

## Prediction of Elastic Modulus of Intact Rocks Using Artificial Neural Networks and non-Linear Regression Methods

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**Abstract:** The purpose of this study is developing of a model for estimation of elastic modulus of intact rocks. Mechanical rock excavation projects require static modulus of elasticity (E) of the intact rock material. High-quality core specimens of proper geometry are needed for the direct determination of this parameter. However, it is not always possible to obtain suitable specimens from highly fractured and/or weathered rocks for this purpose. Therefore, models predicting E based on rock index tests and intact rock properties have become alternative methods. For this reason, in this study, the advantage of artificial neural network (specifically multilayer perceptron and radial basis function networks) and non-linear regression were examined. In addition to correlation coefficient (R), root mean square error (RMSE), mean absolute error (MAE) and mean bias error (MBE) were also used for evaluation of prediction accuracy of both ANNs and non-linear regression methods between the measured and predicted parameter values. Finally, the results of this study indicated that MLP-ANN had better performance in prediction of elastic modulus of rocks rather than RBF-ANN and non-linear regression models.

**Key words:** Artificial neural network; Non-linear regression; Elastic modulus.

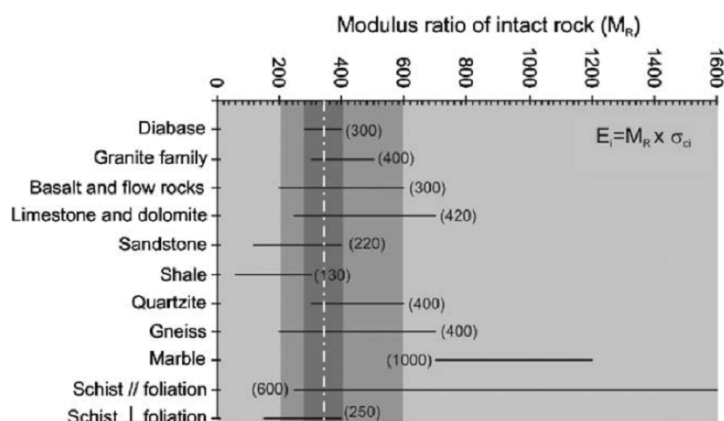
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### INTRODUCTION

One of the most important of intact rock properties for rock engineering projects is the modulus of elasticity (E). Several research (Sachpazis, 1990; Katz *et al*, 2000; Sonmez *et al*, 2006) have emphasized on the importance of the modulus of elasticity Young's modulus (E) and poisson's ratio ( $\nu$ ) are important parameters for understanding the stress- strain behaviour. These parameters are crucial in tunnel design, or rock blasting and drilling, slope stability, pillar design, embankments and many other civil and mining operations (Karakus *et al*, 2005). Young's modulus is also important in determination of the breakage behaviour of rocks under the action of rock cutting picks in mechanical excavation (Tiryaki and Dikmen, 2006). In many rock engineering applications, the elastic modulus of intact rock is not actually determined by laboratory tests-due to the requirements of high quality core samples and sophisticated test equipment such as linear variable Differential Transformer (LVDT), electrical resistance strain gauges and compress meters (Sonmez *et al*, 2006). High-quality core specimens are needed for the direct determination of E. However, high-quality cores in sufficient amount cannot always be extracted from weak, highly fractured, weathered, and/or thinly bedded rocks. To overcome this difficulty, various predictive models using the results of simple index tests and/or mineralogical-petrographic analysis were developed in the past (Bell, 1978; Romana, 1999; Singh *et al*, 2001; Gokecegu, Zolu, 2004; Sonmez *et al*, 2004; Tiryaki, 2008).

Deere performed some well-known studies to repose relation between uniaxial compressive strength and the elastic modulus of intact rock (Deere *et al*, 1968). The relations obtained by Deere depend on the type of rock and are summarized in Table 1.

However, the ranges of the modulus ratios of intact rock ( $M_R$  = uniaxial compressive strength/elastic modulus) are wide and generally overlap for most of the rock types (Fig.1). Therefore, the use of average values of  $M_R$  considering rock type includes considerable uncertainty, and the estimation of  $E_i$  by using the mean value of  $M_R$  can be too rough. Also, there is no generally accepted empirical equation or approach to estimate the elastic modulus of intact rock based on multi-input parameters.



**Fig. 1:** The ranges of modulus ratio of intact rock ( $M_R$ ) for different type of intact rocks // parallel and  $\perp$  perpendicular to the foliation (Deere *et al*, 1968; Sonmez *et al*, 2006).

**Table 1:** the ranges and average value of  $M_R$  for different type of intact rocks (Deer *et al*, 1968; Sonmez *et al*, 2006).

Type of rock	Average value of $M_R^a$	Range of $M_R$
Diabase	300	280-400
Granite family	400	300-500
Basalt and flow rocks	300	200-600
Limestone and dolomite	420	250-700
Sandstone	220	120-400
Shales	130	60-300
Quartzite	400	300-600
Gneiss	400	200-700
Marble	1000	700-1200
Schist//foliation	600	250-1600
Schist $\perp$ forliation	250	150-400

<sup>a</sup> $E_i = MR \times \sigma_{ci}$

The basic idea behind the current research is statement of artificial neural networks and non-linear regression capability to estimate the elastic properties of intact rocks. For this objective, several samples have been supplied from the Seymareh Dam site core boxes. This dam is under construction across Seymarch River in the Zagros zone. Two main formations are exposed in the Seymareh Dam site: Gachsaran (Miocene) and Asmari (Oligocene- Miocene) Formations (Fig. 2). The Gachsaran Formation, composed of salts, anhydrite, marl and gypsum, outcrops in some areas at the lowest elevation of anticlines. The Asmari formation is characterized by layered limestone with inter bedded marl. The Asmari Formation is divided into lower, middle and upper units from different litho-stratigraphical characteristics. Carbonate rocks samples from Asmari formation have been tested in saturation condition according to the ISRM testing standards. These tests consist of the Point Load Index test, Density ( $\rho$ ), Porosity ( $n$ ) Primary wave velocity ( $V_p$ ) and UCS test ( $\sigma_c$ ).

## 2. Previous Studies:

Artificial Neural Networks (ANNs) have received considerable development in recent years, with a wide range of applications in Rock Mechanics and Engineering Geology. Chang and Chao (2006) investigated Application of back-propagation networks in debris flow prediction and those work results indicated that ANN prediction have acceptable accuracy. Zorlu *et al*, (2008) used two different prediction models such as multiple regression and ANN for prediction of uniaxial compressive strength of sandstones. Lee *et al*, (2004) used artificial neural network methods for assessing landslide susceptibility in a chosen study area. Gomez and Kavzoglu (2008) used artificial neural networks to assessment of shallow landslide susceptibility in Jabonosa River Basin. Nefeslioglu *et al*, (2008) employed logistic regression and artificial neural networks for preparation of landslide susceptibility maps. Moosavi *et al*, (2006) used artificial neural networks for modeling the cyclic swelling pressure of mudrock. Das and Basudhar (2008) used artificial neural networks for prediction of residual friction angle of clays.

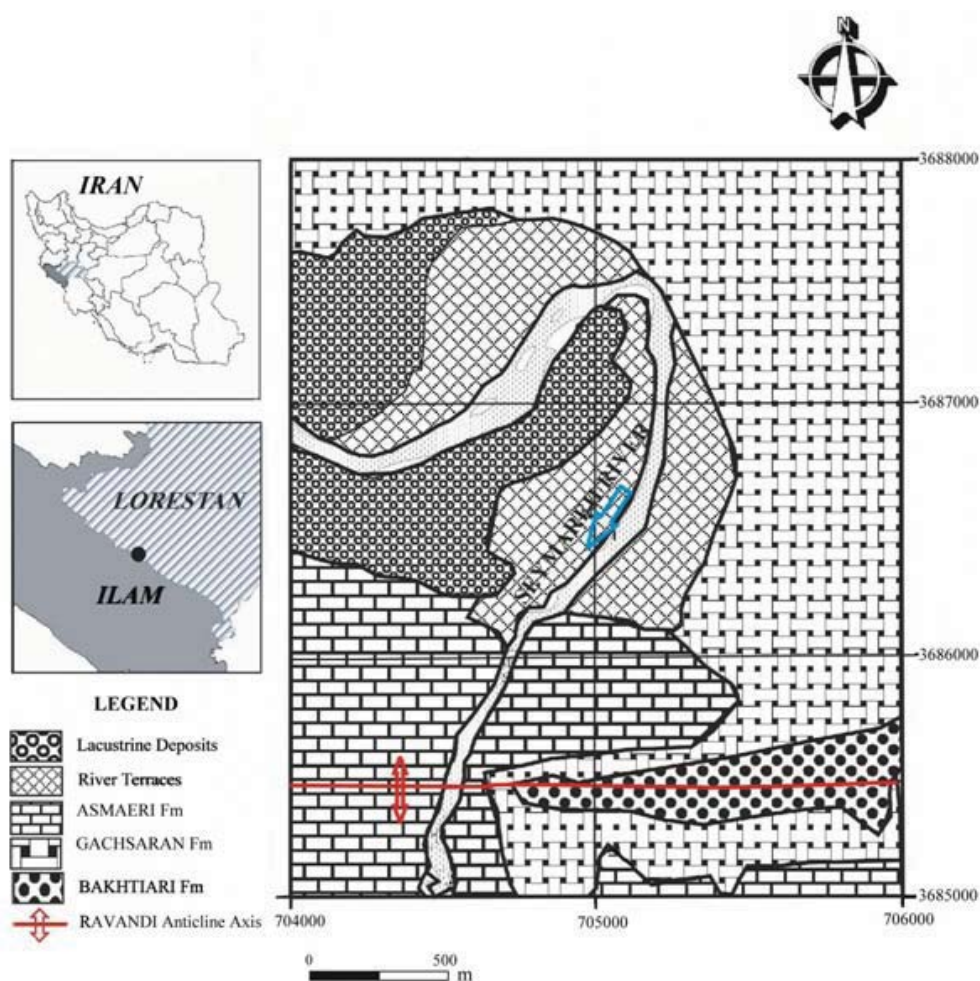


Fig 2: Geology map of the studied area.

Tiryaki (2008) used multivariate statistics, artificial neural networks, and regression trees to predict intact rock strength for mechanical excavation. Maji and Sitharam (2008) used artificial neural networks to predict elastic modulus of jointed rock mass. They selected two artificial neural network models for prediction of elastic modulus of jointed rock mass from the elastic modulus of corresponding intact rock and joint parameters such as joint frequency, joint inclination and joint roughness. The results this research showed that artificial neural networks can be used for the successful prediction of elastic properties of jointed rocks. Sonmez *et al.*, (2006) estimated intact rocks modulus with an artificial neural network. Those work results showed that ANN has a strong prediction capacity and can be used to estimate the modulus of elasticity of intact rock for practical purposes.

### 3. Data Analysis:

The present work uses ANNs and non-linear regression for efficient prediction of the elastic modulus of intact rocks from Primary wave velocity ( $V_p$ ) Density ( $\rho$ ), Porosity ( $n$ ) and UCS test ( $\sigma_c$ ) parameters. These parameters were measured on 92 samples and statistical analysis that were carried out in this study on the relationships between E and other intact rock properties have been based on the data obtained from Seyamreh arc dam project.

According to Fig 2, tested samples were carbonate rocks type (limestone, marl and dolomite) which have been obtained from Asmari formation. Statistical description of examined parameters is given in Table 2. It can also be seen from Table 2 that density  $V_p$ , and UCS distribution of mean and average values are close together and this object represented that distribution of this parameter for experimented samples are normal.

Whereas the ranges of elastic modulus values varies between 2.21-78.96 GPa.

#### **4. Artificial Neural Network Approach:**

A neural network consists of simple synchronous processing elements, Called neurons, which are inspired by biological nerve system, that is artificial neural network is inspired by neuron biology to perform brain-like computation (Malinova and Guo, 2004).

In other hand ANN is a relatively new nonlinear statistical technique and the most particular characteristic of a neural network system is its capability to learn from the data being processed. It can be used to solve problems that are not suitable for conventional statistical methods (Aqil *et al*, 2007). The most commonly used neural network structure is the feed forward hierarchical architecture. Feed forward neural network (FFNN) has a parallel and distributed processing structure. It is composed of three layers. Each layer in a network contains sufficient number of neurons depending on the specific application. The neurons in a layer are connected to the neurons in the next successive layer and each connection carries a weight (Atkinson and Tatnall, 1997).

*Input layer*, which is used to present data to the network. In fact, the input layer receives the data from different sources. Hence, the number of neurons in the input layer depended on the number of input data sources.

*Hidden layer(s)*, which are used to act as a collection of feature detectors. In ANN algorithms, construction of network architecture requires both optimum number of hidden layers between the input and output layers and optimum number of neurons in each layer. This is one of the most important and difficult task, Since there is no unified theory for the optimal architecture (Shahin *et al*, 200; Gullu and Ercelebi, 2007). The number of hidden layers and their neurons are often determined by trial and error.

*Output layer*, which is used to produce an appropriate response to the given input. The output layer contains a single neuron representing elastic modulus.

In this research, several neural models have been used and result indicated that multilayer perceptron (MLP) and radial basis function (RBF) are the best models.

#### **4.1. Architecture of Multilayer Perceptron Network:**

Multilayer perceptron (MLP) is the most common type of neural network used for supervised prediction. A MLP is a feed-forward neural network. In this type network reaction routes always proceed feed-forward and permit for signal to transform only one route that is input to output. By varying number of neurons in the hidden layer, the neural networks are run several times to identify the most appropriate neural network architecture based on training and testing accuracies. A schematic diagram of the best MLP-ANN is given in Fig 3.

In this neural model, we have used to different activation function that included sigmoid and hyperbolic tangent. Furthermore, Levenberg-Marquardt is used as a learning rule. A three-layer artificial neural network with the description of input and output nodes, which was employed in this study is shown in Fig 3.

##### **4.1.1. Activation Functions:**

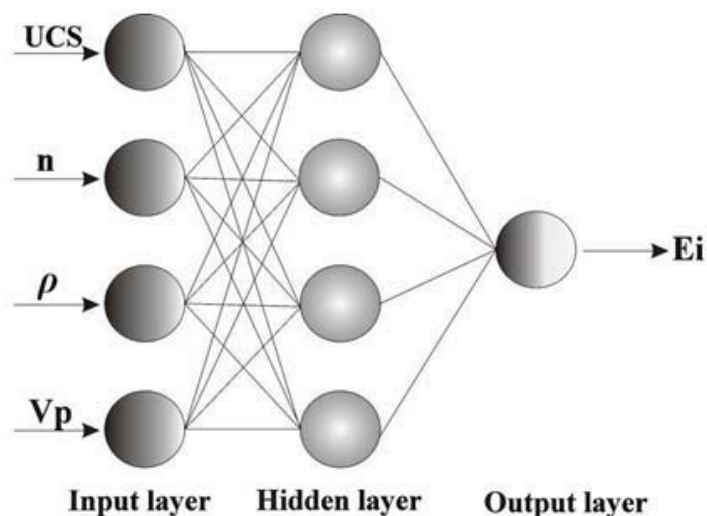
Activation functions are needed to describe the input-output relation at each neuron layer. Nonlinear activation functions are usually employed to introduce nonlinearity into the network. Without this nonlinearity, neurons would perform in a linear fashion and the ANN would not be able to map nonlinear input-output relations.

For the output neuron(s), one should choose a suitable activation Functions to the distribution of the target values an activation functions. Bounded activation functions are particularly useful when the target values have a bounded range. If the target values have unbounded ranges, it is preferable to use an unbounded activation function. Many activation functions have been introduced over the last few years by researchers specialized in ANNs. Two of these activation functions are commonly used (Moosavi *et al*, 2006):

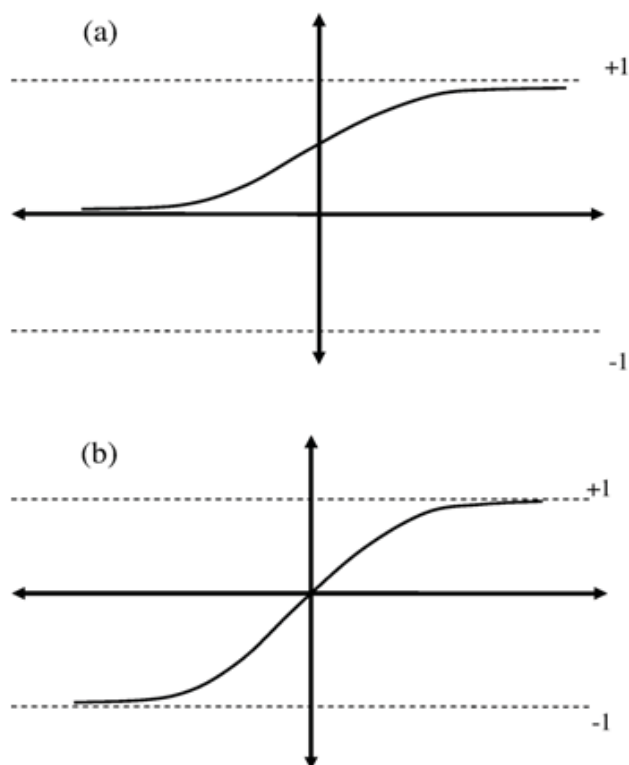
- (a) The log-sigmoid activation function-this activation function, shown in Fig 4a, takes the input, which may have an arbitrary value, and squashes the output into the range 0-1.
- (b) The hyperbolic tangent activation function-this activation function is similar to the log-sigmoid activation function except for the output, which can take values between -1 and +1, as presented in Fig 4b.

##### **4.1.2. Learning Rules:**

The learning rule is a procedure for modifying the weights and biases of the network. This procedure may also be referred to as a training algorithm. The learning rule is applied to train the network to perform some particular task. Learning rules fall into two broad categories, supervised learning and unsupervised learning.



**Fig 3:** Multilayer perceptron artificial neural network architecture constructed in this study.



**Fig 4:** Transfer functions: (a) log-sigmoid transfer function, and (b) hyperbolic tangent transfer function.

**4.2. Architecture of Radial Basis Function Networks:**

RBF is entirely different approach for functional approximation. Development of radial basis network

traces its foundation from interpolation theory. Radial basis learning is equivalent to find a surface in a multidimensional space that provides a best fit to the training data, with a criterion for “best fit” being measured in some statistical sense (Maji and Sitharam,2008). Correspondingly, generalization is equivalent to the use of this multidimensional surface to interpolate the test data. The construction of the RBF network, in its most basic form, involves three layers with entirely different roles.

**Table 2:** Description statistics of examination parameters.

Statistics	UCS (Mpa)	n (%)	$\rho$ (gr/cm3)	E (GPa)
Min	11.5	0.22	2.32	2.21
Max	168.20	20.91	2.73	78.96
Mean	57.35	5.65	2.59	21.58
Medium	48.14	2.49	2.62	16.02

**Table 3:** List of the performance measures.

Statistical parameter	Expression
Correlation coefficient (R)	$R = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2}}$
Root mean square error (RMSE)	$RMSE = \sqrt{\frac{\sum_{i=1}^n (x_i - y_i)^2}{n}}$
Mean absolute error (MAE)	$MEA = \frac{\sum_{i=1}^n  x_i - y_i }{n}$
Mean bias error (MBE)	$MBE = \frac{\sum_{i=1}^n (x_i - y_i)}{n}$

Where  $X_i$  and  $Y_i$  are the observed and predicted data, respectively ;  $\bar{X}$  and  $\bar{Y}$  are the mean of the observed and predicted elastic modulus ; and n is the number of data points.

The input layer is made up of source nodes (sensory units) that connect the network to its environment. The second layer, the only hidden layer in the network applies a nonlinear transformation from the input space to the hidden space. In most applications the hidden space is of high dimensionality. The output layer is linear, supplying the response of the network to the activation pattern (signal) applied to the output layer. The dimension of the hidden space is directly related to the capacity of the network to approximate a smooth input-output mapping (Mhasker,1996; Niyogi and Girosi, 1996).

The higher the dimension of the input space, the more accurate the approximation will be. But it requires larger time to solve and also affect the generalization ability of the network. Therefore getting optimum number of Radial basis network parameters i.e. hidden units and overlap parameter is very important for developing efficient RBF network (Maji and Sitharam, 2008). After several trial and error runs, with different combinations, chosen values of hidden unit and overlap parameter is 25 centers. A schematic diagram of the best RBF-ANN is given in Figs 5. In additions sigmoid and hyperbolic tangent are used as activation function. Furthermore, momentum and step are used as a learning rule. The best architecture of RBF model is shown in Fig 6.

**4.3. Network Performance Evaluation:**

The performances of the models developed in this study were assessed using various standard statistical performance evaluation criteria. The statistical measures considered were correlation coefficient (R), root mean square error (RMSE) mean absolute error (MAE) and mean bias error (MBE). List of the statistical measures are depicted in Table 3.

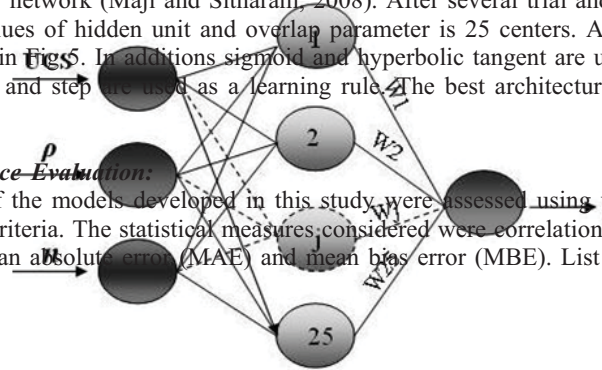


Fig. 5: Radial basis function artificial neural network architecture constructed in this study.

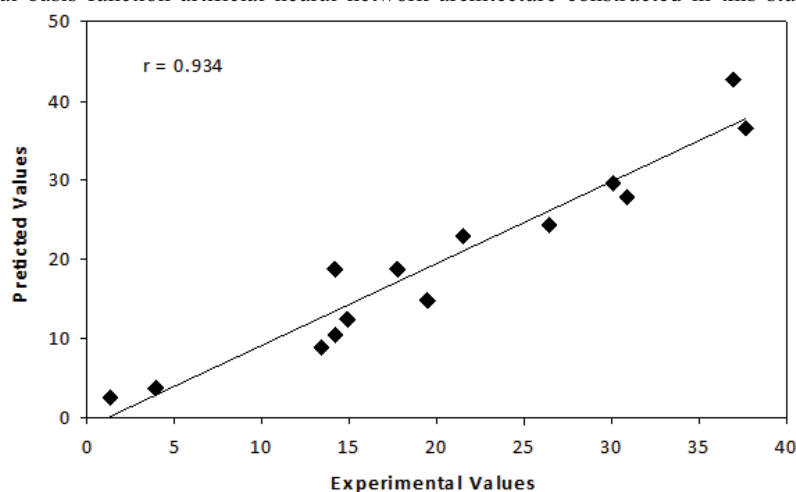


Fig. 6: Correlation diagram for the experimental and predicted values of elastic modulus ratio.

**5. Non-linear Regression Approach:**

In statistics, nonlinear regression is a form of regression analysis in which observational data are modelled by a function which is a nonlinear combination of the model parameters and depends on one or more independent variables. The data are fitted by a method of successive approximations. Some nonlinear regression problems can be moved to a linear domain by a suitable transformation of the model formulation. The equation representing this model can be written in the following form (Tiryaki, B. 200):

$$Y = aX_1^{b_1} X_2^{b_2} \dots X_n^{b_n} \tag{1}$$

Where Y is the predicted value corresponding to the response, a is the intercept,  $X_1, X_2$ , and  $X_n$  are the predictors and  $b_1, b_2$  and  $b_n$  are the regression coefficients of  $X_1, X_2$  and  $X_n$ . Taking logarithms of both sides of Eq. (1) converts the model into the following linear form:

$$\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \dots + b_n \log X_n \tag{2}$$

Eq.(2) can be written as the linear regression function as follows:

$$Y' = a' + b_1 X'_1 + b_2 X'_2 + \dots + b_n X'_n \tag{3}$$

where  $Y'$ ,  $X'_1, X'_2$ , and  $X'_n$  are the logarithms of the observations for variables Y,  $X_1, X_2$  and  $X_n$  respectively whereas a' is the logarithm of value a.

Therefore, values of each variable used in nonlinear regression study have been transformed by taking their logarithms before performing a multiple linear regression that resulted in E model as given in Eq.(4):

$$\log E = -0.85448 + 0.91326 \log \sigma_c + 0.03198 \log n + 0.16123 \log Vp - 0.22327 \log \rho \quad (4)$$

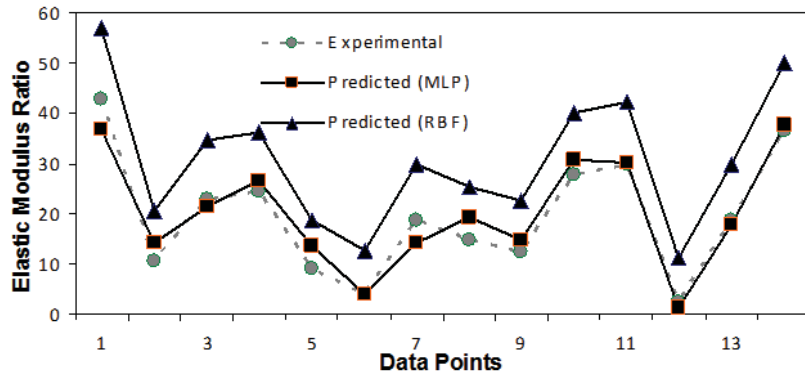
When the regression equations of the models include nonlinear terms. Such as exponential, logarithmic, or power functions, the regression models are nonlinear (Lin *et al*, 2007). In this research, accuracy of the regression equations was evaluated using R, RMSE, MAE, and MBE between the measured or fitted and predicted values.

**RESULT AND DISCUSSION**

In this research MLP and RBF algorithm have been used. By means of trial and error, an optimum Network and parameter configuration for two networks was derived Table 4. summarize the results of the four better architecture networks. As can be seen from the results in Table 4,the MLP-ANN performed better than RBF-ANN model. The best network was proposed with multilayer perceptron model, sigmoid activation function. Levenberg-Marquardt learning rule with 4-4-1 architecture neurons for hidden layer in 1000 numbers of epochs. In addition, from 92 experimental data sets. 70 percent of data sets are used for training the networks for both cases. Furthermore, 15 percent of data sets have been used for testing and 15 percent for crust validation. The accuracy of prediction is also estimated by drawing a line to represent the 100% accuracy of prediction (Fig 6). The points closer to the 45 degree line, show the better prediction.

Data points which are touching the line, show the predictions which are exactly same as experimental ones. The error distribution for the 14 individual test data has also been shown in the comparison plots for the two models with experimental results sets (Fig 7).

Here, it could be observed that the line which represents the MLP results is most closely matching with the actual experimental results.

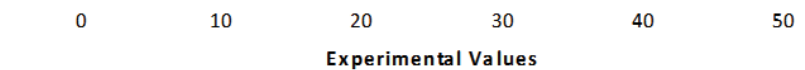


**Fig. 7:** Comparison of MLP and RBF in prediction of elastic modulus.

**Table 4:** Properties of the optimum ANNs architectures.

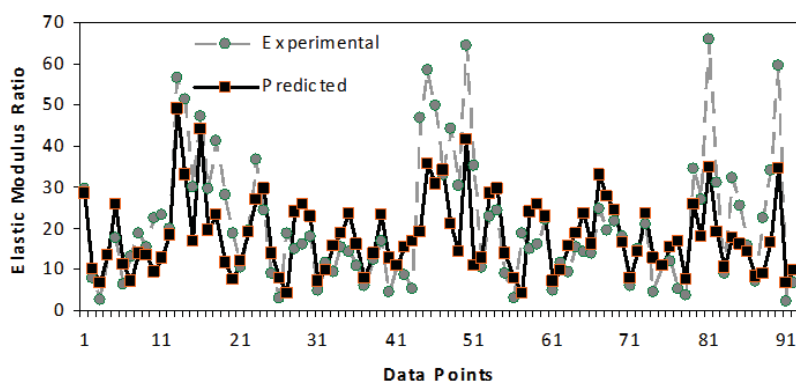
Neural model	Transfer function	Learning rule	Array	Learning stage			Testin stage			
				r	RMSW	MAE	MBE	r	RMSW	MAE
RBF	Sigmoid	Momentum	3-4-1	0.88	0.177	2.975	2.526	0.77	0.192	3.2112.725
RBF	Tan.h	70 step	3-4-1	0.81	0.194	3.325	3.124	0.73	0.288	3.5573.189
MLP	Sigmoid	Leven berg-marquardt	4-4-1	0.95	0.128	2.576	-0.566	0.93	0.132	2.758-0.655
MLP	Tan h	Levenberg-marquardt	4-4-1	0.91	0.140	2.673	-0.624	0.820	0.160	2.924-0.781

Several models of non-linear regression that make from various combination of independent variables with summary of statistics are represented in Table 5. In this models density ( $\rho$ ), porosity ( $n$ ), primary wave velocity ( $Vp$ ) and Uniaxial compressive Strength test ( $\sigma_c$ ) was independent variables. It can also be seen from the Table 5 that model 4 with R, RMSE, MAE and MBE 0.73, 0.451, 0.199, -0.004 respectively has better performance. In addition, correlation diagram for the experimental and predicted values of elastic modulus ratio are shown in Figs.8-9.





**Fig. 8:** Correlation diagram for the experimental and predicted values of elastic modulus ratio in non-linear regression model.



**Fig. 9:** Comparison of MLP and RBF in prediction of elastic modulus.

**Table 5:** Comparison of the goodness of fit of non-linear regression models.

Models	Parameters	R	RMSE	MAE	MBE
Model 1	( $\sigma_c$ )	0.58	0.577	2.758	-0.026
Model 2	( $\sigma_c, \rho$ )	0.61	0.569	2.312	-0.016
Model 3	( $\sigma_c, \rho, n$ )	0.70	0.516	1.124	-0.012
Model 4	( $\sigma_c, \rho, n, Vp$ )	0.73	0.461	0.199	-0.004

**Conclusions:**

In this research, ANNs and non-linear regression capability for predicting of elastic modulus of intact rocks was investigated. The results indicated that ANNs and non-linear models are acceptable approach for Predicting of elastic modulus. Although ANN had better performance in predicting of elastic modulus of intact rocks than regression model, the best network was proposed with multilayer perceptron model, sigmoid activation function, Levenberg-Marquardt learning rule with 4-4-1 architecture.

The ANN and non-linear regression models that proposed in this research, can be used for overcome on problems was to measure of elastic modulus of intact rocks.

**ACKNOWLEDGMENTS**

The authors would like to thanks the college of science of Bu-Ali Sina University for financial support. Collaboration of the Iran Water and Power Resources Development Company is gratefully acknowledged.

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