Position Control of Permanent Magnet Stepper Motor Considering Model Uncertainty Using Type-2 Fuzzy Robust Control Method

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Abstract: Permanent Magnet (PM) stepper motors are widely used in precise systems which are affected by external disturbances, parameters uncertainty and saturated control. Also, an appropriate nonlinear controller is required when the problem is to track reference signal. This paper presents a robust adaptive controller to control rotor angular position in stepper motors. The main idea to make a robust controller is to use an adaptive control system based on type-2 fuzzy sets. Type-2 fuzzy set estimates nonlinear and uncertain functions existing in the stepper motor equations and then, parameters of type-2 fuzzy set are tuned online by adaptive rules obtained from Lyapunov theory. Finally, simulations are performed to control stepper motor position in two cases: certain and uncertain equations. To apply uncertainty into the system model in simulations, values of rotor and load inertia (J) and viscous friction coefficient (f) are considered as a range of numbers. Actually, a limited parametric uncertainty is applied to the system. Simulation results show that the proposed controller has a better performance in tracking and robustness compare to type-1 fuzzy controller.

Key words: Adaptive–fuzzy control, Parametric uncertainty, Permanent Magnet stepper motor (PM stepper motor), Robust control, Type-2 fuzzy sets

INTRODUCTION

Stepper motor is an electromechanical nonlinear motor which has been designed to rotate in specific angular position. Stepper motors require simple and cheap controllers for position and speed control. Therefore, these motors are very popular in industrial applications and are widely used in different industries. DC motors were used in the past for positioning systems due to their linear specifications. Permanent magnet stepper motors (PM Stepper Motors) have become a popular alternative to the traditionally used Brushed DC Motors (BDCM) for many high performance motion control applications for several reasons: better reliability because of the elimination of mechanical brushes, better heat dissipation as there are no rotor windings, higher torque-to-inertia ratio because of a lighter rotor, lower price and easy interfacing with digital systems. The shaft or spindle of a stepper motor rotates in discrete step increments when electrical command pulses are applied to it in the proper sequence. The motors rotation has several direct relationships to these applied input pulses. The sequence of the applied pulses is directly related to the direction of motor shafts rotation. These changes in shaft position can generate oscillations or cause a long delay in the output (torque) which is related to selected controller. Now, PM stepper motors are widely used in numerous motion control applications such as robotics, printers, digital control circuits and so on. The PM stepper motors operate on the reaction between a permanent magnet field in the rotor and an electromagnetic field in the stator. The number of teeth on the rotor and stator determine the step angle that will occur each time the polarity of the windings is reversed. Also, the greater the number of teeth, the smaller the step angle. Recently, various methods have been introduced for rotor position control and determination of the proper control signals in PM stepper motors. It is important that a nonlinear controller will be required due to the nonlinear structure of PM stepper motors while output tracking problem is represented. In recent decades, adaptive algorithms have been applied to the PM stepper motors more than before (R.C. Speagle et al., 1993). On the other hand, the other methods such as sliding-mode control (F. Nollet et al., 2008, M. Zribi et al., 2001) and adaptive robust control have been developed specially for uncertain systems. (S. Ge et al, R. Marino et al., 1995). Also, many of the papers

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focus on system diagnosis and control based on the neural networks. Neural networks are capable in learning input-output mapping rules of nonlinear and complicated systems and they are very popular due to this capability (M. D. Minkov et al., 1998, M. Fallahi et al., 2002). Another method to control PM stepper motors is based on linearization and combination of this method with neural networks. The neural network is designed based on the RBF model and trained for stepper motor diagnosis. In addition, combination of fuzzy systems and adaptive control is used to design controllers specially in canonical systems (P. A. Phan et al., 2008, H. Abid et al., 2007). This paper presents an adaptive robust controller for angular position control in PM stepper motors. In this paper, we used type-2 fuzzy systems in order to make a robust controller, because type-2 fuzzy systems have the capability to cover and minimize all uncertainties of the model. According to this method, the type-2 fuzzy system is first used to estimate nonlinear and uncertain functions existing in PM stepper motor equations and then, type-2 fuzzy system parameters are updated through an online process using adaptive rules obtained from Lyapunov theory. Design of this controller has been described in (J. M. Mendel, 2007), completely. The type-2 fuzzy system has a capability to handle and minimize the effect of both linguistic and random uncertainties, simultaneously. A wide range of applications related to type-2 fuzzy systems show that these systems provide much better solutions specially in handling uncertainties (J. M. Mendel, 2001). There is a set of applications and papers about type-2 fuzzy systems are presented in (Castillo et al., 2008). The procedure in this paper will be as follows:

In section 2, a mathematical model for PM stepper motor is presented and converted to the canonical form. In section 3, we apply the adaptive fuzzy controller of (J. M. Mendel, 2001) to canonical form of the system. It must be noted that the stability analysis based on Lyapunov method has been presented in (J. M. Mendel, 2007). In section 4, simulation results will be investigated and finally, conclusion will be presented in section 5.

II. Mathematical Model of the PM Stepper Motor:

In this paper, PM stepper motor has a two-winding stator and a permanent magnet rotor. In fact, it is a two-phase stepper motor. The PM stepper motor operates on reaction between magnetic flux of the rotor and electromagnetic field in the stator. The strength of electromagnetic field in the stator is proportional to the amount of current sent to the stator windings and the number of turns in the windings.

The mathematical model of PM stepper motor is given below:

\[
\begin{align*}
  i_a &= \frac{1}{L}(V_a - R I_a + k_m \omega \sin(N_r \theta_r)) \\
  i_b &= \frac{1}{L}(V_b - R I_b + k_m \omega \sin(N_r \theta_r)) \\
  \omega &= \frac{1}{J}(-k_m I_a \sin(N_r \theta_r) + k_m I_b \cos(N_r \theta_r) - \beta \omega) \\
  \dot{\theta}_r &= \omega
\end{align*}
\]

Where, \( I_a \) is the current in winding A, \( I_b \) is the current in winding B, \( \omega \) is the angular velocity of the motor’s shaft, \( \dot{\theta}_r \) is the angular displacement of the shaft, \( N_r \) is the number of rotor teeth, \( V_a \) is the voltage across winding A, \( V_b \) is the voltage across winding B, \( J \) is the rotor and load inertia, \( \beta \) is the viscous friction coefficient, \( L \) and \( R \) are the inductance and resistance, respectively, of the phase windings, \( k_m \) is the motor torque constant. Above equations include nonlinear factors which make difficulties for the controller design. Another method for describing the system model is called “DQ model” which can transform equation (1) to more simple equation due to better controller design. DQ model is obtained from the transfer matrix as follows (J. M. Mendel, 2007):

\[
\begin{bmatrix}
  I_a \\
  I_b \\
  \omega \\
  \dot{\theta}_r
\end{bmatrix} =
\begin{bmatrix}
  \cos(N_r \theta_r) & \sin(N_r \theta_r) & 0 & 0 \\
  -\sin(N_r \theta_r) & \cos(N_r \theta_r) & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  I_a \\
  I_b \\
  \omega \\
  \dot{\theta}_r
\end{bmatrix}
\]

(2)
Voltage transfer matrix will be obtained from:

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} =
\begin{bmatrix}
\cos(N, \theta_r) & \sin(N, \theta_r) \\
-\sin(N, \theta_r) & \cos(N, \theta_r)
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b
\end{bmatrix}
\]

(3)

Where, \(V_d\) and \(I_d\) are the direct-axis (d-axis) voltage and current, \(V_q\) and \(I_q\) are the quadrature-axis (q-axis) voltage and current. According to equations (2) and (3), equation (1) can be rewritten as follows with respect to the new variables:

\[
i_d = \frac{1}{L} (V_q - RI_d + N_r \omega L I_q)
\]

\[
i_q = (V_q - RI_q + N_r \omega L I_d - k_n \omega)
\]

\[
\omega = \frac{1}{J} (k_n I_q - B \omega)
\]

\[
\dot{\theta}_r = \omega
\]

are defined as:

\[
D_1 = \frac{R}{L}, D_2 = \frac{k_m}{L}, D_3 = \frac{k_n}{J}
\]

\[
D_4 = \frac{B}{J}, D_5 = N_r, u_q = \frac{V_q}{L}, u_d = \frac{V_d}{L}
\]

(5)

According to above factors, equation (4) can be written as:

\[
\dot{\theta}_r = \omega
\]

\[
\omega = D_1 I_q - D_3 \omega
\]

\[
\dot{I}_q = -D_1 I_q - D_5 \omega I_d - D_2 \omega + u_q
\]

\[
\dot{I}_d = -D_3 I_d + D_5 \omega I_q + u_d
\]

(6)

We have from equation (6) that:

\[
\dot{I}_q = -\frac{D_1}{D_1} I_q - \frac{D_5}{D_1} \omega I_d - \frac{D_2}{D_1} \omega + \frac{1}{D_1} u_q
\]

\[
\dot{I}_d = -\frac{D_3}{D_1} I_d + \frac{1}{D_1} u_d
\]

(7)

Equations (8) and (9) are obtained by substituting equation (7) into (6):

\[
\dot{\theta}_r = \omega
\]

\[
\omega = F + \frac{D_1}{D_1} (u_q - u_d)
\]

(8)
By defining $x = [x_1, x_2]^T = [\theta_d, \dot{\theta}_d]^T$, equation (8) can be rewritten as:

$$
\begin{align*}
\cdot & \quad x_1 = x_2 \\
\cdot & \quad x_2 = F(x) + \frac{D_2}{D_1} (u_q - \frac{D_2 x_2}{D_1} u_u) \\
\end{align*}
$$

Adaptive fuzzy controller design must provide two following objectives in the presence of uncertainty and without it:

- System output follows $\theta_d$ reference value.
- Closed-loop system be stable and all closed-loop variables be bounded.

To design such a controller, the following assumptions will be considered:

1) Reference vector, $\theta_d = [\theta_d, \dot{\theta}_d]^T$, is defined such that $||\ddot{\theta}_d|| < \theta_0$ and $||\theta_d|| < \theta_1$; where, $\theta_0$ and $\theta_1$ are known positive constants. Note that we need to determine the sign of $g(x)$ in order to design the controller. The sign of $x$ must be constant over all domain. $\psi$ is defined to determine the sign of $g(x)$ and its value is chosen by designer. Also, $u_q$, $u_d$ are considered as the inputs, and defined as follows:

$$
\begin{align*}
\psi u_q = \psi u_d = \psi u_c \\
\end{align*}
$$

Where, $u_c$ is the control signal generated by control system. Equation (12) can be obtained from equations (10) and (11).

$$
\begin{align*}
\frac{D_2}{D_1} (u_q - \frac{D_2 x_2}{D_1} u_u) = \frac{D_2}{D_1} (\psi - \frac{D_2 x_2}{D_1} u_c) \\
= g(x)u_c \\
\end{align*}
$$

2) It is assumed that $g(x)$ is a continuous function and its sign, is definite for $x \in \Omega$. Where $x$ is controllability area. In our proposed method, $\psi$ is chosen such that $g(x) > 0$. Therefore, we obtain:

$$
\begin{align*}
\frac{D_2}{D_1} (\psi - \frac{D_2 x_2}{D_1} u_c) > 0 \Rightarrow \psi - \frac{D_2 x_2}{D_1} u_c > 0 \\
\psi - \frac{N_x \omega L}{R} > 0 \Rightarrow \psi > \frac{N_x \omega L}{R} \\
\end{align*}
$$

Also, the state space equation can be written as follows:

$$
\begin{align*}
\cdot & \quad x_1 = x_1 \\
\cdot & \quad x_2 = f(x) + g(x)u_c \\
\end{align*}
$$

III. Proposed Adaptive-fuzzy System:

In previous section, the equations of PM stepper motor were transformed to canonical form. In this section, the objective is applying a feedback controller, $u = u(x, \theta)$, based on type-2 fuzzy system. Also, we present an adjustment rules to regulate the vector of parameters such that the system output achieves the desirable output (J.M. Mendel, 2007). However, it is desired that the system output achieves to the desirable output as much as possible, it is much better that the system output converges toward the desirable output asymptotically. In specific cases, it is assumed that there is an accessible set of fuzzy if-then rules which describes the input-
output behavior of \( f(x) \) and \( g(x) \). These rules can be rewritten as a form of interval type-2 fuzzy rules (J.M. Mendel, 2007) as follows:

\[
f x_i \text{ is } F_i \text{ and } \ldots \text{ and } x_n \text{ is } F_n \text{ then } f(X) \text{ is } C
\]  
\[\text{(15)}\]

Which describe \( f(x) \) and

\[
f x_i \text{ is } G_i \text{ and } \ldots \text{ and } x_n \text{ is } G_n \text{ then } g(X) \text{ is } D
\]  
\[\text{(16)}\]

Which describe \( g(x) \). If nonlinear functions, \( f(x) \) and \( g(x) \), are specified, then we can choose the control vector \( (u) \), to omit the nonlinear part and design a controller based on linear control theory. In specific cases, it is assumed that \( e = y_m - y \), \( e = (e, e', \ldots, e^{(n-1)})^T \), \( K = (k_n, \ldots, k_1)^T \) and \( K \) is determined in such a way that all roots of the characteristic equation \( (s^n + k_n s^{(n-1)} + \ldots + k_1) \) lie in the left-half S plane (left hand side of imaginary axis). Then, we can select the control rules as follows:

\[
u^* = \frac{1}{g(x)} [-f(x) + y_m^{(n)} + k^T e]
\]  
\[\text{(17)}\]

The closed-loop system dynamic is obtained by substituting equation (17) into (14), as:

\[e^{(n)} + k_n e^{(n-1)} + \ldots + k_1 e = 0\]  
\[\text{(18)}\]

If the value of \( K \) is selected properly, then \( \lim_{t \to \infty} e(t) \to 0 \). It means that the system output converges to the desirable output, asymptotically. Equation (17) which is related to ideal controller cannot be used if \( f(x) \) and \( g(x) \) are unknown. Under these circumstances, only fuzzy if-then rules can be used to describe input-output behavior of \( f(x) \) and \( g(x) \) (Equations (15), (16)). Therefore, a reasonable idea is to replace \( f(x) \) and \( g(x) \) by fuzzy functions, \( \hat{f}(x) \) and \( \hat{g}(x) \), which have been obtained from equations (15) and (16), respectively. Equations (15) and (16) just provide approximate information about \( f(x) \) and \( g(x) \) functions. Therefore, the fuzzy functions, \( \hat{f}(x) \) and \( \hat{g}(x) \), are not accurate enough to estimate \( f(x) \) and \( g(x) \). To improve the accuracy, it is recommended to release some parameters which change online during the operation in such a way that approximation accuracy improves after a period of time. Assume that \( \theta_0 \in R^{M_p} \) and \( \theta_f \in R^{M_f} \) are free parameters in \( \theta_f \in R^{M_f} \) functions, respectively. Hence, we have \( \hat{f}(x) = f(x, \theta) \) and \( \hat{g}(x) = g(x, \theta) \). In equation (17), by substituting and \( f(x) \) and \( g(x) \) with \( \hat{f}(x, \theta) \) and \( \hat{g}(x, \theta) \), the fuzzy controller can be presented as:

\[
u = u_f = \frac{1}{g(X, \theta)} [-\hat{f}(X, \theta) + y_m^{(n)} + K^T] \]
\[\text{(19)}\]

This type-2 fuzzy controller is called “certainty equivalent”. After modeling \( \hat{f}(x, \theta) \) and \( \hat{g}(x, \theta) \) using type-2 fuzzy system (the details of which can be found in [J.M. Mendel, 2007]), and defining parameters such as: \( Q, P, \gamma_1, \gamma_2, b, \Lambda, \omega, \theta_0^\gamma, \theta_f^\gamma \) and considering positive definite Lyapunov function as equation (20),
Voltage-time derivative (\(v\)) through closed-loop path (6) is obtained as follows:

\[
V = \frac{1}{2} e^T P e + \frac{1}{2} \gamma_1 (\theta_f - \theta_f^*)^T (\theta_f - \theta_f^*) + \ldots
\]

\[
\frac{1}{2} \gamma_2 (\theta_e - \theta_e^*)^T (\theta_e - \theta_e^*)
\]

To minimize tracking error (\(e\)), adaptation rules must be chosen such that \(\dot{v}\) becomes negative definite. \(\frac{1}{2} e^T P e\) is a negative term and we are able to choose fuzzy systems in such a way that minimum approximation error (\(w\)) becomes small. Therefore, a good strategy is to select the adjustment rule such that the last two terms of equation (21) become zero. Hence, the adaption rules can be written as follows:

\[
\theta_f^* = -\frac{\gamma_2}{2} e^T P (\xi_e(X) + \xi_e(X)) \quad (22)
\]

\[
\theta_e^* = -\frac{\gamma_1}{2} e^T P (\eta_e(X) + \eta_e(X)) u_i \quad (23)
\]

Effective quantities in above equations have been presented in [P. A. Phan et al., 2008]. Moreover, indirect adaptive type-2 fuzzy control system has been shown in Fig. 1.

**IV. Simulation Results:**

In this section, we apply adaptive-fuzzy tracking control to canonical equation which obtained from section 2. Also, we compare the performance of type-1 and type-2 fuzzy systems in rotor angle tracking control. For this purpose, we need to use equation (14) in system block of block-diagram which has been shown in Fig. 1. For the purpose of simulation, we use the following values for the system parameters, obtained from (H. Abid et al., 2007):
It must be noted that the friction torque (has not been considered in equation (1)). The purpose of control process is to force the rotor angular position follow the desirable path shown in Fig. 2 with minimum error.

Fig. 2: Desirable path for rotor angular position

Fig. 2 shows that the rotor angle first needs to increase from 0 to 2 degrees in 0.16 sec. Then, the same increase in rotor angle has to take place in 0.16 sec, so that the rotor angle reaches to 4 degrees. Finally, it decreases again to 2 degrees in the same period of time. To show the efficiency of proposed controller, in addition to certain model of the system, the experiments have been implemented in the presence of uncertainty. Moreover, controller parameters are the same as presented in (Li-xin Wang, 1997). The membership functions, have been shown in Fig. 3. these type-2 fuzzy membership functions are triangular. For more information about type-2 fuzzy systems, see the (J.M. Mendel, 2007, J.M. Mendel, 2001).

Fig. 3: Type-2 fuzzy membership functions

Tracking performance for type-1 and type-2 fuzzy system, under certain model condition, is shown in Fig. 4. Also, model parameters are constant in both type-1 and type-2 fuzzy systems during the simulation process.
It is observed from Fig. 4 that the proposed type-2 fuzzy controller reaches to the desirable output faster than other controller. As shown in Fig. 4, the time constant of type-1 and proposed type-2 fuzzy controllers are 0.029 and 0.017, respectively. In addition, simulation results with the presence of system parameters uncertainty have been shown in Figs. 5 and 6. It must be noted that uncertainty has been applied to rotor and load inertia ($J$) and viscous friction coefficient ($\beta$), separately. Also, a random noise with normal distribution around zero point has been applied to $J$ and $\beta$ in the scale of $57 \times 10^{-6}$ and $10^{-4}$, respectively. In figs 5 and 6, uncertainty is applied to $J$ and, respectively and the tracking performance is compared for both adaptive type-1 and type-2 fuzzy controllers. It is obvious that the proposed adaptive type-2 fuzzy controller has better performance in tracking control and also faster response compare to adaptive type-1 fuzzy controller. Moreover, the proposed type-2 fuzzy controller is more robust against changes of the model. The reason is type-2 fuzzy modeling. Therefore, type-2 fuzzy logic systems are more robust against uncertainties.
V. Conclusion:

According to the simulation results and figs in the previous section, it was clear that the proposed adaptive type-2 fuzzy controller has a better tracking control and more robust response in comparison with type-1 fuzzy controller in the presence of both uncertainties and certainties because, type-2 fuzzy logic systems have better performance in the presence of system parameters uncertainties. In fact, using of indefinite (uncertain) membership functions and type-2 fuzzy methods to model nonlinear and indefinite functions, results in handling uncertainties much better than before. Therefore, the effect of uncertainties becomes minimum. Finally, it was shown that the adaptive type-2 fuzzy controller provides more robust performance around operating point and the simulation results verified the main objective of the proposed controller which was accurate angular position control.

REFERENCES


