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Using Gaussian Elimination for Determination of Structure Index in Euler Deconvolution

[1]Reza.toushmalani and [2]M.ghanbari

[1]Islamic azad university,Hamedan branch
[2]Islamic azad university, khoramabad branch

Abstract: Euler's homogeneity relation has attracted sporadic interest from geophysicists over the years. The choice of structural index in Euler homogeneity equation remains a vexing problem, because structures are poorly imaged and depths are biased if the wrong index is used for any given feature. In this paper we assume that structural index is one of my unknowns and with solving system of equation with Gaussian elimination find value of structure index.

Key words: structure index, Euler Deconvolution, Gaussian Elimination

1)Euler deconvolution:

Euler's homogeneity relation has attracted sporadic interest from geophysicists over the years. It may be stand succinctly in the form

$$(x-x_o)\frac{\partial T}{\partial x}+(y-y_o)\frac{\partial T}{\partial y}+(z-z_o)\frac{\partial T}{\partial z}=-NT$$

Where x0,y0,z0 is the position of a source whose total field T is detected at (x,y,z).the total field has a regional or background value B. where $\partial T/\partial x$, $\partial T/\partial y$ and $\partial T/\partial z$ represent first-order derivative of the magnetic (gravity) field along the x-, y- and z- directions, respectively, *N* is known as a structural index and related to the geometry of the magnetic(gravity) source(si:Thompson,1982). Taking into account a base level for the regional magnetic field (B) equation (1) can be rearranged and written as

$$x_\circ\frac{\partial T}{\partial x}+y_\circ\frac{\partial T}{\partial y}+z_\circ\frac{\partial T}{\partial z}+NB=x\frac{\partial T}{\partial x}+y\frac{\partial T}{\partial y}+z\frac{\partial T}{\partial z}+NT.,$$

Assigning the structural index *(N)* a system of linear equations can be obtained and solved for estimating the location and depth of the magnetic and gravity body. The choice of structural index in euler homogeneity equation remains a vexing problem, because structures are poorly imaged and depths are biased if the wrong index is used for any given feature (Reid *et al.*, 1990). In this paper we assumed N=v , B=0 and obtain the valu of N with solving system of linear equation by Using Gaussian Elimination.

2) Gaussian Elimination:

2-1) How is a Set of Equations Solved Numerically?:

One of the most popular techniques for solving simultaneous linear equations is the Gaussian elimination method. The approach is designed to solve a general set of *n* equations and *n* unknowns

Corresponding Author: Reza.toushmalani, Islamic azad university,Hamedan branch
E-mail:Atomicboy_reza@yahoo.com

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n$$

Gaussian elimination consists of two steps

1. Forward Elimination of Unknowns: In this step, the unknown is eliminated in each equation starting with the first equation. This way, the equations are *reduced* to one equation and one unknown in each equation.
2. Back Substitution: In this step, starting from the last equation, each of the unknowns is found.

2-2)Forward Elimination of Unknowns:

In the first step of forward elimination, the first unknown, x_1 is eliminated from all rows below the first row. The first equation is selected as the pivot equation to eliminate x_1. So, to eliminate x_1 in the second equation, one divides the first equation by a_{11} (hence called the pivot element) and then multiplies it by a_{21}. This is the same as multiplying the first equation by a_{21}/a_{11} to give

$$a_{21}x_1 + \frac{a_{21}}{a_{11}}a_{12}x_2 + \ldots + \frac{a_{21}}{a_{11}}a_{1n}x_n = \frac{a_{21}}{a_{11}}b_1$$

Now, this equation can be subtracted from the second equation to give

$$\left(a_{22} - \frac{a_{21}}{a_{11}}a_{12}\right)x_2 + \ldots + \left(a_{2n} - \frac{a_{21}}{a_{11}}a_{1n}\right)x_n = b_2 - \frac{a_{21}}{a_{11}}b_1$$

or

$$a'_{22}x_2 + \ldots + a'_{2n}x_n = b'_2$$

where

$$a'_{22} = a_{22} - \frac{a_{21}}{a_{11}}a_{12}$$

$$\vdots$$

$$a'_{2n} = a_{2n} - \frac{a_{21}}{a_{11}}a_{1n}$$

This procedure of eliminating x_1, is now repeated for the third equation to the n^{th} equation to reduce the set of equations as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2$$

$$a'_{32}x_2 + a'_{33}x_3 + \ldots + a'_{3n}x_n = b'_3$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$a'_{n2}x_2 + a'_{n3}x_3 + \ldots + a'_{nn}x_n = b'_n$$

This is the end of the first step of forward elimination. Now for the second step of forward elimination, we start with the second equation as the pivot equation and a'_{22} as the pivot element. So, to eliminate x_2 in the third equation, one divides the second equation by a'_{22} (the pivot element) and then multiply it by a'_{32}. This is the same as multiplying the second equation by a'_{32}/a'_{22} and subtracting it from the third equation. This makes the coefficient of x_2 zero in the third equation. The same procedure is now repeated for the fourth equation till the n^{th} equation to give

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2$$

$$a''_{33}x_3 + \ldots + a''_{3n}x_n = b''_3$$

$$\vdots \qquad \vdots$$

$$a''_{n3}x_3 + \ldots + a''_{nn}x_n = b''_n$$

The next steps of forward elimination are conducted by using the third equation as a pivot equation and so on. That is, there will be a total of n - 1 steps of forward elimination. At the end of n - 1 steps of forward elimination, we get a set of equations that look like

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2$$

$$a''_{33}x_3 + \ldots + a''_{3n}x_n = b''_3$$

$$\vdots \qquad \vdots$$

$$a_{nn}^{(n-1)}x_n = b_n^{(n-1)}$$

2-3)Back Substitution:

Now the equations are solved starting from the last equation as it has only one unknown.

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$$

Then the second last equation, that is the $(n - 1)^{th}$ equation, has two unknowns: x_n and x_{n-1}, but x_n is already known. This reduces the $(n - 1)^{th}$ equation also to one unknown. Back substitution hence can be represented for all equations by the formula

$$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)}x_j}{a_{ii}^{(i-1)}} \quad \text{for } i = n-1, n-2, \ldots, 1$$

and

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$$

3)Using Gaussian Elimination in Euler Deconvolution

T=[-0.3690 -0.3860 -0.1440 -0.1630];

x=[1 2 3 4];

y=[2 4 6 8];

$\partial T / \partial x$ =[-0.017 0.112 0.111 0.041]

$\partial T / \partial y$ =[-0.0085 0.056 0.055 0.020]

$\partial T / \partial z$ =[0.003 -0.087 0.061 -0.012]

Input the coefficients for 4 equations

369	V +	-17	X +	-8	Y +	+3	Z =	-33
386	V +	112	X +	56	Y +	-87	Z =	448
144	V +	111	X +	55	Y +	61	Z =	663
163	V +	41	X +	20	Y +	-12	Z =	324

1) Doing forward-elimination phase: column 1 of 4

Found the first row that has nonzero in column 1

369.0000 V	-	17.0000 X	-	8.0000 Y	+	3.0000 Z	=	-33.0000
386.0000 V	+	112.0000 X	+	56.0000 Y	-	87.0000 Z	=	448.0000
144.0000 V	+	111.0000 X	+	55.0000 Y	+	61.0000 Z	=	663.0000
163.0000 V	+	41.0000 X	+	20.0000 Y	-	12.0000 Z	=	324.0000

2) Doing forward-elimination phase: column 1 of 4

Divided entire row 1 by 369.0000

Now subtract 386.0000 times row 1 from row 2

V	-	0.0461 X	-	0.0217 Y	+	0.0081 Z	=	-0.0894
386.0000 V	+	112.0000 X	+	56.0000 Y	-	87.0000 Z	=	448.0000
144.0000 V	+	111.0000 X	+	55.0000 Y	+	61.0000 Z	=	663.0000
163.0000 V	+	41.0000 X	+	20.0000 Y	-	12.0000 Z	=	324.0000

3) Doing forward-elimination phase: column 1 of 4

Now subtract 144.0000 times row 1 from row 3

V	-	0.0461 X	-	0.0217 Y	+	0.0081 Z	=	-0.0894
		129.7832 X	+	64.3686 Y	-	90.1382 Z	=	482.5203
144.0000 V	+	111.0000 X	+	55.0000 Y	+	61.0000 Z	=	663.0000
163.0000 V	+	41.0000 X	+	20.0000 Y	-	12.0000 Z	=	324.0000

4) Doing forward-elimination phase: column 1 of 4

Now subtract 163.0000 times row 1 from row 4

V	-	0.0461 X	-	0.0217 Y	+	0.0081 Z	=	-0.0894
		129.7832 X	+	64.3686 Y	-	90.1382 Z	=	482.5203
		117.6341 X	+	58.1220 Y	+	59.8293 Z	=	675.8780
163.0000 V	+	41.0000 X	+	20.0000 Y	-	12.0000 Z	=	324.0000

5) Doing forward-elimination phase: column 1 of 4
Done with column 1. Move on to next column

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ 129.7832\,X + 64.3686\,Y - 90.1382\,Z &= 482.5203 \\ 117.6341\,X + 58.1220\,Y + 59.8293\,Z &= 675.8780 \\ 48.5095\,X + 23.5339\,Y - 13.3252\,Z &= 338.5772 \end{aligned}$$

6) Doing forward-elimination phase: column 2 of 4
Found first row on or after row 2 that has nonzero in colum 2

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ 129.7832\,X + 64.3686\,Y - 90.1382\,Z &= 482.5203 \\ 117.6341\,X + 58.1220\,Y + 59.8293\,Z &= 675.8780 \\ 48.5095\,X + 23.5339\,Y - 13.3252\,Z &= 338.5772 \end{aligned}$$

7) Doing forward-elimination phase: column 2 of 4
Divided entire row 2 by 129.7832
Now subtract 117.6341 times row 2 from row 3

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ 117.6341\,X + 58.1220\,Y + 59.8293\,Z &= 675.8780 \\ 48.5095\,X + 23.5339\,Y - 13.3252\,Z &= 338.5772 \end{aligned}$$

8) Doing forward-elimination phase: column 2 of 4
Now subtract 48.5095 times row 2 from row 4

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ -0.2210\,Y + 141.5296\,Z &= 238.5266 \\ 48.5095\,X + 23.5339\,Y - 13.3252\,Z &= 338.5772 \end{aligned}$$

9) Doing forward-elimination phase: column 2 of 4
Done with column 2. Move on to next column

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ -0.2210\,Y + 141.5296\,Z &= 238.5266 \\ -0.5254\,Y + 20.3660\,Z &= 158.2241 \end{aligned}$$

10) Doing forward-elimination phase: column 3 of 4
Found first row on or after row 3 that has nonzero in colum 3

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ -0.2210\,Y + 141.5296\,Z &= 238.5266 \\ -0.5254\,Y + 20.3660\,Z &= 158.2241 \end{aligned}$$

11) Doing forward-elimination phase: column 3 of 4
Divided entire row 3 by -0.2210
Now subtract -0.5254 times row 3 from row 4

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ Y - 640.2657\,Z &= -1079.0705 \\ -0.5254\,Y + 20.3660\,Z &= 158.2241 \end{aligned}$$

12) Doing forward-elimination phase: column 3 of 4
Done with column 3. Move on to next column

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ Y - 640.2657\,Z &= -1079.0705 \\ -316.0108\,Z &= -408.6879 \end{aligned}$$

13) Doing forward-elimination phase: column 4 of 4
Found first row on or after row 4 that has nonzero in colum 4

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ Y - 640.2657\,Z &= -1079.0705 \\ -316.0108\,Z &= -408.6879 \end{aligned}$$

14) Doing forward-elimination phase: column 4 of 4
Divided entire row 4 by -316.0108
Done with column 4. That was the last column, move on to back-elimination

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ Y - 640.2657\,Z &= -1079.0705 \\ Z &= 1.2933 \end{aligned}$$

15) Doing back-elimination phase: column 4
Subtract -640.2657 times row 4 from row 3

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ Y - 640.2657\,Z &= -1079.0705 \\ Z &= 1.2933 \end{aligned}$$

16) Doing back-elimination phase: column 4
Subtract -0.6945 times row 4 from row 2

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y - 0.6945\,Z &= 3.7179 \\ Y &= -251.0327 \\ Z &= 1.2933 \end{aligned}$$

17) Doing back-elimination phase: column 4
Subtract 0.0081 times row 4 from row 1

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y + 0.0081\,Z &= -0.0894 \\ X + 0.4960\,Y &= 4.6161 \\ Y &= -251.0327 \\ Z &= 1.2933 \end{aligned}$$

18) Doing back-elimination phase: column 3
Subtract 0.4960 times row 3 from row 2

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y &= -0.0999 \\ X + 0.4960\,Y &= 4.6161 \\ Y &= -251.0327 \\ Z &= 1.2933 \end{aligned}$$

19) Doing back-elimination phase: column 3
Subtract -0.0217 times row 3 from row 1

$$\begin{aligned} V - 0.0461\,X - 0.0217\,Y &= -0.0999 \\ X &= 129.1208 \\ Y &= -251.0327 \\ Z &= 1.2933 \end{aligned}$$

20) Doing back-elimination phase: column 2
Subtract -0.0461 times row 2 from row 1

$$\begin{aligned} V - 0.0461\,X &= -5.5424 \\ X &= 129.1208 \\ Y &= -251.0327 \\ Z &= 1.2933 \end{aligned}$$

Doing back-elimination phase: column 1
All done!

V				=	0.4063
	X			=	129.1208
		Y		=	-251.0327
			Z	=	1.2933

With the calculated value of the structural index (V=N) we have achieved our goal and we don't need a specific geological assumption to determine the structural index. If we also have more points this method will Extended and we can use this method.

Conclusion:

The choice of structural index in euler Deconvolution remains a vexing problem, because structures are poorly imaged and depths are biased if the wrong index is used for any given feature . In this paper we assume that structural index is one of my unknowns and with solving system of equation with Gaussian elimination find Value of structure index. With the calculated value of the structural index we have achieved our goal and we don't need a specific geological assumption to determine the structural index. If we also have more points this method will Extended and we can use this method.

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