Australian Journal of Basic and Applied Sciences, 4(12): 6461-6469, 2010
ISSN 1991-8178

Using Data Envelopment Analysis and Analytical Hierarchy Process Model to Evaluate Flexible Manufacturing Systems

[1]Kamran Rezaie, [1]Salman Nazari-Shirkouhi, [1]Seyed Mostafa Alem, [2]Loghman Hatami-Shirkouhi

[1]Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.
[2]Islamic Azad University, Roudbar branch, Iran.

Abstract: Nowadays most of manufacturing firms in accordance with continuous changes in the market and the today's competitiveness world are competing for meeting demand, increasing quality and decreasing costs. For reaching these goals, it is necessary to select a suitable flexible manufacturing system (FMS) for the most of manufactures. The aim of this paper is applying a method for evaluating the flexible manufacturing systems based on a model incorporating two decision models namely "Data Envelopment Analysis (DEA)" and "Analytical Hierarchy Process (AHP)". DEA helps us to categorize the Decision Making Units (DMUs) to only two classes of efficient and inefficient units, whereas by using an AHP, we can have a full rank of DMUs. Input and output factors considered for ranking FMSs are Capital and Operating Costs, Throughput Time, Work in Process, Labor Requirements, Required Floor Space, Product Mix Flexibility, Yield, and Volume Flexibility. The proposed approach is applied in a Vehicle Manufacturing Company.

Key words: Flexible Manufacturing System, Data envelopment analysis, Analytical hierarchy process.

INTRODUCTION

A flexible manufacturing system (FMS) is designed to combine the efficiency of a mass-production line and the flexibility of a job shop to produce a variety of work pieces on a group of machines (Chan *et al*., 1997). Flexibility concept in FMS is defined as the capability of a workstation to response quickly to the various requirements and expectations of the market. Recently, many of manufacturers have been investing in FMS. The decision to invest in FMS and other advanced manufacturing technologies has been an issue in the practical and academic literature for over two decades (Venkata, 2008). The gradual development of FMS provide great flexibility based on the competition principles by focusing on both factors of cost-effective and customized manufacturing, simultaneously. Investment in FMS is a multi-attribute decision and includes many quantitative and qualitative attributes. Deciding attributes for selecting a FMS differ from application to application. These attributes include capital and operating costs, throughput time, work in process, labor requirements, required floor space, product mix flexibility, yield, volume flexibility, number of tardy jobs, worker approval, quality improvement, product flexibility, expansion flexibility, lead-time, etc.

Many research works have been focused on various models of evaluation and selection of FMS from simple financial analysis methods, such as Net Present Value (NPV) method, Return on Investment (ROI), and Internal Rate of return (IRR) Canada and Sullivan (1989), Sullivan *et al*.(2000), and Primrose (1991) for example), to more complex multi-criteria mathematical programming methods. However, the need for a structured methodology for evaluation FMSs is felt.

The insufficiency of traditional financial analysis and appraising measures lies on their non-stochastic nature. The conventional financial analysis methods do not appear to be suitable on their own for the evaluation of advanced manufacturing technologies investments due to the nonmonetary impacts posed by the manufacturing System (Duran and Aguilo, 2008). Anyway, financial analysis (NPV, ROI, IRR, and etc.) can lead to incorrect results in most of real-world applications. Mathematical models have been developed to determine performance criteria such as quality and flexibility for evaluating of FMSs. Nelson (1986) introduced a scoring model for FMS project selection by complementing the traditional capital budgeting procedures with the treatment of project interdependence and non-economic criteria. Wabalickis (1988) developed a justified

Corresponding Author: Kamran Rezaie, Department of Industrial Engineering, College of Engineering, University of Tehran, P.O. Box 11155-45632, Tehran, Iran.
E-mail: krezaie@ut.ac.ir.

methodology based on the AHP to evaluate many tangible and intangible benefits of an investment on FMS. In addition, Bayazit (2005) presented an AHP approach for selecting a FMS. Stam and Kuula (1991) developed a two-phase decision procedure that uses the AHP method and multi-objective mathematical programming to select an FMS. Although AHP is variously used in selection problems of FMS, but it suffers from a number of disadvantages. Boucher *et al*. (1997) argued that AHP is often criticized for the way the criteria weights are elicited, rank reversal problem, inappropriateness of the crisp ratio representation, and problems faced in the comparison process when the number of criteria and/or the number of alternatives increase. In addition, Karsak and Kuzgunkaya (2002) presented a fuzzy approach with multi-objective programming for selection of a flexible manufacturing system. Data envelopment analysis (DEA) has been used severally as a tool for evaluation of manufacturing technologies and FMSs. Rezaie and Ostadi (2007) introduced an integrated dynamic programming model in order to analyze the optimal and phased implementation of flexible technology in a manufacturing system. The objectives of the model are minimizing the cost network flow and represented capital investment decisions, aggregate production decisions, and the functional of the production costs.

Shang and Sueyoshi (1995) proposed a selection framework of a FMS using AHP, simulation, and data envelopment analysis. They integrated AHP and simulation to generate "input" data and used computing techniques to provide "output" data for analysis using DEA. Sarkis (1997) applied DEA technique for evaluating FMSs. He presented a number of DEA models to aid in the investment and adoption decision process. Shiang (2008) integrated Fuzzy Data Envelopment analysis (FDEA) and Assurance Region (AR) approach for selection of FMSs when the input and output data is represented as crisp and fuzzy data. He compared twelve FMS alternatives (DMUs) by considering two inputs and four outputs. He considered capital and operating costs, and floor space requirement as input factors and improvements in qualitative factor, work-in-process (WIP), numbers of tardy jobs, and yield as output factors.

In crisp environment all of the above mentioned models categorize FMSs to efficient and inefficient. So it is necessary to a methodology to evaluate and select FMSs based on their competencies. Therefore integrating both DEA and AHP model is applied for ranking FMSs. DEA/AHP is capable to represent ranking of FMSs based on their competencies which are dependent on considered inputs and outputs factors.

In this paper a quantitative/qualitative decision support system for the evaluation and selection of FMS alternatives is proposed and is applied for a Vehicle Manufacturing Company in Iran. We perform this evaluation by integrating Date employment analysis (DEA) and Analytical Hierarchy Process (AHP) and from now on call it DEA/AHP method.

The structure of the paper is as follows: the DEA model is described in section 2. AHP technique is presented in section 3. DEA/AHP method is presented in section 4. In Section 5 DEA/AHP model is applied to evaluate performance of FMSs. Finally, section 6 includes conclusion and further research directions.

Data Envelopment Analysis (DEA) Model:

Data Envelopment Analysis (DEA) by Charnes *et al*. (1978), is a method for evaluating the relative efficiency of comparable entities referred to as Decision Making Units (DMU). Also DEA is a method to identify the performance frontiers. In data envelopment analysis, production function is defined with no assumptions. The DMUs are identified by several inputs and outputs. The efficiency score according to multiple input and output factors is defined as:

$$E = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}} \quad (1)$$

In DEA we use a mathematical model for evaluate relative efficiency of each DMU. The basic idea of this mathematical model based on equation 1. Assuming that there are n DMUs, each with 'm' different inputs and's' different outputs. The relative efficiency score of the DMU_p is obtained by solving the following model proposed by Banker *et al*. (1984):

$$Max \frac{\sum_{k=1}^{s} u_k y_{kp}}{\sum_{j=1}^{m} v_j x_{jp}}$$

$$s.t.$$
$$\frac{\sum_{k=1}^{s} u_k y_{ki}}{\sum_{j=1}^{m} v_j x_{ji}} \le 1 \qquad (2)$$
$$v_k, u_k \ge 0$$

Where

k = 1 to s,
j = 1 to m,
i = 1 to n,

y_{ki} = amount of output k produced by DMU_i

x_{ji} = amount of input j utilized DMU_i

v_k = weight given to input k,

u_k = weight given to output j.

The nonlinear program shown as (2) can be converted to a linear program as shown in (3).

$$Max \sum_{k=1}^{s} u_k y_{kp}$$
$$s.t.$$
$$\sum_{j=1}^{m} v_j x_{jp} = 1$$
$$\sum_{k=1}^{s} u_k y_{ki} - \sum_{j=1}^{m} v_j x_{ji} \le 0 \qquad (3)$$
$$v_k, u_j \ge 0$$

The model (3) should run n times to calculate the efficiency of n DMUs. In this model u and v vectors are the variables. The u and v vectors assume that prices and values respectively. These prices and values are not real prices and values rather they are virtual prices and values. The basic idea in DEA is finding best combination of prices and values to maximize the efficiency score of DMUs. Based on this model the DMU take a weight of inputs and outputs factor that maximize the relative efficiency.

Analytical Hierarchy Process (AHP):

Saaty defines analytic hierarchy process as a decision method that decomposes a complex multi-criteria decision problem into a hierarchy (Saaty, 1980; Saaty, 1986). Analytical hierarchy process has been used extensively for solving different multi-criteria decision-making (MCDM) problems. The AHP method proposed by Saaty (1980) uses pair-wise comparisons shown in (4). Number a_{ij} shows the relative importance of criterion i (C_i) in comparison with criterion j (C_j) in the scale of Saaty (1986).

$$A = \left[a_{ij} \right] = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ C_1 & 1 & a_{12} & \cdots & a_{1n} \\ C_2 & 1/a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{matrix} \tag{4}$$

Where

$$a_{ij} = 1 : \forall i = j;\ a_{ij} = \frac{1}{a_{ji}} : \forall i \neq j$$

Saaty (1980) suggests consistency index (C.I.) and consistency ratio (C.R) to verify the consistency of the matrix. Random index R.I. represents the average consistency index over numerous random entries of the same order reciprocal matrices. If *C.R* < 0.1, the estimate is accepted; otherwise, a new comparison matrix is solicited. The value of *R.I* depends on the value of *n* and should be selected from Table 1.

To find the Consistency Index (*C.I*), eigen-value of the matrix A should be found first. The number $\lambda_{\max}$ is defined as the eigen-value of the matrix calculated by (5-6):

$$A.W = \lambda_{\max}.W \tag{5}$$

$$(A - \lambda_{\max}).W = 0 \tag{6}$$

Table 1: Random index used to compute consistency ratio (C.R.)

n	1	2	3	4	5	6	7	8	9	10
R.I.	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

In which *W* is the eigenvector of matrix *A*, and final weighting of C_i (*i=1*to *n*). After finding $\lambda_{\max}$, values of *C.I.* and *C.R.* can be calculated from (7-8)

$$C.I. = \frac{\lambda_{\max} - n}{n-1} \tag{7}$$

$$C.R. = \frac{C.I.}{R.I.} \tag{8}$$

DEA/AHP Model:

According to results of DEA, DMUs can classify to efficient and inefficient. If DMUs could be classified to more than two classes, then they are evaluated more than better. In addition, a fully ranking of DMUs could be achieved. In order to handle this defect AHP methodology is used for ranking and classifies DMUs, and designing an AHP/DEA model for evaluating and ranking the relative efficiency of DMUs. DEA/AHP methodology for the first time was introduced by Sinuany-Stern *et al*. (2000) that provide full ranking in the DEA context for all units, efficient and inefficient. DEA/AHP method applied for different application such as facilities layout design problem, supply chain performance evaluation, risk assessment, warehouse operator selection, and etc. (Jing-yuan et al., 2006; Korpela et al. 2007; Wang, 2008; Yang and Kuo, 2003). DEA/AHP approach divides to following steps.

First step: for each pair of DMU, such as DMU_i and DMU_j, E_{ii}, E_{ji}, E_{jj}, E_{ij} are defining as follows and calculating them. Note that the equation (10) must calculate after equation (9) and equation (12) must calculate after equation (11).

$$E_{ii} = Max\sum_{r=1}^{s} u_r y_{rj}$$

$$\sum_{p=1}^{m} v_p x_{pi} = 1$$

$$\sum_{r=1}^{s} u_r y_{ri} \leq 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{p=1}^{m} v_p x_{pj} \leq 0 \tag{9}$$

$$u_r, v_p \geq 0$$

$$E_{ji} = Max\sum_{r=1}^{s} u_r y_{rj}$$

$$\sum_{p=1}^{m} v_p x_{pj} = 1 \tag{10}$$

$$\sum_{r=1}^{s} u_r y_{rj} \leq 1$$

$$\sum_{r=1}^{s} u_r y_{ri} - E_{ii}\sum_{p=1}^{m} v_p x_{pi} \leq 0$$

$$u_r, v_p \geq 0$$

$$E_{jj} = Max\sum_{r=1}^{s} u_r y_{ri}$$

$$\sum_{p=1}^{m} v_p x_{pj} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} \leq 1$$

$$\sum_{r=1}^{s} u_r y_{ri} - \sum_{p=1}^{m} v_p x_{pi} \le 0 \tag{11}$$
$$u_r, v_p \ge 0$$

$$E_{ij} = Max \sum_{r=1}^{s} u_r y_{ri}$$
$$\sum_{p=1}^{m} v_p x_{pi} = 1$$
$$\sum_{r=1}^{s} u_r y_{ri} \le 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - E_{jj} \sum_{p=1}^{m} v_p x_{pj} \le 0 \tag{12}$$
$$u_r, v_p \ge 0$$

Second step: After calculate, E_{ii} , E_{jj} , E_{ij} , E_{ji} we can calculate the relative efficiency of DMU_i and DMU_j as follows:

$$\alpha_{ij} = \frac{E_{ii} + E_{ij}}{E_{jj} + E_{ji}} \tag{11}$$

α_{ij} is comparative efficiency of DMU_i and DMU_j. Also α_{ij} is generally defined as *i-th* row *j-th* column element a_{ij} in the AHP judging matrices that shows by 'A'. With according to above formula we have two relations between comparative efficiency of DMU_i and DMU_j .

$$\alpha_{ii} = 1 \quad ; \; i = j \tag{12}$$

$$\alpha_{ij} = \frac{1}{\alpha_{ji}} \tag{13}$$

Third step: in this step, we calculate fully ranking of DMUs using AHP technique. Using matrix A that is constructed in before step, we can calculate weights of DMUs using formula 5, 6.

FMS Performance Evaluation:

In this paper, we have considered ten alternatives of FMSs. In this section, we implement DEA approach and proposed approach for evaluations of FMSs. Minimizing Factors is considered as inputs, whereas maximizing factors as outputs. Eight factors for DMUs are regarded. Input factors includes Capital and Operating Costs, Throughput Time, Work in Process, Labor Requirements, Required Floor Space and output factors includes Product Mix Flexibility, Yield, Volume Flexibility. Firm can select other factors such as number of tardy, worker approval, quality improvement, product flexibility, expansion flexibility, lead-time, etc. required data are achieved by simulation and estimation. Table 2 indicates used data for calculating DAE/AHP. Now, we implement DEA and DEA/AHP approach for evaluating of FMSs.

Table 2: Data used to evaluating of FMS alternatives

FMS Alternative	Input Factors					Output Factors		
	COC	TT	WIP	LR	RFS	PMF	Y	VF
1	7	22	46	5	9	38	5.54	13.2
2	4	19	43	17	6.3	15	2.24	2.3
3	4.55	18.25	45	14	5.6	18	2.95	2.11
4	4.67	20.32	45	16	6.9	17	2.21	1.4
5	8.77	20.1	50	6	8.3	31	4.87	18.7
6	3.93	5.38	40	3	6.2	27	1.85	6.8
7	5.85	36.2	58	11	4.8	30	2.23	7.2
8	6.16	7.82	112	8	7.5	21	3.15	15
9	6.94	22.3	65	6	8	35	4.25	12.7
10	6.53	28.62	81	12	4.9	22	2.86	13.2

COC, Capital and Operating Costs (million \$); TT, Throughput Time (Hours per Unit Production); WIP, Work in Process (units);LR, Labor Requirements (units); RFS, Required Floor Space (100m^2); PMF, Product Mix Flexibility (Product types);Y, Yield(Yield is defined as throughput minus scrape and rework multiply 1000); VF, Volume Flexibility (Average Range of Production Capacity per product Type multiply 10).

Using of DEA to Evaluate FMSs:

Table 3 shows the efficiency value of FMSs by using DEA model in (3). It is notable of table 3 that (FMS1, FMS5, FMS6, FMS7, FMS8, FMS9, and FMS10) are efficient and (FMS2, FMS3, FMS4) are inefficient. So selection of FMSs according to their values is difficult. But using DEA/AHP model can present full rank of FMSs based on weight obtained from AHP.

Table 3: Results of DEA model for ranking of FMSs

FMS	1	2	3	4	5	6	7	8	9	10
FMS value	1	0.708	0.856	0.642	1	1	1	1	1	1
FMS ranking	1	9	8	10	1	1	1	1	1	1

Using of DEA/AHP to evaluate FMSs:

In this section, DEA/AHP model is described in section 4 have been applied for evaluating FMSs. after performing step 1, 2 from DEA/AHP model, table 4 shows the results of these steps in. This Table shows the comparative efficiency of each pair of DMUs that is the same as 'A' matrix. Where $\lambda_{\max} = 10.0215$, $C.I. = 0.00239$ and $C.R. = 0.0016$ shows that 'A' matrix is consistent.

Table 4: Comparative efficiency of each pair of DMUs that is the same as 'A' matrix

	DMU1	DMU 2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
DMU1	1	1.4133	1.1685	1.4913	1	1	1	1	1	1
DMU2	0.7076	1	1	1	1	1	1	1	0.9144	1
DMU3	0.8558	1	1	1.0588	1	1	1	1	1	1
DMU4	0.6706	1	0.94	1	1	1	1	1	0.7728	1
DMU5	1	1	1	1	1	1	1	1	1	1
DMU6	1	1	1	1	1	1	1	1	1	1
DMU7	1	1	1	1	1	1	1	1	1	1
DMU8	1	1	1	1	1	1	1	1	1	1
DMU9	1	1.0936	1	1.2941	1	1	1	1	1	1
DMU10	1	1	1	1	1	1	1	1	1	1

In the third step, we can use AHP approach and calculate full ranking of DMUs that is shown in Table 5 using equations (5-6). These results show that DMU 1 has maximum weight, and DMU 4 has minimum weight, therefore FMS 1 is the best alternative and FMS 4 is the worst alternative.

Table 5: Ranking of DMUs

DMU	DMU1	DMU 2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
Weight	0.11	0.0957	0.0988	0.0932	0.0988	0.0988	0.0988	0.0988	0.1034	0.0988
FMS Ranking	1	9	3	10	3	3	3	3	2	3

In accordance with the results of both types of ranking (DEA, DEA/AHP), it is observable that the DEA method is not capable to recognize the best rank of FMS. But the DEA/AHP method overcomes on weakness of DEA. Based on DEA/AHP method the FMS1 is selected as the best FMS, whereas in DEA method FMS1, FMS5, FMS6, FMS7, FMS8, FMS9, and FMS10 have equal chance to be selected. Hence, DEA/AHP method can be used widely for selecting and evaluating problems of FMSs and help who are involved in FMS field. Moreover, it could be used as new method for ranking and evaluating problems in the other work areas.

Conclusion:

In this paper, we presented a new approach for evaluating and selecting Flexible Manufacturing Systems. Our mainspring to develop an integrating DEA/AHP method for evaluating and selecting FMSs, was that the DEA approach categorize the DMUs to efficient and inefficient, and by this categorization may be we can't making a good decision. So we used an integrated DEA/AHP model for evaluating of FMSs. our approach had three steps, which it's summarized consist of: calculation of comparative efficiency elements, calculation of comparative efficiency and judging matrices and finally calculation of fully DMUs ranking. Input factors was considered for DMUs consist of Capital and Operating Costs, Throughput Time, Work in Process, Labor Requirements, Required Floor Space, and output factors consist of Product Mix Flexibility, Yield, and Volume Flexibility. The proposed approach to select FMSs using an integrated DEA and AHP method is a comparatively easy and simple approach that can extensively apply in FMS area.

REFERENCES

Boucher, T.O., O. Gogus and E.M. Wicks, 1997. "A comparison between two multi attribute decision methodologies used in capital investment decision analysis" Engineering Economist, 42: 179-201.

Bayazit, O., 2005. "Use of AHP in decision making for flexible manufacturing systems", Journal of Manufacturing Technology Management, 16(7): 808-19.

Banker, R.D., A. Charnes and W.W. Cooper, 1984. "Some models for estimating technical and scale inefficiencies in data envelopment analysis" Management Science, 30(9): 1078-1092.

Charnes, A., W.W. Cooper and E. Rhodes, 1978. "Measuring the efficiency of decision making units" European Journal of Operational Research, 2(6): 429-444.

Chan, F.T.S., A. Kazerooni, K. Abhary, 1997. "Fuzzy approach to operation selection", Engineering Applications of Artificial Intelligence, 10: 345-56.

Canada, J.R., W.G Sullivan, 1989. "Economic and multi-attribute evaluation of advanced manufacturing systems", Englewood Cliffs, NJ: Prentice-Hall.

Duran, O., J. Aguilo, 2008. "Computer-aided manufacturing-tool selection based on a Fuzzy-AHP approach", Expert Systems with Applications, 34: 1787-94.

Jing-yuan, G., L. Jia and Q. Li, 2006. "Research on Supply Chain performance Evaluation Based on DEA/AHP model" Proceedings of the 2006 IEEE Asia-Pacific Conference on Services Computing.

Korpela, J., A. Lehmusvaara, J. Nisonen, 2007. "Warehouse operator selection by combining AHP and DEA methodologies" International Journal of Production Economics, 108: 13-5142.

Karsak, K.K., O. Kuzgunkaya, 2002. "A fuzzy multiple objective programming approach for the selection of a flexible manufacturing system", International Journal of Production Economics, 79: 101-11.

Nelson, C.A., 1986. "A scoring model for flexible manufacturing systems project selection" European Journal of Operation Research, 24: 346-59.

Primrose, , P.L., 1991. "Investment in manufacturing technology", London, UK, Chapman and Hall.

Rezaie, K., B. Ostadi, 2007. "A mathematical model for optimal and phased implementation of flexible manufacturing systems", Applied Mathematics and Computation, 184(2): 729-736.

Sullivan, W.G., J.A. Bontadelli, E.M. Wicks, 2000. "Engineering economy", Englewood Cliffs, NJ: Prentice-Hall.

Stam, A., M. Kuula, 1991. "Selecting a flexible manufacturing system using multiple criteria analysis", International Journal of Production Research, 28: 1657-72.

Shang, J., T. Sueyoshi, 1995. "A unified framework for the selection of a flexible manufacturing system", European Journal of Operation Research, 85: 297-315.

Sarkis, J., 1997. "Evaluating flexible manufacturing systems alternatives using data envelopment analysis" Engineering Economist, 43(1); 25-47.

Shiang, T.L., 2008. "A fuzzy DEA/AR approach to the selection of flexible manufacturing systems" Computers & Industrial Engineering, 54: 66-76.

Saaty, T.L., 1980. “The analytic hierarchy process”. New York: McGraw Hill.

Saaty, T.L., 1986. “Axiomatic foundationd of the analytical hierarchy process” Management Science, 32: 841-55.

Sinuany-Stern, Z., A. Mehrez and Y. Hadad, 2000. “An AHP/DEA methodology for ranking decision making units”, International Transactions in Operational Research, 7(2): 109-124.

Wang, Y.M., J. Liu, T.M.S. Elhag, 2008. “An integrated AHP–DEA methodology for bridge risk assessment”, Computers & Industrial Engineering, 54: 513-25.

Wabalickis, R.N., 1988. “Justification of FMS with the analytic hierarchy process” Journal of Manufacturing Systems, 7: 175-82.

Venkata, R., 2008. “Evaluating flexible manufacturing systems using a combined multiple attribute decision making method”, International Journal of Production Research, 46(7): 1975-89.

Yang, T., C. Kuo, 2003. “A hierarchical AHP/DEA methodology for the facilities layout design problem” European Journal of Operational Research, 147: 128-36.