

Theory of Constraints and Particle Swarm Optimization Approaches for Product Mix Problem Decision

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Abstract: Theory of constraint (TOC) is a management philosophy that focuses on constraints which limit organization in achieving to its goal. The goal of each organization is to maximize the throughput. The product mix is a major problem and an application of TOC in manufacturing enterprise that includes determination of quantifying and the indentifying of each product to produce. In general, heuristic or meta-heuristic solution methods are used to optimize such problems. Heuristic approaches for these problems include TOC by Goldratt and revised TOC and etc. Sometimes heuristic approaches are inefficient in multi constraints especially in large problems and instead, in these cases meta-heuristic algorithms have been applied extensively. In this paper a proposed particle swarm optimization (PSO) algorithm is applied for solving the product mix optimization problem. Also, the results obtained from the proposed PSO are compared with the results of other approaches.

Key words: Theory of Constraints, Product Mix, Particle Swarm Optimization.

INTRODUCTION

Product mix optimization has become as one of the most important problem in manufacturing enterprise. Because the most manufacturing enterprises have usually more demand than capacity, they intent to select the quantity and type of each product based on the limited recourses. So, there is an optimization problem with an objective function which is to maximize enterprise's throughput and limitations which should be satisfied. This problem is known as a product mix problem. The product mix problem determines the type and quantity of product to produce. Product mix optimization has a direct impact on manufacturing enterprise's financial and non-financial performance, such as its profit, work-in-process (WIP) inventory, customer service, and shop manageability (Lea and Fredendall, 2002). In the literature, two general approaches to solve the product mix problem are argued: heuristic and meta-heuristic algorithms.

One of the well-known heuristics is the Theory of Constraints (TOC) (Goldratt, 1987). TOC is a management philosophy developed by Goldratt. Goldratt and Cox (1992) in their book called "The Goal: A Process of Ongoing Improvement", states that a firm's goal is to make money now and in the future. TOC focuses on constraints which restrict the performance of an organization in obtaining its goal that is making money. TOC is implemented in an enterprise performance measures through three financial criteria: throughput, operating expenses, and inventory. Goldratt and Fox (1986) defined these original performance measures under TOC. Throughput is defined as the rate at which the entire system generates money through sales. Inventory is defined as all the money the system invests in purchasing things the system intends to sell. Operating expenses has defined as all the money the system spends in turning inventory into throughput. Goldratt and Cox (1992) states that there is always at least one constraint that restricts the company's ability to achieve its goal. TOC categorizes the resources into two bottleneck resources and non-bottleneck resources. Goldratt and Cox (1992) developed a five step process to manage constraints and improve the system continuously. These steps are as following:

Step 1: IDENTIFY system constraints, whether physical or political constraints.

Step 2: Decide how to **EXPLOIT** the system constraints. That is, get the most possible within the limit of the current constraints.

Step 3: SUBORDINATE everything else to the above decision.

Step 4: ELEVATE the system constraints. That is, reduce the effects of the current constraints; off-load some

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demand or expand capability; and make everyone aware of the constraints and its effects on per performance of processes.

Step 5: If in the previous steps a constraint has been broken, go back to step 1, but **DON'T** allow inertia to cause a system constraint.

For complete description of these steps can refer to Goldratt and Cox (1992), Motwani and Vogelsang (1996), Dettmer (1997), Stein (1997), Rahman (1998). This process can be applied for increasing throughput, decreasing inventory and operating expenses. In this way the company will reach to its goal, namely making money (Goldratt & Cox, 1992). Based on TOC approach, the throughput, due date, utilization, and other key performance measures of an enterprise can be controlled and optimized by controlling only the bottleneck resources in the enterprise. Various applications of TOC are known to be product mix, logistics, scheduling, performance measurement, problem solving/thinking process, project management, market segmentation and etc. The main objective of TOC is to maximize the output that is obtained by determination and exploitation of the Critically Constrained Resource (CCR) (Onwubolu and Mutingi, 2001).

Nevertheless, researchers have not agreement about finding the best method of the product mix. Some researchers believe that the theory of constraints (TOC) heuristic for product mix problem (Goldratt and Cox, 1992; Low, 1992; Corbett, 1998) present like or better than linear programming (LP).

Countless studies have been performed to determine the product mix to obtaining the optimal profit. Goldratt (1990) proposed traditional TOC algorithm to solving the product mix problem. According to (Plenert, 1993; Fredendall and Lea, 1997; Hsu and Chung, 1998; Balakrishnan and Cheng, 2000), traditional TOC algorithm is not capable provide the optimal feasible solution or might provide an optimal solution (Luebbe and Finch, 1992; Lee and Plenert, 1996) when it is face with multi constraint resource problem. Because of inability the TOC heuristic to obtain the optimal product mix, Fredendall and Lea (1997) and Hsu and Chung (1998) proposed a revised TOC (RTOC) heuristic that identifies the optimal solution for the product mix problems. Fredendall and Lea (1997) in the beginning found a feasible solution that was not optimal solution, after that, through a searching neighborhood, attempted to increase or decrease the product quantity for removing the idle time for each constraint until all constraints could be completely utilized. Hsu and Chung (1998) made the TOC approach explicitly that includes successive iterations for deriving all capacity constraint resource(s) (CCR(s)), a method to update the value of “\$/ (constraint time)”, and quantity adjustment for previously chosen products. The presented algorithm by Hsu and Chung (1998) concludes the similar results at iterations as the dual-simplex method with bounded variables. Aryanezhad and Komijan (2004) presented an improved algorithm on the revised TOC. They also developed an improved algorithm to achieve an optimal solution and their algorithm result was the same result as with Integer linear programming. Two traditional TOC and revised TOC methods have common defects: (first) when there are multiple CCR, their computation workload is very much; (second) these two methods cannot effectively solve the product mix problem with large-scale, which, this case occurs in manufacturing enterprises in practice.

The Integer linear programming (ILP) approach has been examined with TOC product mix heuristic by Luebbe and Finch (1992). Plenert (1993) formulated product mix problem as ILP and concluded proved when there are multiple constraint, TOC heuristic has worse results than the ILP approach and so can be considered as suboptimal.

Meanwhile Some meta-heuristics and intelligent search algorithms is presented by researchers. Onwubolu (2001) proposed a Tabu search (TS) to solve product mix in multiple constraints resource problems. Result of applying TS was better than the Traditional TOC heuristic but is worse in relation to ILP and revised TOC approaches. Also Onwubolu and Mutingi (2001) used genetic algorithms (GA), (Mishra *et al.* 2005) applied hybrid Tabu-simulated annealing approach to solve large-scale product mix optimization problem. It is worth to know that applied approach by (Mishra *et al.* 2005) because resource 40 in their research was overloaded is infeasible. Recently, Wang *et al.* (2008) developed an immune algorithm to solve the TOC product mix problem.

Also, as for large scale product mix problem cannot be solved by heuristic algorithm such as TOC, therefore, this paper proposes another meta-heuristic particle swarm optimization (PSO) method to deal with large typical instances of manufacturing enterprises which have not been dealt with by known literature. With the proposed novel intelligent search approach based on PSO algorithm better optimization solution in the small-scale or large-scale product mix problem is achieved.

Problem Description:

Here, we describe the linear programming (LP) formulation for the product mix problem. In addition, an instance is mentioned related to this problem and instance are based on the work of Fredendall and Lea (1997).

The LP definition of product mix problem can be represented by:

$$\begin{aligned}
 \text{Max } Z &= \sum_{i=1}^n X_i(MP_i - RM_i) \\
 \text{s. t.} & \\
 \sum_{i=1}^n a_{ij}X_i &\leq b_j \quad j \in \{1, 2, \dots, M\} \\
 X_i &\leq MD_i \quad i \in \{1, 2, \dots, n\}
 \end{aligned} \tag{1}$$

Where X_i is the decision variable that indicates the amount of production of product i in the enterprise, MP_i and RM_i are the market price and the raw material cost of product i in the enterprise, respectively. MD_i indicates the market demand of product i . a_{ij} is the processing time for product i in resource j and b_j is the total capacity of resource j in the planning period. We assume that there are M machines in the enterprise and enterprise can produce n products.

The objective function indicates the throughput of enterprise that is resulted from producing products by enterprise production to meet the market demand. The profit of enterprise can be calculated from subtracting costs (raw materials costs, and operating expenses) from incomes resulted from selling the products:

$$\text{profit} = \sum_{i=1}^n [X_i(MP_i - RM_i)] - OE \tag{2}$$

Where, OE represents the operation expense that is constant value.

For applying and illustration of the solving methods problems, here, an instance for the integrated product mix problem is mentioned. This example has been taken from Fredendall and Lea (1997). In this example an enterprise decides to determine the amount of production of each of its products that should produce as it achieve to maximum profit. The enterprise produces three products A, B, C, D and E . The enterprise has six resources (machines) $I, II, III, IV, V,$ and VI that are used for producing these products; the market demand of products A, B, C, D and E is 10, 30, 40, 30, and 60 units per week, respectively. In addition, the capacity of all resources in each week is 2400 minutes except resource 2 which is 1825 minutes. The raw materials cost for products A, B, C, D and E is \$ 10, 42, 25, 25 and 15 per unit. Figure 1 shows the process route, processing time of each product in each resource, market demand (MD) of each product, market price (MP) of each product, and raw material (RM) unit cost of each product.

TOC Approach:

TOC approach controls the throughput and utilization of the only bottleneck resources to control the throughput and other performance measures. TOC divide the resources of enterprise into two categories: bottleneck and non-bottleneck resources. For identifying the bottleneck resource, firstly the amount of time that each resource is loaded for meeting the demand of products should be determined. If the amount of total load on a resource is more than the amount of available time of its resource, this resource is bottleneck, otherwise is non-bottleneck. Based on the TOC, the cost of missing an hour of bottleneck equals to cost of missing of an hour in whole system (Goldratt and Cox, 1992). Now, for abovementioned instance the bottleneck is identified as following:

Table 1 shows how the bottleneck is identified. The second row in Table 1 indicates the total needed processing time for production all of the products A, B, C, D and E that in each resource is needed. The third row shows the total available time for each resource in a week that in this example is 2400 minutes for all the resources except resource 2 which is 1825 minutes. The fourth row indicates the utilization percent of each resource. The utilization percent of each resource is determined by following equation:

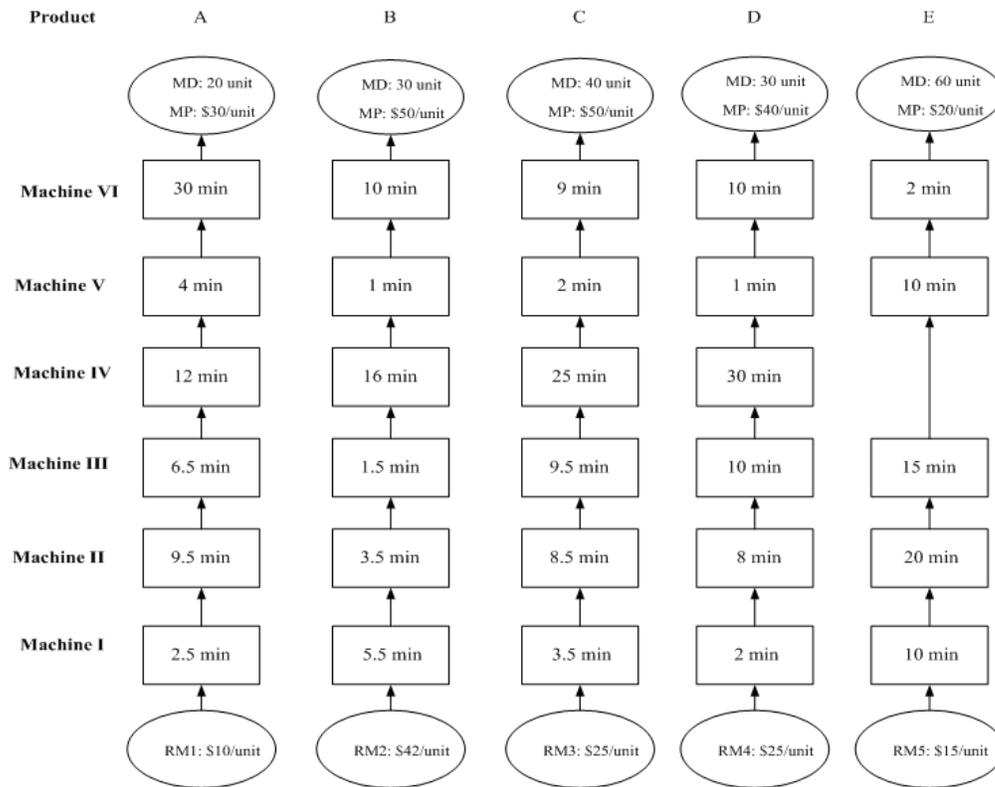


Fig. 1: Product structure and process times taken from Fredendall and Lea (1997)

$$Utilization\ percent = \frac{\sum_{i=1}^n (Processing\ Time_i \times MD_i)}{Total\ Available\ Time} \times 100 \tag{4}$$

For example, for resource *I* based on equation (3) the utilization percent is

$$Utilization\ percent = \frac{2.5 \times 20 + 5.5 \times 30 + \dots + 10 \times 60}{2400} \times 100 = \frac{1015}{2400} \times 100 = 42.3\% \tag{5}$$

Table 1: Bottleneck identification

Resource	I	II	III	IV	V	VI
Total needed processing time (min)	1015	2075	1755	2620	820	1680
Total available time per week (min)	2400	1825	2400	2400	2400	2400
Utilization percent (%)	42.3	113.7	73.1	109.2	34.2	70

Therefore, as in Table 1 for resource *II* and *IV* the utilization percent value is more than 100% then this resource is bottleneck. Herein, most authors in the literature the TOC-based approach intent to the most overloaded constraint, i.e. resource II in this example.

Now, the amount of internal production of each product is determined. Table 2 shows this for each product in its sixth row. As we know, the subtraction of raw material costs from market price for each product indicates the value of throughput resulted from producing one unit of it. Dividing the value of unit throughput of each product on its processing time in the bottleneck resource indicates the unit throughput that is obtained from one minute bottleneck resource for each product. As before mentioned one hour of bottleneck resource equals to one hour of the whole system. Thus, the more value of unit throughput per unit time in the bottleneck is, the more whole system throughput is. Based on Table 2 the priority of internal production of

each product is $C > B > A > D > E$. Thus, the product C should be produced at first. The demand of product C is 40 units; therefore, 40 units of product C are produced at first. Production of 40 units of product C takes $40 \times 8.5 = 340$ minutes in the bottleneck resource II . The rest time for resource F is 1485. With this available time, 30 units of product B , 20 units of product A , and 30 units of product D can be produced. Production of 40 units of product C , 30 units of product B , 20 units of product A , and 30 units of product D takes $40 \times 8.5 + 30 \times 3.5 + 20 \times 9.5 + 30 \times 8 = 875$ minutes in the bottleneck resource II . The rest time for resource F is 950. With this available time, only $950/20 = 47$ units of product E can be produced. With this obtained solution from TOC approach, the whole throughput is \$ 2325.

Table 2: The results of TOC approach

product	A	B	C	D	E
throughput per product unit (\$)	20	8	25	15	5
Processing time in the bottleneck resource (II) (min)	9.5	3.5	8.5	8	20
throughput per unit time in the bottleneck (\$/min)	2.11	2.29	2.94	1.88	.25
Demand	20	30	40	30	60
production	20	30	40	30	47

However, the solution obtained the TOC approach is infeasible since resource IV is overloaded. whole Processing time in resource IV for producing 40 units of product C , 30 units of product B , 20 units of product A , 30 units of product D , and 47 units of product E is 2620 min. The best feasible solution obtained by the revised TOC-based approach is: 20 units of product A , 20 units of product B , 40 units of product C , 28 units of product D , and 50 units of product E with a total throughput of \$2230 (Fredendall and Lea, 1997). This solution also is equal to the optimal ILP solution. Onwubolu (2001) applied TS approach for solving the same product mix problem. result was 20 units of product A , 27 units of product B , 40 units of product C , 24 units of product D , and 49 units of product E the throughput of \$2221. In this paper, we suggest the process of using particle swarm optimization approach for solving the multi-constraint TOC product mix problem.

Particle Swarm Optimization Approach:

Particle Swarm Optimization (PSO) as a method for nonlinear optimization is introduced by Kennedy and Eberhart (1995). This method uses a model of flight of birds (or motion of particles) to solve the optimization problem in which any potential solution in search space is considered as a potential position for particles. Swarm of particles move through search space under a defined dynamics of flight and find the best solution as the optimum solution. The position of i -th particle is presented by an n -dimensional vector for an n -dimensional search space

$$x_i = (x_{i1}, x_{i2}, \dots, x_{in}) \tag{1}$$

Also, every particle possess a velocity vector as

$$v_i = (v_{i1}, v_{i2}, \dots, v_{in}) \tag{2}$$

In any iteration, positions and velocities are updated by means of equations below

$$v_{ij}(t+1) = I_{ij}v_{ij}(t) + R_{1ij}c_1(P_{ji} - x_{ij}(t)) + R_{2ij}c_2(P_{gj} - x_{ij}(t)) \tag{3}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \tag{4}$$

Where P_{ji} and P_{gj} are the j -th coordinate of best previous positions of i -th particle and global best previous position associated with cost function. R_{1ij} and R_{2ij} are two random variables in $[0,1]$, c_1 and c_2 are acceleration constants. I_{ij} is the parameter which is used to model the inertia effect of particle's previous velocity Using best previous positions, PSO needs memory to save these previous states. Algorithm of PSO method has steps below:

1. Problem definition: This step includes the definition of cost function and determining the range of search space. The minimum and maximum values which the particles could take as coordinates of their position should be determined.
2. Initialization: Initialize positions and velocities of particles randomly. Also, best position of each particle set to be its initial position and global best position is set to be the best initial position associated with cost function.
3. Particles motion: Find new position and velocities of particles using equations 3 and 4.
4. Evaluation: Evaluate cost function for each particle position and update best positions of particles and best global position.
5. Repeat: Go to step 3 again and repeat till a stopping criterion is satisfied. The stopping criterion could be considered as taking a predefined number of iterations.

The steps of PSO algorithm are shown in Figure 2.

Simulation Results:

In this section, PSO is applied to the problem of product mix. A sample product mix problem which is taken from Fredendall and Lea (1997) is used to show how PSO can be applied to these kinds of problems. In the following each part of the algorithm is restated to show the applicability of PSO to the product mix problems.

The Steps of PSO for Solving Product Mix Problem:

The problem studied in previous section consists of 5 parameters, X_A, X_B, X_C, X_D, X_E for which $0 \leq X_A \leq 20, 0 \leq X_B \leq 30, 0 \leq X_C \leq 40, 0 \leq X_D \leq 30, 0 \leq X_E \leq 60$. the objective is to

maximize profit satisfying the constraints. The search space consists of $(21 \times 31 \times 41 \times 31 \times 61 = 50472681)$ set of solutions. To use the PSO to solve this problem, at first a proper definition of a particles position should be stated. As mentioned before, the particle position is a set of unknown parameters of the problem. In this problem a particle position includes 5 parameters as following.

Particle position = $[X_A, X_B, X_C, X_D, X_E]$

For example $[X_A, X_B, X_C, X_D, X_E] = [15, 30, 22, 17, 53]$ indicate a particle position. The algorithm looks for a particle position which has the minimum value of cost function. To convert the maximization problem of product mix to a minimization one, the cost function is considered to be the minus of Throughput value. That is,

Cost (Particle position) = - Throughput $(X_A, X_B, X_C, X_D, X_E)$

The constraints of the problem are considered in the definition of the cost function by defining a penalty value for the solutions which are out of the feasible solution space. Other ways of constraint handling can be used which might enhance the convergence of the algorithm.

To start the algorithm a set of random solutions (random particles positions) is generated. The costs of all particles positions are calculated and the best position among them (with respect to cost) is determined. The velocities of each particle are calculated, and then particles move with these velocities toward their next positions.

The velocities of particles are functions of best experienced positions of particles and the best experienced position among all particles, so that in successive iterations of algorithm the particles are getting closed to the best position in the search space. Parameters of PSO algorithm used in this study are shown in Table 3.

Table 3: Parameters of PSO approach

Parameter	Value
Number of Particles	200
Num Of Iterations	60
Inertia	0.285
C_1	1.5
C_2	2.5

The convergence plot of PSO algorithm is depicted in Figure 3 resulting the best solution as $[X_A, X_B, X_C, X_D, X_E] = [20, 29, 40, 23, 50]$ with throughput of 2343.

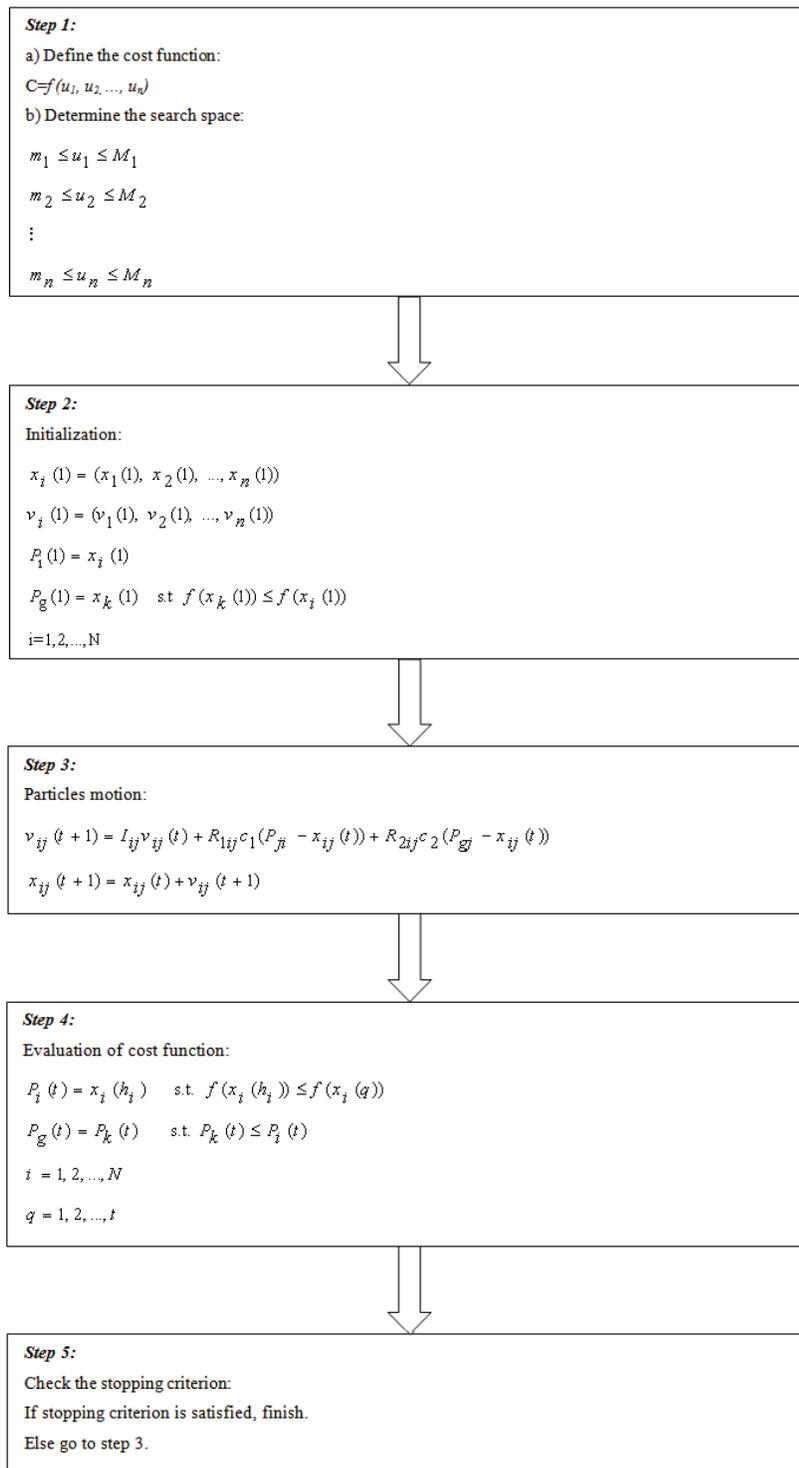


Fig. 2: Flow chart of PSO algorithm

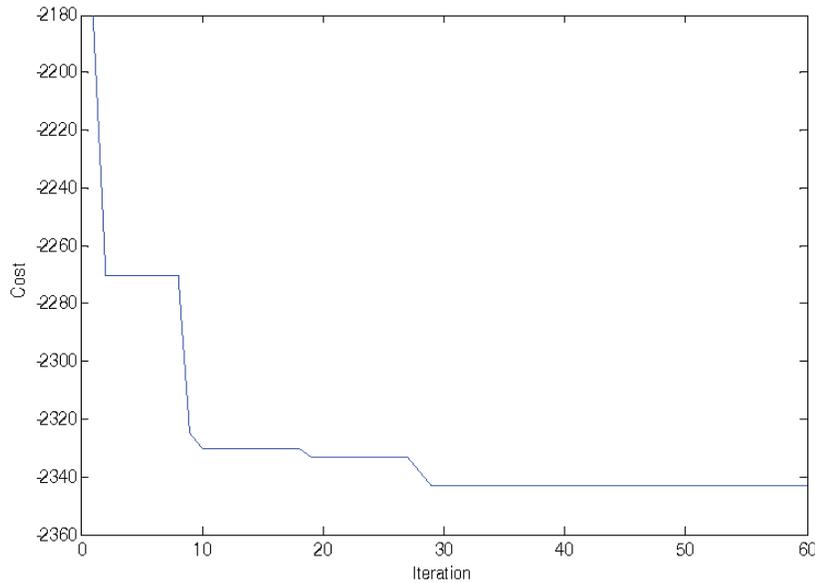


Fig. 3: Convergence of the PSO to the optimal solution.

Simulation Result Comparison Between Proposed PSO and Other Approaches:

To validate the proposed PSO algorithm for solving product mix problem decision, PSO approach used for this research has comprised with the other approaches that previously applied in the literature. Existence approaches in literature are TOC, RTOC, ILP, TS, and TS-SA. Figure 4 indicates all of the comparisons in related to proposed PSO method with other approaches. Also, this Figure shows that proposed PSO algorithm leads to the maximum throughput among other methods it is notable that traditional TOC approach and TS-SA approach by Mishra et al. (2005) are infeasible.

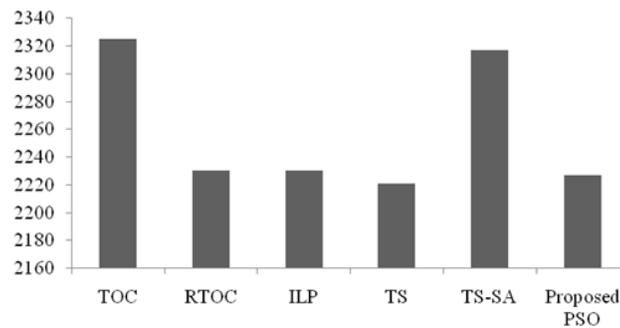


Fig. 4: Comparisons of proposed PSO with other approaches

Conclusion:

Theory of constraints introduced by Goldratt can be applied in various fields such as production, project management, market, and problem solving. One of the most important applications of TOC in production field is product mix problem decisions. Product mix problem was examined by the various authors using different methods such as revised TOC, integer linear programming, tabu search, and etc. In this paper, we presented new approach to decision making about product mix problem. This approach is entitled PSO. Then, we examined PSO approach against another approaches in the literature (TOC, RTOC, ILP, TS, TS-SA) and obtained better solution in example from Fredendall and Lea (1997). Also, we solved other example to further test the proposed PSO algorithm. All of the cases had similar results and demonstrated the superiority of the proposed PSO in relation to other approaches. Achieving optimal Product mix helps managers to make a decision in hard situation. The proposed approach to solve product mix problem can used by managers that are involved in manufacturing areas.

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