Processing of Information Security Risks with Ordered Weighted Averaging Operators

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Abstract: In 1988 Ronald R. Yager introduced a new aggregation technique based on the ordered weighted averaging (OWA) operators. The aim of this paper is to present a short survey of OWA operators and illustrate their applicability for risk processing. In this article, we use OWA operators for decision making on selection of counter-measures for reduction of risks.

Key words: Decision making; OWA operator; Probabilities; risk modification

INTRODUCTION

Provision of information security in modern information systems is based on information security risk management. Risk management process contains risk analysis, risk assessment, risk evaluation, risk processing and informing the users about risks (ISO Guide, 2009). Risk processing is a process of selection and realization of actions by modification of risk. Risk processing actions can include acceptance, rejection, reduction, transfer or insurance of risk. One of the processing mechanisms of information security risks is reduction of risks by using correct selection of counter-measures against threats. While choosing counter-measures it’s necessary to consider several criterions. In this article ordered weighted averaging operators are used for risks processing of information security (Yager. R.R., 1988). OWA operators consider decision making person’s behavior (risk avoidance or risk acceptance) and interaction among criterions and from this perspective OWA method has a supremacy in comparison with other multi-criteria decision making models (Multi Criteria Decision Making), also TOPSIS (Technique for Order Preferences by Similarity to Ideal Solution) and AHP (Analytic Hierarchy Process).

A very efficient for information combination method OWA was suggested by R. Yager (1988). Since then OWA operators are studied from different aspects, and applied in engineering and different fields of artificial intellect (Jiang, H., 2000; Xu, Z.S., 2003).

Owa Operators:
Definition:
An OWA operator of dimension n with an associated vector \( W = (w_1, \ldots, w_n) \) is a mapping

\[ F : R^n \rightarrow R \]

defined as

\[ F(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \]

Where \( b_j \) is the \( j^{th} \)-th largest element of the of the bag \( <a_1, \ldots, a_n> \), \( w_j \in [0,1] \), \( \sum_{j=1}^{n} w_j = 1 \)

For example, the value of OWA operator which is given with the vector for the bag \( <0.7, 1.0, 0.2, 0.6> \) will be calculated as following:

\[ F(0.7, 1.0, 0.2, 0.6) = 0.4 \times 1.0 + 0.3 \times 0.7 + 0.2 \times 0.6 + 0.1 \times 0.2 = 0.75 \]

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The fundamental aspect of this operator is the re-ordering step, in particular an aggregate \( a_i \) is not associated with a particular weight \( w_i \) but rather a weight is associated with a particular ordered position of aggregate.

It is noted that different OWA operators are distinguished by their weighting function. R.Yager pointed out three important types of OWA operators:

1) \( F^* : W = W^* = (1; 0; \ldots; 0)^T \) and \( F^*(a_1, \ldots, a_n) = \max \{a_1, \ldots, a_n\} \)

2) \( F : W = W_1 = (0; 0; \ldots; 1)^T \) and \( F(a_1, \ldots, a_n) = \min \{a_1, \ldots, a_n\} \)

3) \( F_{\text{mean}} : W = W_2 = (1/n; 1/n; \ldots; 1/n)^T \) and \( F_{\text{mean}}(a_1, \ldots, a_n) = \frac{a_1 + \ldots + a_n}{n} \)

There are several important properties (commutative, monotonicity, idempotency and limitation) of OWA operators. Let’s have a short look on limitation characteristics. Each OWA operator meets an inequality

\[ F(a_1, \ldots, a_n) \leq F^*(a_1, \ldots, a_n) < F^*(a_1, \ldots, a_n) \]

In other words, value of operator is between \( \min \{a_1, \ldots, a_n\} \) and \( \max \{a_1, \ldots, a_n\} \)

OWA operators have an important parameter identified by orness function; it can be also defined as a degree of risk acceptance. R.Yager defined orness function for \( W \) weight vector as following (Yager, R.R., 1988):

\[
\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-1)w_i
\]  

(2)

It can be shown that, \( 0 \leq \text{orness} \leq 1 \) A little value of orness illustrates risk avoidance, big value illustrates order acceptance of risk. As we can see from definition of OWA operator, identification of aggregate weights \( w_i \) is an essential issue (R.R Yager, 1993). There are several methods for calculation of aggregate weights; the most used is a method suggested by R. Yager based on linguistic quantifier. Decision makers identify Q linguistic quantifier (for example, "many"). Linguistic quantifier \( Q \) can be illustrated as a fuzzy subset of \( I \) single interval, for every \( r \in I \) value of \( Q(r) \) shows in what degree \( Q \) meets a concept marked as \( Q \). If \( Q \) is a regularly growing monotone qualifier, then aggregate weights can be calculated with following formula:

\[
w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i=1,\ldots,n
\]  

(3)

Following formula is widely used as \( Q \) linguistic quantifier:

\[ Q(r) = r^a, a \geq 0 \]  

(4)

The orness function of calculated aggregate weights is as following:

\[
\text{orness}(w) = \int_0^1 Q(r)dr = \int_0^1 r^adr = \frac{1}{\alpha + 1}
\]  

(5)

If \( a > 1 \), it will be orness \( (w) < 0.5 \) and it illustrates the avoidance of decision makers from risk. If \( a = 1 \), it will be orness \( (w) < 0.5 \) and illustrates neutrality of decision maker against risk. If \( a < 1 \), it will be orness \( (w) < 0.5 \) and it illustrates secure risk acceptance of decision maker.

**OWA Approach for Risk Processing:**

Risk processing is a process of selection and realization of actions on risk modification. Actions on risk processing can include keeping the risk as before, rejection of risk, reduction, transfer and insurance of risk. In this article, we use OWA operators for decision making on selection of counter-measures for reduction of risks.
It’s advisable to express selection of counter-measures as multi-dimensional decision making problem. Let’s presume that, there are \( a_i, i = 1, \ldots, n \) alternatives for counter-measures. There alternatives are estimated by \( f_j, j = 1, \ldots, k \) criterions. Let us consider the estimation of \( a_i \) alternative by \( f_j \) criterion as \( v_{ij} \). Using these marks, multi-dimensional problem of decision making can be written in matrix form (for example, rows are alternatives, columns are criterions). It is required to choose alternatives by this method, which meets as many criterions as possible. \( v_{ij} \) Values can be precise and fuzzy as well. For example, \( f_j \) criterions are considered as fuzzy sets and \( V_{ij} \) value illustrates belonging degree of \( a_i \) alternative to this fuzzy set, in this case \( V_{ij} \in [0,1] \).

In this issue linguistic version of OWA operator – LOWA will be used (Herrera, F., 1996). In this method, arithmetic scale relevant to linguistic scale is used and it is presumed that \( v_{ij} \) takes values from the ordered scale \( S = \{ S_1, \ldots, S_n \} \). Linguistic values of \( a_i \) alternative are recursively identified with LOWA operator according to aggregate weights \( W \) as following:

\[
C^m(W, v_i) = C^2 \left( (w_1, 1-w_1), (v_{i,(1)}, v_{i,(2)}), \ldots, (v_{i,(m)}, v_{i,(m-1)}), \ldots, (v_{i,(n)}, v_{i,(n-1)}) \right), \quad m > 2
\]

\[
v' = \left( v_{i,(2)}, \ldots, v_{i,(n)} \right), \quad W' = \left( w_2/(1-w_1), \ldots, w_n/(1-w_1) \right)
\]

\[
C^2 \left( (w_1, w_2), (v_{i,1}, v_{i,2}) \right) = s_k
\]

\[
k = \min \left( r, \text{height}(v_{i,(2)}) + \text{round} \left( w_i \cdot (\text{height}(v_{i,(1)}) - \text{height}(v_{i,(2)})) \right) \right)
\]

In these expressions, \( \sigma \) is a permutation of \( v \), where \( v_{i,(1)} \geq v_{i,(2)} \). \( \text{height}(v_{i,j}) \) Function shows the position of \( V_{ij} \) in the scale \( L \).

**Result:**

OWA operators consider decision making person’s behavior (risk avoidance or risk acceptance) and interaction among criterions and from this perspective OWA method has a supremacy in comparison with other multi-criteria decision making models (Multi Criteria Decision Making), also TOPSIS (Technique for Order Preferences by Similarity to Ideal Solution) and AHP (Analytic Hierarchy Process).

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