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Higher Ternary Jordan Derivations on Ternary Algebras

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Abstract: We prove the Hyers-Ulam stability of higher ternary Jordan derivations in ternary algebras associated with the Cauchy functional equation by applying a version of the fixed point Theorem.

Key words: Hyers-Ulam stability; ternary algebra; higher ternary Jordan derivation.

INTRODUCTION

A ternary algebra is a real or complex linear space, endowed with a linear mapping, the so-called a ternary product, $(x,y,z)\to[xyz]$ of $A\times A\times A$ into A such that $[[xyz]tu]=[x[yxt]u]=[xy[ztu]]$ for all $x,y,z,t,u\in A$.

If $(A,.)$ is a usual binary algebra, then an induced ternary multiplication can be defined by $[xyz]=(x,y).z$ Hence, the ternary algebra is a natural generalization of the binary case. If a ternary algebra $(A,[])$ has a unit, i.e., an element $e\in A$ such that $x=[xee]=[eex]$ for all $x\in A$, then A with the binary product $x.y=[xey]$ is a usual algebra.

The ternary algebras were studied in the nineteenth century. Their structures appeared more or less naturally in various domains of mathematical physics and data processing. The discovery of the Nambu mechanics and the progress of quantum mechanics Nambu, (1973), as well as work of S. Okubo Okubo, (1993) on Yang-Baxter equations resulted in significant development on ternary algebras (see also [1,4,29,35,36,37,50,52]).

A normed ternary algebra is a ternary algebra with a norm $\|.\|$ such that $\|[xyz]\|\le\|x\|\|y\|\|z\|$ for all $x,y,z\in A$. A Banach ternary algebra is a normed 2000 Mathematical Subject Classification. 39B52; 39B82; 46B99; 17A40. ternary algebra such that the normed linear space with norm $\|.\|$ is complete. Assume that A *and* B are real or complex ternary algebras. A linear map $h:A\to B$ is said to be a ternary homomorphism if $h[xyz]=[h(x)h(y)h(z)]$ holds for all $x,y,z\in A$.

A linear mapping $D:A\to B$ is called a ternary Jordan derivation if

$D([xxx])=[d(x)xx]+[xD(x)x]+[xxD(x)]$ for all $x\in A$.

Let $\mathbb{N}$ be the natural numbers. For $m\in\mathbb{N}\cup\{0\}=\mathbb{N}_0$, a sequence $H=\{h_0,h_1,...,h_m\}$ $(\text{resp. } H=\{h_0,h_1,...,h_n,...\})$ of linear mappings from A into B is called a higher

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ternary Jordan der ivation of rank m (resp. infinite rank) from A into B if

$$h_n[xxx] = \sum_{i+j+k=n} [h_i(x)h_j(x)h_k(x)]$$

holds for each $n \in \{0,1,...,m\}$ (resp. $n \in \square_0$) and all $x \in A$.

We say that a functional equation (ξ) is stable if any function g satisfying the equation (ξ)

approximately is near to true solution of (ξ) . We say that a functional equation is superstable if every approximate solution is an exact solution of it.

The stability of functional equations was first introduced by (S.M. Ulam, 1940; Hyers, 1941) gave a

partial solution of Ulam s problem for the case of approximate additive mappings in the context of Banach spaces. In T. Aoki, (1950) was the second author to treat this problem for additive mappings (see also Bourgin, 1949). In Th. M. Rassias, (1978) generalized the Theorem of Hyers by considering the stability problem with unbounded Cauchy differences

$\|f(x+y)-f(x)-f(y)\| \le \varepsilon(\|x\|^p + \|y\|^p)$, $(\varepsilon \succ 0,\ p \in [0,1).)$ This phenomenon of stability that was

introduced by Th. M. Rassias, (1978) is called the Hyers-Ulam-Rassias stability (Eshaghi Gordji, 2010; 2009; Eshaghi Gordji and Savadkouhi, 2010; Eshaghi Gordji, *et al.,* 2009; Eshaghi Gordji and Khodaei, 2009; Eshaghi Gordji, *et al.,* 2009; Eshaghi Gordji, *et al.,* 2010; Eshaghi Gordji, *et al.,* 2008; Eshaghi Gordji, *et al.,* 2010; Eshaghi Gordji, *et al.,* 2009; 2009; Eshaghi Gordji and Najati, 2010; Eshaghi Gordji, *et al.,* 2009; Eshaghi Gordji, *et al.,* 2009; Eshaghi Gordji, *et al.,* 2009; 2009; Eshaghi Gordji, M., Savadkouhi, 2009; Eshaghi Gordji, *et al.,* 2009; 2009; Eshshgi Gordji, M., Savadkouhi, 2009; Eshaghi Gordji *et al.,* 2010; 2010; 2010; Eshaghi Gordji and Bavand Savadkouhi, 2009; G vruta, 2010; Ghaemi and Alizadeh, 2010; Khodaei and Rassias, 2010; Khodaei and Kamyar, 2010; Park, *et al.,* 2010; Park and Najati, 2010; Park and Rassias, 2010; Rassias, 2000; 2000; 1993; Shakeri, 2010). In Gavruta, (1992). generalized the Th. M. Rassias Theorem as follows:

Suppose $(G,+)$ is an ablian group, X is a Banach space $\varphi : G \times G \to [0,\infty)$ satisfying

$\tilde{\varphi}(x,y) = \frac{1}{2}\sum 2^{-n}\varphi(2^n x, 2^n y) \prec \infty$ for all $x,y \in G$. If G→X is a mapping with

$\|f(x+y)-f(x)-f(y)\| \le \varphi(x,y)$ for all $x,y \in G$, then there exists a unique mapping $T : G \to X$ such that

$T(x+y) = T(x) + T(y)$ and $\|f(x) - T(x)\| \le \tilde{\varphi}(x,x)$ for all $x, y \in G$.

Throughout this paper, A denote a ternary algebra, B is a Banach ternary algebra, X denote a linear space, and Y represents a Banach space.

Main results:

We start our work with a known fixed point Theorem.

Theorem 2.1. Let (X,d) be a complete generalized metric space and $J : X \to X$ be a strictly contractive mapping, that is,

$$d(Jx,Jy) \le Ld(x,y) \quad (x,y \in X)$$

for some $L \prec 1$. Then, for each fixed element $x \in X$, either

$$d(J^n x, J^{n+1}x) = +\infty, \quad \text{for all } n \ge 0$$

or

$$d(J^n x, J^{n+1}x) \prec +\infty, \quad \text{for all } n \ge n_0$$

for some n_0. Moreover, if the second alternative holds, then:
(i) The sequence $J^n x$ is convergent to a fixed point y^* of j;

(ii) y^* is the unique fixed point of J in the set $Y = \left\{ y \in X : d\left(J^{n_o} x, y\right) \prec +\infty \right\}$ and

$d\left(y, y^*\right) \le \frac{1}{1-L} d\left(y, Jy\right) \quad \left(x, y \in Y\right)$

We need the following lemma in the proof of our main result.

Lemma 2.2. Let $0 \le L \prec 1$ and $\lambda \ge 0$ be given numbers, and $\psi : X \to [0, +\infty)$ has the property

$$\psi\left(x\right) \le \lambda L \psi\left(\frac{x}{\lambda}\right)$$

for all $x \in X$. Assume that $S = \left\{ g : X \to Y : g\left(0\right) = 0 \right\}$ and the generalized metric d on S is defined by

$$d\left(g, h\right) = \inf\left\{ c \in \left(0, \infty\right) : \left\| g\left(x\right) - h\left(x\right) \right\| \le c\psi\left(x\right), \forall x \in X \right\}$$

Then the mapping $J: S \to S$ given by $Jg\left(x\right) = \frac{1}{\lambda} g\left(\lambda x\right)$ is a strictly contractive mapping.

Proof. It is easy to see that (S,d) is complete. For arbitrary elements $g,h \in S$, we have

$$d\left(g, h\right) \prec c \Rightarrow \left\| g\left(x\right) - h\left(x\right) \right\| \le c\psi\left(x\right), x \in X \Rightarrow \left\| \frac{1}{\lambda} g\left(\lambda x\right) - \frac{1}{\lambda} h\left(\lambda x\right) \right\|$$
$$\le \frac{1}{\lambda} c\psi\left(\lambda x\right), x \in X \Rightarrow \left\| \frac{1}{\lambda} g\left(\lambda x\right) - \frac{1}{\lambda} h\left(\lambda x\right) \right\|$$
$$\le Lc\psi\left(x\right), x \in X \Rightarrow d\left(Jg, Jh\right) \le Lc.$$

Therefore, $d\left(Jg, Jh\right) \le Ld\left(g, h\right), g, h \in S.$

Hence, J is a strictly contractive mapping on S with the Lipschits constant L.

Theorem 2.3. Let $\varphi : A^5 \to [0, \infty)$ be a control function such that

$$\lim_{x \to \infty} \frac{\varphi\left(2^n x, 2^n y, 2^n u, 2^n u, 2^n u\right)}{2^n} = 0$$

for all $x,y,u \in A$. Suppose that $F = \left\{ f_0, f_1, \ldots, f_n, \ldots \right\}$ is a sequence of mapping from A into B such that

fn (0)=0 and

$$\left\| f_n\left(\lambda x + y + [uuu]\right) - \lambda f_n\left(x\right) - f_n\left(y\right) - \sum_{i+j+k=n} \left[f_i\left(u\right) f_j\left(u\right) f_k\left(u\right) \right] \right\| \le \varphi\left(x, y, u, u, u\right) \tag{2.1}$$

for all $x, y, u \in A, n \in \square_0, \lambda \in \Delta = \left\{ z \in \square : |z| = 1 \right\}$ Assume that there exists $0 \le L \prec 1$ such that the

mapping $\psi\left(x\right) = \varphi\left(\frac{x}{2}, \frac{x}{2}, 0, 0, 0\right)$ has the property

$$\psi\left(x\right) \le 2L\psi\left(\frac{x}{2}\right) \tag{2.2}$$

for all $x \in A$. Then there exists a unique higher ternary Jordan derivation $H = \{h_0, h_1, ..., h_n, ...\}$ of any rank from A into B such that

$$\|f_n(x) - h_n(x)\| \le \frac{L}{1-L}\psi(x)$$

for each $n \in \mathbb{N}_0$ and for all $x \in A$.

Proof. Settiny $\lambda=1, y=x$ and $u=0$ in (2.1) to get

$$\|f_n(2x) - 2f_n(x)\| \le \varphi(x, x, 0, 0, 0). \quad (2.3)$$

It follows from (2.2) and (2.3) that

$$\left\|\frac{1}{2}f_n(2x) - f_n(x)\right\| \le \frac{1}{2}\psi(2x) \le L\psi(x)$$

For all $n \in \mathbb{N}_0$ and $x \in A$. So, $d(f_n, Tf_n) \le L \prec \infty$, where the mapping T is defined on $S = \{g_n : A \to B : g_n(0) = 0\}$ by $(Tg_n)(x) = \frac{1}{2}g_n(2x)$ and is a strictly contractive function as in lemma 2.2. Applying the fixed point alternative, we deduce the existence of a mapping $h_n : A \to B$ such that h_n is a fixed point of T that is $h_n(2x) = 2h_n(x)$ for all $x \in A$. Since $\lim_{m\to\infty} d(T^m f_n, h_n) = 0$, it follows that

$$\lim_{m\to\infty}\frac{f_n(2^m x)}{2^m} = h_n(x)$$

for all $x \in A, n \in \mathbb{N}_0$. The mapping h_n is the unique fixed point of T in the set $U = \{g_n \in S : d(f_n, g_n) \prec \infty\}$. Hence, h_n is the unque fixed point of T such that $\|f_n(x) - h_n(x)\| \le K\psi(x)$ for some $K \succ 0$ and for all $x \in A$. Again, by applying the fixed point alternative Theorem, we infer that

$$d(f_n, h_n) \le \frac{1}{1-L}d(f_n, Tf_n) \le \frac{L}{1-L},$$

so

$$\|f_n(x) - h_n(x)\| \le \frac{L}{1-L}\varphi\left(\frac{x}{2}, \frac{x}{2}, 0, 0, 0\right)$$

for all $x \in A, n \in \mathbb{N}_0$. It follows from (2.1) that

$$\|f_n(\lambda x + y) - \lambda f_n(x) - f_n(y)\| \le \varphi(x, y, 0, 0, 0)$$

By replacing x and y in the previous inequality with $2^m x$ and $2^m y$, respectively, dividing both side by 2^m and taking $m \to \infty$, we get

$$h_n(\lambda x + y) = \lambda h_n(x) + h_n(y)$$

for all $\lambda \in \Delta$ and all $x, y \in A$.

Now, let $\lambda \in \mathbb{C}(\lambda \ne 0)$ and let K be a natural number greater than $4|\lambda|$. Then $\left|\frac{\lambda}{K}\right| \prec \frac{1}{4} \prec 1 - \frac{2}{3} = \frac{1}{3}$.

According to Kadison and Pedersen, (1985) Theorem 1, there exists numbers $\lambda_1, \lambda_2, \lambda_3 \in \Delta$ such that $3\frac{\lambda}{K} = \lambda_1 + \lambda_2 + \lambda_3$.

By the additivity of each $h_n, n \in \square_0$, we get $h_n\left(\frac{1}{3}x\right) = \frac{1}{3}h_n(x)$ for each $n \in \square_0$ and all $x \in A$.

Therefore,

$$h_n(\lambda x) = h_n\left(\frac{K}{3}.3.\frac{\lambda}{K}x\right) = \frac{K}{3}h_n\left(3.\frac{\lambda}{K}x\right) = \frac{K}{3}h_n(\lambda_1 x + \lambda_2 x + \lambda_3 x)$$

$$= \frac{K}{3}\left(h_n(\lambda_1 x) + h_n(\lambda_2 x) + h_n(\lambda_3 x)\right)$$

$$= \frac{K}{3}(\lambda_1 + \lambda_2 + \lambda_3)h_n(x) = \lambda h_n(x)$$

for each $n \in \square_0$ and all $x \in A$, so that h_n is $\square-$ linear for each $n \in \square_0$

Next, we need to show that the sequence $H = \{h_0, h_1, ..., h_n, ...\}$ satisfies the identity $h_n([uuu]) = \sum_{i+j+k=n}\left[h_i(u)h_j(u)h_k(u)\right]$ for each $n \in \square_0$ and all $u \in A$. Putting $x=y=0$ in (2.1) and

$$D_n(u,u,u) = f_n([uuu]) - \sum_{i+j+k=n}\left[f_i(u)f_j(u)f_k(u)\right] \tag{2.5}$$

for each $n \in \square_0$ and all $u \in A$, we see that

$$\lim_{r\to\infty}\frac{D_n(2^r u, 2^r u, 2^r u)}{2^r} = 0 \tag{2.6}$$

$n \in \square_0$ and all $u \in A$. Using (2.4), (2.5), and (2.6), we get

$$h_n([uuu]) = \lim_{r\to\infty}\frac{f_n(2^r[uuu])}{2^r} = \lim_{r\to\infty}\frac{f_n\left(\left[(2^r u)(2^r u)(2^r u)\right]\right)}{2^{3r}}$$

$$= \lim_{r\to\infty}\frac{\sum_{i+j+k=n}\left[f_i(2^r u)f_j(2^r u)f_k(2^r u)\right] + D_n(2^r u, 2^r u, 2^r u)}{2^{3r}}$$

$$= \lim_{r\to\infty}\sum\left[\frac{1}{2^r}f_i(2^r u)\frac{1}{2^r}f_j(2^r u)\frac{1}{2^r}f_k(2^r u)\right] + \lim_{r\to\infty}\frac{D_n(2^r u, 2^r u, 2^r u)}{2^{3r}}$$

$$= \sum\left[h_i(u)h_j(u)h_k(u)\right].$$

This completes the proof of the Theorem.

As consequence of the previous Theorem, we show the Hyers-Ulam-Rassias stability of higher ternary Jordan derivations.

Corollary 2. 4. Let $0 \le p \prec 1, \alpha, \beta \succ 0$ and $F = \{f_0, f_1, ..., f_n, ...\}$ be a sequenve of mapping from A into B satisfying $f_n(0) = 0$ and

$$\left\| f_n(\lambda x + y + [uuu]) - \lambda f_n(x) - f_n(y) - \sum [f_i(u) f_j(u) f_k(u)] \right\|$$
$$\le \alpha + \beta\left(\|x\|^p + \|y\|^p + 3\|u\|^p\right)$$

for all $\lambda \in \Delta$ and all $x, y, u \in A$. Then there exists a unique higher ternary Jordan derivation $H = \{h_0, h_1, ..., h_n, ...\}$ of any rank from A into B such that

$$\|f_n(x) - h_n(x)\| \le \frac{\alpha + \beta 2^{1-p}\|x\|^p}{2^{1-p} - 1}$$ for all $x \in A$.

Proof. Put $\varphi(x, y, u, u, u) = \alpha + \beta\left(\|x\|^p + \|y\|^p + 3\|u\|^p\right)$ and let $L = \frac{1}{2^{1-p}}$ in the previous Theorem. Then $\psi(x) = \alpha + 2^{1-p}\beta\|x\|^p$ and there exists a sequence with the required property.

$$H = \{h_0, h_1, ..., h_n, ...\}$$

In a similar fashion to Theorem 2.3, we can prove the following Theorem.

Theorem 2.5. Let $\varphi : A^5 \to [0, \infty)$ be a control function such that

$$\lim_{n\to\infty} 2^n \varphi\left(2^{-n}x, 2^{-n}y, 2^{-n}u, 2^{-n}u, 2^{-n}u\right) = 0$$

for all $x,y,u \in A$. Suppose that $F = \{f_0, f_1, ..., f_n, ...\}$ is a sequence of mapping from A into B such that $f_n(0) = 0$ and

$$\left\| f_n(\lambda x + y + [uuu]) - \lambda f_n(x) - f_n(y) - \sum_{i+j+k=n} [f_i(u) f_j(u) f_k(u)] \right\| \le \varphi(x, y, u, u, u), \tag{2.7}$$

for all $x, y, u \in A, n \in \square_0, \lambda \in \Delta = \{z \in \square : |z| = 1\}$. Assume that there exists $0 \le L \prec 1$ such that the mapping $\psi(x) = \varphi\left(\frac{x}{2}, \frac{x}{2}, 0, 0, 0\right)$ has the property

$$\psi(x) \le \frac{1}{2} L \psi(2x),$$

for all $x \in A$. Then there exists a unique higher ternary Jordan derivation $H = \{h_0, h_1, ..., h_n, ...\}$ of any rank from A into B such that

$$\|f_n(x) - h_n(x)\| \le \frac{1}{1 - L}\psi(x)$$

for each $n \in \square_0$ and for all $x \in A$.

Proof. Setting $\lambda = 1, y = x$, and $u=0$ in (2.7) implies

$$\|f_n(2x) - 2f_n(x)\| \le \varphi(x, x.0.0.0). \tag{2.8}$$

Replacing x with $\frac{x}{2}$ in (2.8), we obtain

$$\left\| f_n(x) - 2f_n\left(\frac{x}{2}\right) \right\| \le \psi(x)$$

for each $n \in \mathbb{N}_0$ and $x \in A$. Thus, $d(f_n, Tf_n) \le L \prec \infty$, where the mapping T is defined on $S = \{g_n : A \to B : g_n(0) = 0\}$ by $(Tg_n)(x) = 2g_n\left(\frac{1}{2}x\right)$ and is a strictly contractive function as in lemma 2.2. Applying the fixed point alternative, we deduce the existence of a mapping $h_n : A \to B$ such that h_n is a fixed point of T that is $h_n\left(\frac{1}{2}x\right) = \frac{1}{2}h_n(x)$ for all $x \in A$. Since $\lim_{m\to\infty} d(T^m f_n, h_n) = 0$, it follows that $\lim_{m\to\infty} 2^m f_n(2^{-m}x) = h_n(x)$ for all $x \in A, n \in \mathbb{N}_0$ The mapping h_n is the unique fixed point of T in the set $U = \{g_n \in S : d(f_n, g_n) \prec \infty\}$. Hence, h_n is the unique fixed point of T such that $\|f_n(x) - h_n(x)\| \le K\psi(x)$ for some $K \succ 0$ and for all $x \in A$. Again, by applying the fixed point alternative Theorem, we infer that

$$d(f_n, h_n) \le \frac{1}{1-L} d(f_n, Tf_n) \le \frac{1}{1-L},$$

so

$$\|f_n(x) - h_n(x)\| \le \frac{1}{1-L}\varphi\left(\frac{x}{2}, \frac{x}{2}, 0, 0, 0\right)$$

for all $x \in A, n \in \mathbb{N}_0$. The rest is similar to the proof of Theorem 2.3.

The following corollary is similar to corollary 2.4 for the case where $p \succ 1$.

Corollary 2.6. Let $p \succ 1, \alpha, \beta \succ 0$ and $F = \{f_0, f_1, ..., f_n, ...\}$ be a sequence of mapping from A into B satisfying $f_n(0) = 0$ and

$$\left\| f_n(\lambda x + y + [uuu]) - \lambda f_n(x) - f_n(y) - \sum \left[f_i(u) f_j(u) f_k(u) \right] \right\|$$
$$\le \alpha + \beta\left(\|x\|^p + \|y\|^p + 3\|u\|^p\right)$$

for all $\lambda \in \Delta$ and all $x, y, u \in A$. Then there exists a unique higher ternary Jordan derivation $H = \{h_0, h_1, ..., h_n, ...\}$ of any rank from A into B such that

$$\|f_n(x) - h_n(x)\| \le \frac{\alpha 2^{p-1} + \beta\|x\|^p}{2^{p-1} - 1} \quad \text{for all } x \in A.$$

Proof. Put $\varphi(x, y, u, u, u) = \alpha + \beta\left(\|x\|^p + \|y\|^p + 3\|u\|^p\right)$ and let $L = \frac{1}{2^{p-1}}$ in the previous Theorem. Then $\psi(x) = \alpha + 2^{1-p}\beta\|x\|^p$ and there exists a sequence with the required property.

$H = \{h_0, h_1, ..., h_n, ...\}$

It is natural to ask whether there exists an approximately higher ternary Jordan derivation that is not a higher ternary Jordan derivation. The following example is a slight modification of an example of Park and Jung Park and Jung, (2008).

Example 2.7. Let X be a compact Hausdorff space and let (A[]) be the Banach ternary algebra of complex-valued continuous functions on X under the usual addition of complex-valued continuous functions, the ternary operation $\left[\rho_1\rho_2\rho_3\right] = \rho_1 * \rho_2 * \rho_3$, and the supremum norm $\|.\|_\infty$, where * denotes the usual multiplication of complex-valued continuous functions. Assume that $\tau : A \to A$ is a continuous ternary homomorphism. We define $f : A \to A$ by

$$f\left(x\right)\left(a\right) = \begin{cases} \tau\left(x\right)\left(a\right)\log\left|\tau\left(x\right)\left(a\right)\right| & \text{if} \quad \tau\left(x\right)\left(a\right)\neq 0 \\ 0 & \text{if} \quad \tau\left(x\right)\left(a\right)=0 \end{cases}$$

for all $x \in A$ and all $a \in A$. It is easy to see that

$$f\left(\left[xxx\right]\right) = \left[f\left(x\right)\tau\left(x\right)\tau\left(x\right)\right] + \left[\tau\left(x\right)f\left(x\right)\tau\left(x\right)\right] + \left[\tau\left(x\right)\tau\left(x\right)f\left(x\right)\right]$$

for all $x \in A$ Let $h_0 = 0, h_n = 0, 1 \le n \le m-1$ and $h_m = f$. Then we see that the sequence $H = \left\{h_0, h_1, \ldots, h_n, \ldots\right\}$ satisfies the relation

$$h_n\left(\left[xxx\right]\right) = \sum_{i+j+k=n} \left[h_i\left(x\right)h_j\left(x\right)h_k\left(x\right)\right]$$

for all $x \in A$ H is not a higher ternary Jordan derivation on A Park and Jung, (2008). That is, we may regard H as an approximately higher ternary Jordan derivation of rank m on A.

REFERENCES

Abramov, V., R. Kerner and B.L. Roy, Hypersymmetry a Z_3 graded generalization of supersymmetry, J. Math. Phys., 38: 1650-1669.

Aoki, T., 1950. On the stability of the linear transformation in Banach spaces, J. Math. Soc. Japan, 2L 64-66.

Bourgin, D.G., 1949. Approximately isometric and multiplicative transformations on continuous function rings, Duke Math. J. 16: 385-394.

Cayley, A., 1881. On the 34 concomitants of the ternary cubic, Amer. J. Math., 4: 1-4: 1-15.

Eshaghi Gordji, M., 2010. Stability of a functional equation deriving from quartic and additive functions, Bull. Korean Math. Soc., 47(3): 491-502.

Eshaghi Gordji, M., 2009. Stability of an additive-quadartic functional rquation of two variables in F-space, Journal of nonlinear Sciences and Applications, 2(4): 251-259.

Eshaghi Gordji, M. and M.B. Savadkouhi, 2010. Stability of a mixed type cubicquartic functional equation in non-Archimedean spaces, Applied Mathematics Letters, 23(10): 1198-1202.

Eshaghi Gordji, M., T. Karimi, S. Kaboli Gharetapeh, 2009. Approximately Jordan homorphisms on Banach algebras, J. Ineq. Appl. Article ID 870843, 8.

Eshaghi Gordji, M., H. Khodaei, 2009. On the generalized Hyers-Ulam stability of quadratic functional equations, Abstract and Applied Analysis, Article ID 923476, 11.

Eshaghi Gordji, M., M.B. Ghaemi, H. Majani, 2010. Generalized Hyers-Ulam-Rassias theorem in Menger probabilistic normed spaces, Discrete Dynamics in Nature and Society, Article ID 162371, 11.

Eshaghi Gordji, M., M.B. Ghaemi, H. Majani, C. Pakk, 2010. Generalized Ulam-Hyers Stability of Jensen Functional Equation in erstnev PN Spaces, Journal of Inequality and Applications, Article ID 868193, 14.

Eshaghi Gordji, M., A. Ebadian and S. Zolfaghari, 2008. Stability of a functional equation deriving from cubic and quartic functions, Abstract and Applied Analysis, Article ID 801904, 17.

Eshaghi Gordji, M., M.B. Ghaemi, S. Kaboli Gharetapheh, S. Shams, A. Ebadian, 2010. On the stability of derivations, Journal of Geometry and Physics, 60(3): 454-459.

Eshaghi Gordji, M., H. Khodaei, 2009. Solution and stability of generalized mixed type cubic, quadratic and additive functional equation in quasi-Banach spaces, Nonlinear Analysis.-TMA 71: 5629-5643.

Eshaghi Gordji, M., S. Zolfaghari, J.M. Rassias and M.B. Savadkouhi, Solution and Stability of a Mixed type Cubic and Quartic functional equation in Quasi-Banach spaces, Abstract and Applied Analysis, Art. ID 417473: 1-14.

Eshaghi Gordji, M. and A. Najati, 2010. Approximately homomorphisms: A fixed point approach, Journal of Geometry and Physics, 60: 809-814.

Eshaghi Gordji, M., J.M. Rassias, N. Ghobadipour, 2009. Generalized Hyers-Ulam stability derivations, Abs. Appl. Anal., Article ID 437631, 8.

Eshaghi Gordji, M., S. Kaboli Gharetapeh, C. Park and S. Zolfaghari, Stability of an additive-cubic-quartic functional equation, Advances in Difference Equations, Article ID 395693, 20.

Eshaghi Gordji, M., S. Kaboli Gharetapeh, J.M. Rassias and S. Zolfaghari, Solution and stability of a mixed type additive, quadratic and cubic functional equation, Advance difference equations, Article ID 826130, 17 pages, doi:1155/2009/826130.

Eshaghi Gordji, M., M. Bavand Savadkouhi, J.M. Rassias and S. Zolfaghari, 2009. Solution and stability of a mixed type cubic and quartic functional equation in quasi-Banach spaces, Abstract and Applied Analysis, Article ID 417473, 14 pages, doi: 1155/2009/417473.

Eshaghi Gordji, M., M.B. Savadkouhi, 2009. Stability of a mixed type cubic and quartic functional equations in random noemed spaces, Journal of Inequalities and Applications, Article ID 527462, 9 pages.

Eshahgi Gordji, M., S. Abbaszadeh, C. Park, On the stability of generalized mixed type quadratic and qurtic functional equation in quasi-Banach spaces, Journal of Inequality and Applications, Article ID 153084, 26.

Eshshgi Gordji, M., M.B. Savadkouhi and C. Park, 2009. Quadratic-quartic functional equations spaces, Journal of Inequality and Applications, Article ID 868423, 14.

Eshshgi Gordji, M., M.B. Savadkouhi, 2009. Approximation of generalized homomorphisms in quasi-Banach algebras, Analele Univ. Ovidius Constata, Math Series, 17(2): 203-214.

Eshshgi Gordji, M., M.B. Savadkouhi and M. Bidkham, 2010. Stability of a mixed type additive and quadratic functional equation in non-Archimedean spaces, Journal of Computational Analysis and Applications, 12(2): 454-462.

Eshshgi Gordji, M., S. Kaboli Gharetapeh, T. Karimi, M. Aghaei, 2010. Ternary Jordan derivations on ternary algebras, Journal of Computational Analysis and Applications, 12(2): 463-470.

Eshaghi Gordji, M., M.S. Moslehian, 2010.A trick for investigation of approximate derivations, Math. Commun., 15(1): 99-105.

Eshaghi Gordji, M., M. Bavand Savadkouhi, 2009. On approximate cubic homomorphisms, Advance difference equations, Article ID 618463, 11 pages, doi: 10.1155/2009/618463.

Farokhza Rostami, R., A.R. Hosseinion, 2010. Perturbation of Jordan higher derivations in Banach ternary algebras: An alternative fixed point approach, Int. J. Nonlinear Anal. Appl., 1(1): 42-63.

Gvruta, P., 1994. A generalization of the Hyers-Ulam-Rassias stability of approximately additive mapping, J. Math. Anal. Appl., 184: 431-436.

Gvruta, P., L. Gvruta, 2010. A new method for the generalized Hyers-Ulam-Rassias stability, Int. J. Nonlinear Anal. Appl., 1(2): 11-18.

Ghaemi, M.B., B. Alizadeh, 2010. Approximately higher Hilbert module derivations, Int. J. Nonlinear Anal. Appl., 1(2): 36-43.

Hyers, D.H., 1941. On the stability of linear functional equation, Proc. Natl. Acad. Sc., 27: 222-224.

Kadison, R.V., G.K. Pedersen, 1985. Means and convex combinations of unitary operators, Math. Scan., 57: 249-266.

Kapranov, M., I.M. Gelfand and A. Zelevinskii, 1994. Discrimininants, Resultnts and Multidimensional Determinants, Birkhauser, Berlin.

Kerner, R., 1997. The cubic chessbord: Geometry and physics, Class. Quantum Grav., 14: A203-A225.

Kerner, R., 2000. Ternary algebras structures and their applications in physics, Univ. P. M. Curie preprint, Paris, http://arxive.org/list/math-ph/0011.

Khodaei, H., M. Rassias, Th. 2010. Approximately generalized additive functions in several variables, Int. J. Nonlinear Anal. Appl., 1(1): 22-41.

Khodaei, H., M. Kamyar, 2010. Fuzzy approximately additive mapping, Int. J. Nonlinear Anal. Appl., 1(2): 44-53.

Nambu, Y., 1973. Generalized Hamiltonian mechanics, Pys. Rev D7: 2405-2412.

Okubo, S., 1993. Triple products and Yang-Baxter equation (1): Octonional and quaternionic triple systems, J. Math. Phys., 34: 3273-3291.

Park, C., M. Eshaghi Gordji, 2009. Comment on Approximate ternary Jordan derivations on Banach ternary algebras [Bavand Savadkouhi et al. J. Math. Phys., 50: 042303 J. Math. Phys. 51, 044102 (2010) (7 pages).

Park, K.H., Y.S. Jung, 2008. Perturbations of higher ternary derivations in Banach ternary algebras, Commum. Korean Math. Soc., 23(3): 387-399.

Park, C., A. Najati, 2010. Generalized additive functional inequalities in Banach algebras, Int. J. Nonlinear Anal. Appl., 1(2): 54-62.

Park, C., M. Rassias, Th, 2010. Isomorphisms in unital algebras, Int. J. Nonlinear Anal. Appl., 1(2): 1-10.

Rassias, Th. M., 1978. On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc., 72: 297-300.

Rassias, Th. M., 2000. On the stability of functional equations in Banach spaces, J. Math. Anal. Appl., 251: 264-284.

Rassias, Th. M., 2000. On the stability of functional equations and a problem of Ulam, Acta Appl. Math., 62: 23-130.

Rassias, Th. M.P., 1993. On the Hyers-Ulam stability of linear mappings, J . Math. Anal. Appl., 173: 325-338.

Sewell, G.L., 2002. quantum Mechanics and its Emergent Macrophysics, Princeton Univ. Press, Princeton, NJ, MR1919619 (2004b:82001).

Shakeri, S., R. Saadati and C. Park, Stability of the quadratic functional equation in non-Archimedean Ł-fuzzy normed spaces, Int. J. Nonlinear Appl., 1(2): 72-83.

Takhtajan, |L., 1994. On foundation of the generalized Nambu mechanics, Comm. Math. Phys., 160(2): 295-315.

Ulam, S.M., 1940. Problem in Modern Mathematics, Chapter VI, science ed. Wiley, New York.