makes the code a random code which has binomial
distance distribution that approaches Gaussian as the length of interleaver approaches infinity. The performance of the Turbo-Hadamard code is assessed here and compared with another CDMA system which uses simply the Turbo Code as depicted in Figure 9 below.

![Fig. 9: Performance Comparison of two CDMA Systems. Interleaver = 10240](image)

It can be observed from the results reported in Figure 9 that the modified CDMA system with Turbo codes affords better performance in terms of achieving BER targets at lower $E_b/N_0$ values. For example, at an $E_b/N_0$ value of 2.5 dB, the BER achieved is $10^{-4}$ and $10^{-5}$ respectively for the conventional and modified CDMA systems. This demonstrates that the use of Turbo-Hadamard codes results in BER performance improvement by an order of magnitude in all the three cases considered here. Further, from another point of view, the results indicate that the modified CDMA system offers a coding gain of 0.7dB at a BER of $10^{-4}$ over the conventional system.

V. Conclusions:

In this work, the objective has been to assess the performance of a modified CDMA system that employs Turbo-Hadamard codes for forward error correction as well as data spreading. Simulation results have shown that the modified CDMA system affords a coding gain of 0.7dB at a BER of $10^{-4}$ over the conventional CDMA system. Moreover, performance improvement of an order in BER is achieved with Turbo-Hadamard codes as compared with simple Turbo codes. This performance can be ameliorated if the order of the Hadamard matrix and the number of component codes are increased in the Turbo-Hadamard code. There may be several other parameters which can affect the system performance. High interleaver size, low code rate and number of iterations can also play a vital role in performance improvement. However the cost will be paid in terms of hardware complexity, large memory size and high processing time.

REFERENCES


