

Mhd Effects on Convective Flow of Dusty Viscous Fluid with Fraction in a Porous Medium

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Abstract: This paper deals with laminar convective flow of an incompressible, conducting, viscous fluid embedded with non –conducting dust particles through a porous medium in the presence of uniform magnetic field and constant pressure gradient taking volume fraction of a dust particles into account when one plate of the channel is fixed and the other is oscillating in time and magnitude about a constant non-zero mean. Solutions of the equations governing the flow are obtained for the velocity of fluid and dust particles. The skin friction and heat transfer are also obtained. The effects of various parameters on the velocities, skin friction and heat transfer are discussed. It is found that both the velocity of the liquid and dust particles decreases with the increase in the porous parameter (ϵ_3).

Key words: Magnetohydrodynamics, Convective Flow, Viscous Fluid and Skin Friction

INTRODUCTION

The influence of dust particles on convective flow of dusty viscous fluids has its importance in many application such as wastewater treatment, power plant piping, combustion and petroleum transport. Particularly, the flow and heat transfer of electrically conducting fluids in channels under the effect of a transverse magnetic field occur in magnetohydrodynamic (MHD) accelerators, pumps and generators. This type of flow has uses in nuclear reactors, geothermal systems and filtration, among others. The possible presence of dust particles in combustion MHD generators and their effect on the performance of such devices led to studies of volume fraction of dust particles in non-conducting walls in the presence of uniform transverse magnetic field.

The study of convective flow of dusty viscous fluid under the influence of different physical conditions has been carried out by several authors like: Nag and Jana (1979) have studied unsteady couette flows of a dusty gas between two infinite parallel plates, when one plate is kept fixed and the other plate moves in its own plane. The problem has been solved with the help of Laplace Transform technique. It is found that the dust velocity in the case of accelerated start of the plate is less than the fluid velocity, for moderate value of the relaxation time of the dust particles become very fine. It is observed that the magnitude of the shear stress is larger when the plate starts with uniform acceleration than when it is impulsively started to move with uniform velocity. The paper of Nag and Jana (1979) did not examined time dependent plane, transient effects and wave structure of the fluid. Kulshretha and Puri (1981), have investigated the couette flow of a dusty gas due to an oscillatory motion of the plate. The time dependent plate and transient effects have been included. The dusty gas contained between two parallel plates is disturbed by the motion of the lower plate with an

arbitrary velocity $F(t)$. When $F(t)$ contains a factor of the type $\exp \{-(\lambda^2 - i\omega)t\}$, two distinct types of waves

are generated, one of which is oscillatory and the other is non-oscillatory which disappears for $\lambda = 0$. Reflections of these waves are studied and graphs for the wave speeds are presented. Long time approximations for this type of $F(t)$ are evaluated and steady state solutions are obtained for $F(t)$ of the type $\exp(i\omega t)$.

Bratsun and Teplov (2000), also studied two phase flow in a tall vertical slot differently heated from the side walls, where one of the phase is fluid and another phase consists of small solid particles. The particles

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are subjected to downward drag exerted by the gravity and the drag exerted by finite frequency horizontal vibrations along the layer. In the framework of the generalized Boussinesq approximations non-contradictive set of governing equations describing the dynamics of suspension were derived. The pulsed base flow is obtained and its linear stability is analysed. The comparison of the numerical results with the recent experimental findings is given. In this study, the magnetic field and temperature field were not considered.

Attia (2002) studied the effects of variable viscosity on the unsteady flow of an electrically conducting, viscous, incompressible dusty fluid and heat transfer between parallel non-conducting porous plates. The fluid is driven by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The governing non-linear partial differential equations are solved numerically using finite differences. The effect of the variation in the viscosity and electric conductivity of the fluid and the uniform magnetic field on the velocity and temperature fields for both the fluid and dust particles is discussed.

Attia and Aboul-Hassan (2002), studied the flow of a conducting, viscoelastic fluid between two horizontal porous plates in the presence of transverse magnetic field. The plates are assumed to be non-conducting and maintained at two fixed but different temperatures. The fluid viscosity is assumed to be temperature dependent and the fluid is subjected to a uniform suction from above and injective from below. The motion of the fluid is produced by a uniform horizontal pressure gradient. The equation of motion and energy equation are solved numerically to yield the velocity and temperature distributions.

Attia (2006), investigated the time varying couette flow with heat transfer of a dusty viscous incompressible, electrically conducting fluid under the influence of a constant pressure gradient is studied without neglecting the Hall Effect. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below while the fluid is acted upon by an external uniform magnetic field applied perpendicular to the plates. The governing equations are solved numerically using finite differences to yield the velocity and temperature distribution for both the fluid and dust particles. It is found that both the fluid and the solid particle phases have two components velocity. The main two components of velocity of the fluid and dust particles, u and u_p respectively, are found to increase with an increase in the Hall parameter m . However, the other two components of velocity ω and ω_p , which result due to the Hall Effect, increase with the Hall parameter m for small m and decrease with m for large values of m . It is also found that the temperatures of both fluid and particles phases decrease with the Hall parameter. In these studies, the volume fraction of dust particles is neglected. However, the assumption of ignoring volume fraction of the particles is not justified for high fluid densities or high particle mass fraction where the volume fraction of the particles may become significantly large and cannot be neglected.

Singh and Singh (2002) studied the laminar convective flow of an incompressible, conducting, viscous fluid embedded with non-conducting dust particles through a vertical parallel plate channel in the presence of uniform magnetic field and constant pressure gradient taking volume fraction of dust particles into account when one plate of the channel is fixed and the other is oscillating in time and magnitude about a constant non-zero mean. Solutions of the equation governing the flow are obtained for the skin velocity of the fluid and dust particles. The expression for skin friction and heat transfer is also obtained. The effects of various parameters on the velocities, skin friction and heat transfer are discussed. In this study porous parameter is not considered.

In the present paper we discuss the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of uniform transverse magnetic field with volume fraction and considering porous parameter.

Mathematical Formulation and Solution:

In Cartesian co-ordinate system, we consider unsteady laminar flow of a dusty, incompressible, Newtonian, electrically conducting, viscous fluid through a porous medium of uniform cross section h , when one wall of the channel is fixed and the other is oscillating in time about a constant non-zero mean Ibrahim et al (2010). Initially at ($t \leq 0$) the channel wall as well as the fluid are assumed to be at the same temperature T_0 . When $t > 0$, the temperature of the channel walls is instantaneously raised to T_w which oscillate with time and is thereafter maintained constant. Let x -axis be along the flow of liquid at the fixed wall and y -axis perpendicular to it. A uniform magnetic field of strength $B_0 (= \mu_c H_0)$ is applied perpendicular to the flow region.

Assumptions Ibrahim et al (2010):

The governing equations are written based on the following assumptions:

The dust particles are solid, spherical, non-conducting equal in size and uniformly distributed in the flow region. This means that the dust particles gain heat energy from the fluid by conduction through their spherical surface.

1. The number density of dust particles is constant and the temperature between the particles is uniform throughout the motion. It is an incompressible fluid, therefore the density is constant and also to prevent energy loss between the particles.
2. The interactions between the particles, chemical reaction and radiation between the particles and liquid have not been considered. This is necessary in order to avoid multiple equations.
3. The buoyancy force, induced magnetic field and Hall effects have been neglected. This means that the flow region has uniform temperature, uniform applied magnetic field and a Cartesian coordinate.
4. The volume occupied by the particles per unit volume of the mixture, (i.e., volume fraction of dust particles) and mass concentration have been taken into consideration.
5. The magnetic Reynolds number is taken to be very small so that induced magnetic field is negligible. This means that a uniform magnetic field B_0 is applied in the positive y-direction and is the only magnetic field in the problem.
6. The dust concentration is so small so that it is not disturbing the continuity and hydro magnetic effects. This means that the continuity equation is satisfied.

Governing Equations (Ibrahim et Al (2010):

The fluid flow is governed by the momentum and energy equation under the above assumptions:

$$(1 - \phi) \frac{\partial u}{\partial t} = (1 - \phi) \left[\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} + g \beta^+ (T - T_0) \right] + \frac{KN_0}{\rho} (v - u) + \frac{KN\sigma\mu_c^2 H_0^2}{\rho} u - \frac{\mu}{K_1} u \quad (1)$$

$$N_0 m \frac{\partial v}{\partial t} = \phi \left[\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g \beta^+ (T - T_0) \right] + KN_0 (u - v) \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions to the problem are:

$$\begin{aligned} t \leq 0; \quad u(y,t) = v(y,t) = 0, \quad T(y,t) = 0 \quad \text{for } 0 \leq y \leq 1 \\ t > 0; \quad u(y,t) = v(y,t) = 0, \quad T(y,t) = 0 \quad \text{at } y = 0 \end{aligned} \quad (4)$$

$$u(y,t) = v(y,t) = 1 + \varepsilon e^{\text{int}} \quad T(y,t) = 1 + \varepsilon e^{\text{int}} \quad \text{at } y = 1$$

Where $u(y, t)$ is the velocity of the fluid and $v(y, t)$ velocity of the dust particles, m is the mass of each dust particle, N_0 is the number density of dust particles, T is the temperature. T_0 is the initial temperature, T_w is the raised temperature, β^+ is the volumetric coefficient of thermal expansion. C_p is the specific heat at constant pressure, f is the volume fraction of dust particles (i.e., the volume occupied by the particles per unit volume of the mixture), K is the Stoke's resistance coefficient ($=6\pi\mu r$ for spherical particles of radius r). H_0 is the magnetic field induction, μ_c is magnetic permeability, σ is the electrical conductivity of the liquid, κ is thermal conductivity and K_1 is the porous parameter.

The first term in the right hand side of equation (1) consists of pressure gradient while the second is the viscous flow and the third buoyancy force terms respectively. The last three terms represent the force term due to the relative motion between fluid and dust particles, magnetic and porous terms respectively. While the left hand side represent streamwise velocity unsteady term.

From equation (2) the left hand side signifies unsteady normal velocity expressed in terms of pressure and viscous dissipation terms while equation (3) is the energy balance.

The problem is simplified by writing the equations in the non-dimensional form. The characteristic length is taken to be h and the characteristic velocity is v . We introduce the following non-dimensional variables:

$$x^* = \frac{x}{h}, y^* = \frac{y}{h}, p^* = \frac{h^2 p}{\rho v^2}, t^* = \frac{vt}{h^2}, u^* = \frac{uh}{v}, T^* = \frac{T-T_0}{T_w-T_0} \quad \text{and} \quad v^* = \frac{vh}{v} \tag{5}$$

Substituting equations (5) in equation (1-3), and then removing asterisks, we get

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + G_r T + \varepsilon_1(v-u) - \varepsilon_2 Mu - \varepsilon_3 u \tag{6}$$

$$f \frac{\partial v}{\partial t} = \phi \left[-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + G_r T \right] + \beta(u-v) \tag{7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

Where

$$G_r = \frac{g\beta^+(T_w-T_0)h^3}{\nu^2} \quad (\text{Grashof number}), \quad \varepsilon_1 = \frac{f}{\sigma_1(1-\phi)}, \sigma_1 = \frac{mv}{Kh^2}, \varepsilon_2 = \frac{1}{1-\phi}, M = \mu_c^2 h^2 H_0^2 \frac{\sigma}{\mu} \quad (\text{Magnetic parameter}), f = \frac{mN_0}{\rho} \quad (\text{mass concentration of dust particles}), \varepsilon_3 = \frac{\mu h^2}{K_1(1-\phi)} \quad (\text{Porous Parameter}), \beta = \frac{f}{\sigma_1}.$$

(concentration resistance ratio) and $P_r = \frac{\mu C_p}{\kappa}$ (Prandtl number).

The non-dimensional boundary conditions are:

$$\begin{aligned} t \leq 0; \quad u(y,t) = v(y,t) = 0, \quad T(y,t) = 0 \quad \text{for} \quad 0 \leq y \leq 1. \\ t > 0; \quad u(y,t) = v(y,t) = 0 \quad T(y,t) = 0 \quad \text{at} \quad y = 0. \\ u(y,t) = v(y,t) = 1 + \varepsilon e^{\text{int}}, \quad T(y,t) = 1 + \varepsilon e^{\text{int}}, \quad \text{at} \quad y = 1. \end{aligned} \tag{9}$$

The solution to equations (6) - (9) are in the form of the following Soundalgekar and Bhat (1984) equations.

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y) e^{\text{int}} \\ v(y,t) &= v_0(y) + \varepsilon v_1(y) e^{\text{int}} \\ T(y,t) &= T_0(y) + \varepsilon T_1(y) e^{\text{int}} \end{aligned} \tag{10}$$

$$\frac{\partial p}{\partial x} = P = \text{Const}$$

We solve equations (6-8) under the boundary conditions (9) Substituting equation (2) in equations (6-8), we get

$$u_0''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3) u_0(y) + \varepsilon_1 v_0(y) = P - G_r T_0(y) \tag{11}$$

$$\beta v_0(y) = \beta u_0(y) + \phi [u_0''(y) - P + G_r T_0(y)] \tag{12}$$

$$T_0''(y) = 0 \tag{13}$$

$$u_1''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3 + in)u_1(y) + \varepsilon_1 v_1(y) = -G_r(y)T_1(y) \tag{14}$$

$$(\beta + nif) v_1(y) = \beta u_1(y) + \phi [u_1''(y) + G_r T_1(y)] \tag{15}$$

$$T_1''(y) - in p_r T_1(y) = 0 \tag{16}$$

The corresponding boundary conditions are now

$$\begin{aligned} u_0(y) = u_1(y) = v_0(y) = v_1(y) = 0, T_0(y) = T_1(y) = 0 \quad \text{at } y = 0 \\ u_0(y) = u_1(y) = v_0(y) = v_1(y) = 1, T_0(y) = T_1(y) = 1 \quad \text{at } y = 1 \end{aligned} \tag{17}$$

The solution to (11-16) subject to the boundary conditions (17) are
By solving equation (13), we obtain

$$T_0(y) = y \tag{18}$$

Substituting equation (18) in equation (11), and (12), we obtain

$$u_0''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3)u_0(y) + \varepsilon_1 v_0(y) = P - G_r y \tag{19}$$

$$\beta v_0(y) = \beta u_0(y) + \phi [u_0''(y) - P + G_r y] \tag{20}$$

Substituting equation (20) in equation (19), we obtain

$$u_0''(y) - A^2 u_0(y) = P - G_r y \tag{21}$$

Where $A^2 = \frac{\beta(\varepsilon_2 M + \varepsilon_3)}{\beta + \phi \varepsilon_1}$

By solving equation (2.11), we obtain

$$u_0(y) = \frac{P}{A^2} (\cosh Ay - 1) + \left[1 - \frac{P(\cosh A - 1) + G_r}{A^2} \right] \frac{\sinh Ay}{\sinh A} + \frac{G_r y}{A^2} \tag{22}$$

The First and Second partial derivatives of equation (22) are:

$$u_0'(y) = \frac{P}{A} \sinh Ay + \left[1 - \frac{P(\cosh A - 1) + G_r}{A^2} \right] A \frac{\cosh Ay}{\sinh A} + \frac{G_r}{A^2} \tag{23}$$

$$u_0''(y) = P \cosh Ay + \left[1 - \frac{P(\cosh A - 1) + G_r}{A^2} \right] A^2 \frac{\sinh Ay}{\sinh A} \tag{24}$$

Substituting equations (23) and (24) in equation (12) we obtain

$$v_0(y) = \frac{A_1 P}{A^2} (\cosh Ay - 1) + A_1 \left[1 - \frac{P(\cosh A - 1) + G_r}{A^2} \right] \frac{\sinh Ay}{\sinh A} + \frac{A_1 G_r y}{A^2} \tag{25}$$

Where $A_1 = 1 + \frac{\phi}{\beta} A^2$

By solving equation (16), we obtain

$$T_1(y) = \frac{\sinh L_0}{\sinh L_0} y \tag{26}$$

Substituting equation (26) in equations (14) and (15), we obtain

$$u_1''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3 + in)u_1(y) + \varepsilon_1 v_1(y) = -\frac{G_r \sinh L_0}{\sinh L_0} y \tag{27}$$

$$(\beta + nif)v_1(y) = \beta u_1(y) + \phi \left[u_0''(y) + \frac{G_r \sinh L_0}{\sinh L_0} y \right] \tag{28}$$

Substituting equation (28) in equation (27), we obtain

$$u_1''(y) - B^2 u_1(y) = -\frac{G_r \sinh L_0}{\sinh L_0} y \tag{29}$$

By solving equation (29), we obtain

$$u_1(y) = \left(1 + \frac{G_r}{(L_0^2 - B^2)} \right) \frac{\sinh By}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{\sinh L_0 y}{\sinh L_0} \tag{30}$$

The First and Second partial derivatives of equation (30) are:

$$u_1'(y) = \left(1 + \frac{G_r}{(L_0^2 - B^2)} \right) \frac{B \cosh By}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{L_0 \cosh L_0 y}{\sinh L_0} \tag{31}$$

$$u_1''(y) = \left(1 + \frac{G_r}{(L_0^2 - B^2)} \right) \frac{B^2 \sinh By}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{L_0^2 \sinh L_0 y}{\sinh L_0} \tag{32}$$

Substituting equations (31) and (32) in equation (28), we obtain

$$v_1(y) = A_2 B_0 \left[\left(1 + \frac{G_r}{L_0^2 - B^2} \right) \frac{\sinh By}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{\sinh L_0 y}{\sinh L_0} \right] \tag{33}$$

Where $A_2 = 1 + \frac{\phi B^2}{\beta}$ and $B_0 = \frac{\beta(\beta - nif)}{\beta^2 + n^2 f^2}$

Substituting equations (22) and (30) in equation (10), we obtain

$$u(y, t) = \frac{P}{A^2} (\cosh Ay - 1) + \left[1 - \frac{P(\cosh A - 1) + G_r}{A^2} \right] \frac{\sinh Ay}{\sinh A} + \frac{G_r y}{A^2} + \varepsilon \left[\left(1 + \frac{G_r}{(L_0^2 - B^2)} \right) \frac{\sinh By}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{\sinh L_0 y}{\sinh L_0} \right] e^{\text{int}} \tag{34}$$

Substituting equations (25) and (33) in equation (10), we obtain

$$v(y, t) = \frac{A_1 P}{A^2} (\cosh Ay - 1) + A_1 \left[1 - \frac{P(\cosh A - 1) + G_r}{A^2} \right] \frac{\sinh Ay}{\sinh A} + \frac{A_1 G_r y}{A^2} + \varepsilon A_2 B_0 \left[\left(1 + \frac{G_r}{(L_0^2 - B^2)} \right) \frac{\sinh By}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{\sinh L_0 y}{\sinh L_0} \right] e^{\text{int}} \tag{35}$$

Also, substituting equations (13) and (26) in equation (10), we obtain

$$T(y, t) = y + \varepsilon \frac{\sinh L_0}{\sinh L_0} y e^{\text{int}} \tag{36}$$

Hence equation (34)-(36) represent velocity of the liquid, velocity of the dust particles and temperature respectively.

In order to get the physical insight of the problem, the transients fluid and particles velocities are studied

[Soundalgekar and Bhat, (1984)] for $nt = \frac{\pi}{2}$, we obtain

$$u = u_0(y) - \varepsilon M_{ui} \tag{37}$$

$$v = v_0(y) - \varepsilon M_{ui} \tag{38}$$

Taking the partial derivative in equation (2.24), we obtain

$$\tau_l = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \left[\frac{A^2 - P(\cosh A - 1) - G_r}{A \sinh A} \right] + \frac{G_r}{A^2} + \varepsilon \left[\left(1 + \frac{G_r}{L_0^2 - B^2} \right) \frac{B}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{L_0}{\sinh L_0} \right] e^{\text{int}} \tag{39}$$

Hence equation (39) represents the skin friction (τ_l) of the fluid at the plate ($y = 0$).

Taking the partial derivative in equation (35), we obtain

$$\tau_p = \left[\frac{\partial v}{\partial y} \right]_{y=0} = A_1 \left[\frac{A^2 - P(\cosh A - 1 - G_r)}{A \sinh A} \right] + \varepsilon A_2 B_0 \left[\left(1 + \frac{G_r}{L_0^2 - B^2} \right) \frac{B}{\sinh B} - \left(\frac{G_r}{L_0^2 - B^2} \right) \frac{L_0}{\sinh L_0} \right] e^{\text{int}} \quad (40)$$

Hence equation (40) represents the skin friction of the dust particles (τ_p) at the plate ($y = 0$). Taking the partial derivative of equation (36), we obtain

$$N_0 = \left(\frac{\partial T}{\partial y} \right)_{y=0} = 1 + \varepsilon \frac{L_0}{\sinh L_0} e^{\text{int}} \quad (41)$$

Hence equation (41) represents the rate of heat transfer in terms of Nusset number at the plate ($y=0$).

RESULTS AND DISCUSSIONS

Velocity of the liquid and Dust Particle ($G_r < 0$)

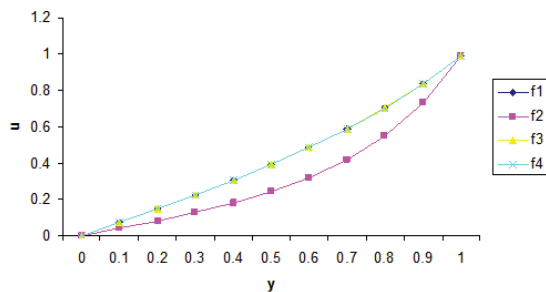


Fig. 1: Showing the primary velocities of the liquid for $\epsilon_3=0$.

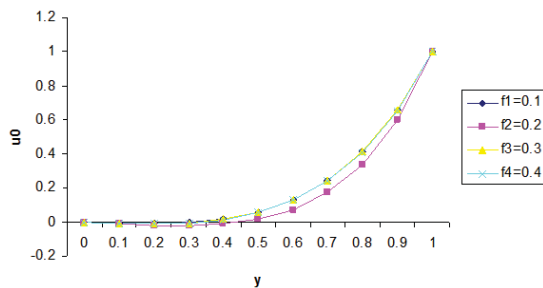


Fig. 2: Showing the Primary velocities of the liquid for various values of ϵ_3 .

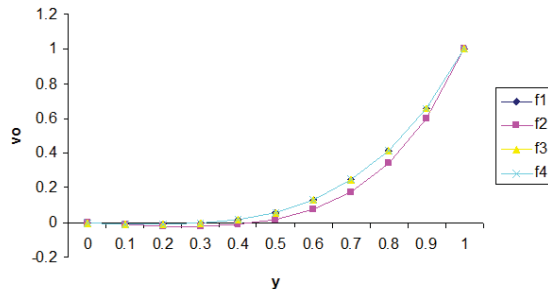


Fig. 3: Showing the primary velocities of the dust particles for $\epsilon_3=0$.

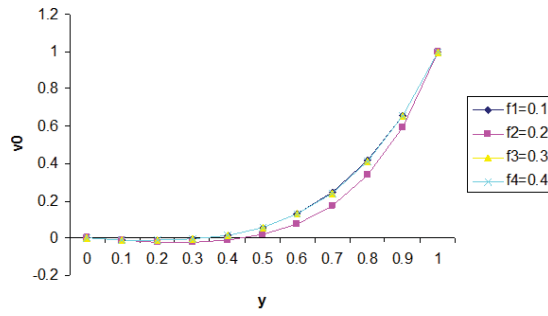


Fig. 4: Showing the Primary velocities of the dust particles for various values of ϵ_3 .

Transient Velocity of the liquid and Dust Particle ($G_r < 0$) .

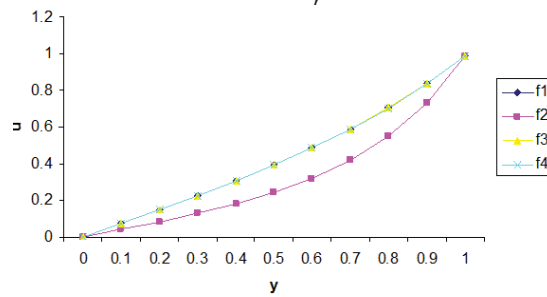


Fig. 5: Showing the transient velocity of the liquid for $\epsilon_3=0$.

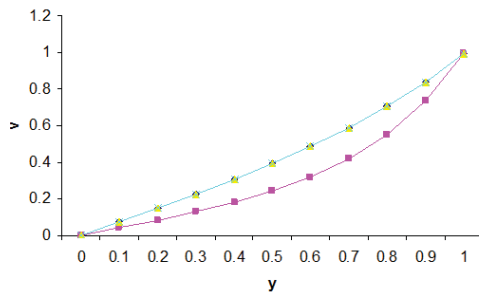


Fig. 6: Showing the transient velocities of the liquid for various values of ϵ_3 .

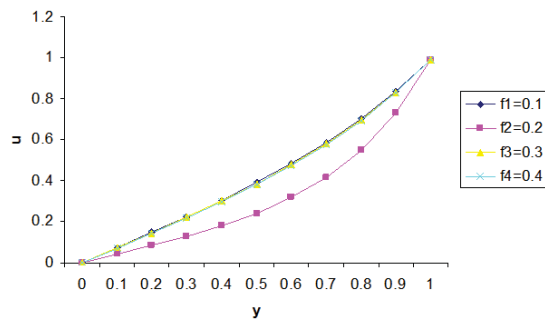


Fig. 7: Showing the transient velocities of the dust particles for $\epsilon_3=0$.

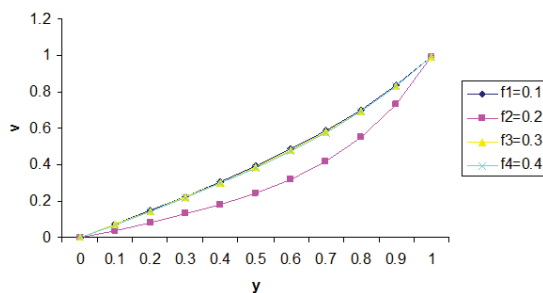


Fig. 8: Showing the transient velocity of the dust particles for various of ϵ_3 .

Values of the Skin Friction and Heat Transfer:

Table I: values of the mean skin friction τ_{l0} , τ_{p0} , skin friction τ_l , τ_p and Nusselt number N_o .

M	n	β	ϕ	f	G_r	τ_{l0}	τ_{p0}	τ_l	τ_p	N_o
8	10	40	0.005	0.5	5	0.74677	0.74752	0.746596	0.747357	1.007019
16	10	40	0.005	0.5	5	0.539686	0.540499	0.539484	0.540273	1.007019
8	20	40	0.005	0.5	5	0.74677	0.74752	0.746766	0.747516	1.005109
8	10	80	0.005	0.5	5	0.746737	0.747337	0.746547	0.747122	1.007019
8	10	40	0.01	0.5	5	0.742739	0.744238	0.7425	0.743984	1.007019
8	10	40	0.005	0.3	5	0.74677	0.74752	0.741837	0.742434	1.007019
8	10	40	0.005	0.5	5	-0.08175	-0.08184	-0.08209	-0.08213	1.007019
16	10	40	0.005	0.5	5	-0.11075	-0.11092	-0.11113	-0.11129	1.007019
8	20	40	0.005	0.5	5	-0.08175	-0.08184	-0.08175	-0.08183	1.005109
8	10	80	0.01	0.5	5	-0.08176	-0.08182	-0.08213	-0.08219	1.007019
8	10	40	0.005	0.5	5	-0.08373	-0.0839	-0.08414	-0.08431	1.007019
8	10	40	0.005	0.3	5	-0.08175	-0.08184	-0.08753	-0.08767	1.007019

From the observations of Figure-1 and Figure-3, Figure-2 and Figure-4, Figure-5 and Figure-7, and Figure-6, Figure-8 and table 1, we arrive at the following conclusions:

An increase in M results in a decrease in the primary velocities $u_0(y), v_0(y)$ as well as in transient velocities $u\left(y, \frac{\pi}{2n}\right), v\left(y, \frac{\pi}{2n}\right)$ for both $\epsilon_3 = 0$ and various values of ϵ_3 .

An increase in β leads to a small change in the primary velocities $u_0(y), v_0(y)$ as well as in transient velocities $u\left(y, \frac{\pi}{2n}\right), v\left(y, \frac{\pi}{2n}\right)$ for both $\epsilon_3 = 0$ and various values of ϵ_3 .

An increase in ϕ leads to a small change in the primary velocities $u_0(y), v_0(y)$ as well as in transient velocities $u\left(y, \frac{\pi}{2n}\right), v\left(y, \frac{\pi}{2n}\right)$ for both $\epsilon_3 = 0$ and various values of ϵ_3 .

The mean skin friction for liquid is less in comparison to mean skin friction of dust particles for $G_r > 0$ and vice-versa for $G_r < 0$.

For an increase in M , there is decrease in $\tau_{l0}, \tau_{p0}, \tau_l$ and τ_p for $G_r > 0$ and $G_r < 0$.

For an increase in n there is no change in τ_{l0} and τ_{p0} for both $G_r > 0$ and $G_r < 0$.

For an increase in n there is an increase in τ_l and τ_p for both $G_r > 0$ and $G_r < 0$.

For an increase in n there is a decrease in N_0 for both $G_r > 0$ and $G_r < 0$.

For an increase in β there is an increase in τ_{l0} for both $G_r > 0$ and $G_r < 0$.

For an increase β there is a decrease in τ_{p0} for both $G_r > 0$ and an increase in τ_{p0} for $G_r < 0$.

For an increase in β there is decrease in τ_l and τ_p for both $G_r > 0$ and $G_r < 0$.

For an increase in ϕ , there is an increase in τ_{l0} , τ_{p0} , τ_l and τ_p for $G_r > 0$ and an increase τ_{l0} , τ_{p0} , τ_l and τ_p for $G_r < 0$.

For a decrease in f , there is no change in τ_{l0} and τ_{p0} for both $G_r > 0$ and $G_r < 0$.

For $G_r > 0$, a decrease in f , there is decrease in τ_l and τ_p but for $G_r < 0$ there is an increase in τ_l and τ_p .

For an increase in M , β or ϕ and a decrease in f , there is no change in N_0 for both $G_r > 0$ and $G_r < 0$.

4. Conclusions:

In this paper the velocities of the fluid, velocities of the particles, skin friction and heat transfer are obtained. The effects of concentration resistance ratio (β), volume fraction of dust particles (ϕ), porous parameter (\mathcal{E}_3), frequency parameter (n) mass concentration of dust particles (f), Grashof number and (G_r) an external magnetic field on the unsteady laminar flow of a dusty, incompressible, Newtonian, electrically conducting viscous fluid through a porous medium are studied. It is found that both the velocity of the liquid and dust particles decreases with the increase in the porous parameter (\mathcal{E}_3).

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