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A Comparative Study of Defuzzification Through a Regular Weighted Function

[1]Rahim Saneifard and [2]Rasoul Saneifard

[1]Department of Mathematics, Islamic Azad University, Oroumieh Branch, Oroumieh, Iran.
[2]Department of Engineering Technology, Texas Southern University, Houston, Texas, USA.

Abstract: Fuzzy systems have gained more and more attention from researchers and practitioners in various fields. In such systems, the output represented by a fuzzy set often requires transformation into a scalar value, and this task is known as the defuzzification process. In this endeavor a new approach to the problem of defuzzification using the parametric metric between two fuzzy numbers is suggested. Some preliminary results on properties of such defuzzification are divulged herein.. Therefore, this article utilizes the concept of a parametric symmetric triangular fuzzy number, and introduces a new approach to defuzzify a fuzzy quantity. The proposed method obtains the nearest parametric symmetric triangular fuzzy number relating to a fuzzy quantity.

Key words: Fuzzy number; Defuzzification; Parametric distance; Regular function.

INTRODUCTION

For the last few years, many researchers have focused their attention to fuzzy systems when the encountered problems become more and more complex and are difficult to solve using traditional methods. For this reason, fuzzy systems theory is more beneficial to solve problems of complex systems, especially humanistic systems. Fuzzy systems theory was developed based on fuzzy logic and other related disciplines so that the relationships among system variables may be expressed via fuzzy logic. Many successful applications have been revealed that employ fuzzy systems theory to solve various problems, from industrial production process control, refuse incineration plant control and mobile robot control to university enrollments forecast. One of the important steps in applying fuzzy systems theory is to transfer the output, in the form of fuzzy sets, into a scalar called defuzzification. Several defuzzification methods have been studied and the properties of this process have been investigated. The centroid method, for example, is one of the most commonly used; however, a different method has been applied in forecasting university enrollments, where the defuzzification task is performed with a procedure of 3 rules.

Moreover, in Ming, M., *et al*, 2000, the researchers used the concept of the symmetric triangular fuzzy number, and introduced an approach to defuzzify a fuzzy number based L_2-distance. In this effort, the authors propose a new approach to the problem of defuzzification using the weighted metric between two fuzzy numbers. Some preliminary results on properties of such defuzzification are reported. Therefore, using defuzzification, this article utilizes the concept of symmetric triangular fuzzy numbers and introduces a new approach that obtains a lower fuzzy value (than that of Ming, *et al*, 2000) to defuzzify a fuzzy quantity. The new method obtains the nearest symmetric triangular fuzzy number to which a fuzzy quantity is related. Unlike other methods, this study defuzzifies the fuzzy number, and at the same time retains the fuzziness of the original quantity.

2. Fundamentals of Fuzzy Numbers and Theories:

The basic definition of a fuzzy number is given in Grzegorzewski, 2009; Dubois, *et al*., 1987; Heilpern, 1992; Kauffman, *et al*. 1992 and Saneifard, 2009 are as follows:

Definition 1:

A fuzzy number A is a mapping $A(x): R \to [0,1]$ with the following properties:
A is an upper semi-continuous function on R ;
$A(x) = 0$ outside of some interval $[a_1 , b_2] \subset R$;

Corresponding Author: Rahim Saneifard, Department of Mathematics, Islamic Azad University, Oroumieh Branch, Oroumieh, Iran.
Email: Saneifard@iaurmia.ac.ir

There are real numbers a_1 , b_2 such that $a_1 \leq a_2 \leq b_1 \leq b_2$ and
$A(x)$ is a monotonic increasing function on $[a_1 , a_2]$;
$A(x)$ is a monotonic decreasing function on $[b_1 , b_2]$; and
$A(x) = 1$ for all x in $[a_2 , b_1]$.

Let R be the set of all real numbers. The authors selected a fuzzy number A that can be expressed for all $x \in R$ in the form

$$A(x) = \begin{cases} g(x) & when \;\; x \in [a,b), \\ 1 & when \;\; x \in [b,c], \\ h(x) & when \; x \in (c,d], \\ 0 & otherwise. \end{cases} \tag{1}$$

Where a, b, c and d are real numbers such that $a < b \leq c < d$ and g is a real valued function that is increasing and right continuous, and h is a real valued function that is decreasing and left continuous.

Definition 2:
A fuzzy number A in parametric form is a pair $(\underline{A},\overline{A})$ of functions $\underline{A}(r)$ and $\overline{A}(r)$ such that $0 \leq r \leq 1$, which satisfy the following requirements:
$\underline{A}(r)$ is a bounded monotonic increasing left continuous function,
$\overline{A}(r)$ is a bounded monotonic decreasing left continuous function such that
$\underline{A}(r) \leq \overline{A}(r), 0 \leq r \leq 1$.

Definition 3:
The trapezoidal fuzzy number $A = (x_0, y_0, \sigma, \beta)$, with two defuzzifiers x_0, y_0 , and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membership function is as follows:

$$A(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \leq x \leq x_0, \\ \quad 1 & x_0 \leq x \leq y_0, \\ \frac{1}{\beta}(y_0 - x + \beta) & y_0 \leq x \leq y_0 + \beta, \\ \quad 0 & otherwise. \end{cases}$$

if $x_0 = y_0$ and $\sigma = \beta$, a popular fuzzy number is obtained. It is the symmetric triangular fuzzy number
$S[x_0, \sigma]$ centered at x_0 with basis 2σ in following form

$$A(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \leq x \leq x_0, \\ \quad 1 & x = x_0, \\ \frac{1}{\sigma}(x_0 - x + \sigma) & x_0 \leq x \leq x_0 + \sigma, \\ \quad 0 & otherwise. \end{cases}$$

The parametric form of a symmetric triangular fuzzy number is
$\underline{A}(r) = x_0 - \sigma(1 - r)$, $\overline{A}(r) = x_0 + \sigma(r - 1)$.

Definition 4:
A function $f: [0,1] \rightarrow [0,1]$ is symmetric around $\frac{1}{2}$,[i.e. $f\left(\frac{1}{2} - r\right) = f\left(\frac{1}{2} + r\right)$ for all $r \in [0, \frac{1}{2}]$, which

reaches its minimum in $\frac{1}{2}$], is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if

$$f\left(\frac{1}{2}\right) = 0$$

$$f(0 = f(1) = 1,$$

$$\int_0^1 f(r)dr = \frac{1}{2}.$$

Definition 5:

The support function for fuzzy set *A* is defined as follows:

$$supp\,(A) = \overline{\{x|A(x) > 0\}}.$$

where $\overline{\{x|A(x) > 0\}}$ is closure of set $\{x|A(x) > 0\}$. The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows: For arbitrary fuzzy numbers $A = (\underline{A}, \overline{A})$ and $B = (\underline{B}, \overline{B}$, this article defines addition as $(A + B)$ and multiplication by a scalar $k > 0$ as

$$\left(\underline{A + B}\right)(r) = \underline{A}(r) + \underline{B}(r) \quad , \quad \left(\overline{A + B}\right)(r) = \overline{A}(r) + \overline{B}(r) \tag{2}$$

$$\left(\underline{kA}\right)(r) = k\underline{A}(r) \quad , \quad \left(\overline{kA}\right)(r) = k\overline{A}(r) \tag{3}$$

F, a convex cone, expresses the collection of all fuzzy numbers with addition and multiplication as defined by (2) and (3).

Definition 6:

For arbitrary fuzzy numbers $A = (\underline{A}, \overline{A})$ and $B = (\underline{B}, \overline{B})$ the quantity

$$D(A,B) = \int_0^1 \left(\underline{A}(r) - \underline{B}(r)\right)^2 dr + \int_0^1 \left(\overline{A}(r) - \overline{B}(r)\right)^2 dr, \tag{4}$$

is the distance between *A* and *B* *(Diamond, 1990)*.

Having reviewed the previous methods and another rule for defuzzification introduced in Ming, M., *et al*, 2000, the authors propose the nearest symmetric triangular defuzzification approach associated with the metric *D* in *F* as follows:

Let *A* be a fuzzy number and $A = (\underline{A}(r), \overline{A}(r))$ be its parametric form. To obtain a symmetric triangular fuzzy number $S[x_0, \sigma]$, that is nearest to *A*, the researchers minimized

$$D(A, S[x_0,\sigma]) = \int_0^1 \left(\underline{A}(r) - \underline{S}[x_0,\sigma](r)\right)^2 dr + \int_0^1 \left(\overline{A}(r) - \overline{S}[x_0,\sigma](r)\right)^2 dr,$$

with respect to x_0 and σ. If $S[x_0,\sigma]$ minimizes *D*, it provides a defuzzification of *A* with a defuzzifier x_0 and fuzziness σ. Therefore, to minimize *D*, the authors solved the following set of equations

$$\frac{\partial D(A,S[x_0,\sigma])}{\partial \sigma}=0 \quad , \quad \frac{D(A,S[x_0,\sigma])}{\partial x_0}=0.$$

The solution was

$$\sigma=\frac{3}{2}\int_0^1[\overline{A}(r)-\underline{A}(r)](1-r)dr \tag{5}$$

$$x_0=\frac{1}{2}\int_0^1[\underline{A}(r)+\overline{A}(r)]dr \tag{6}$$

For more details see Ming, M., *et al*, 2000.

Definition 7:
Let A be a fuzzy number and $(\underline{A}(r),\overline{A}(r))$ be its parametric form. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number A:

$$I(A)=\frac{1}{2}\int_0^1[\underline{A}(r)+\overline{A}(r)]dr \tag{7}$$

and

$$D(A)=\int_0^1[\overline{A}(r)-\underline{A}(r)]f(r)dr \tag{8}$$

where $f:[0,1]\to[0,1]$ is a bi-symmetrical (regular) weighted function.

One can, of course, propose many regular bi-symmetrical weighted functions, and hence, obtain different bi-symmetrical weighted distances. Later on, the following function will be considered:

$$f(r)=\begin{cases}1-2r & when \quad r\in\left[0,\frac{1}{2}\right],\\ 2r-1 & when \quad r\in\left[\frac{1}{2},1\right].\end{cases} \tag{9}$$

Definition 8:
For arbitrary fuzzy numbers A and B the quantity

$$d_p(A,B)=\sqrt{[I(A)-I(B)]^2+[D(A)-D(B)]^2} \tag{10}$$

is called the bi-symmetrical (regular) weighted distance between A and B based on F.

3. Proposed Nearest Parametric Symmetric Triangular Defuzzification:
In this section, the researchers propose the nearest weighted symmetric triangular defuzzification approach associated with the weighted metric d_p in F.

Let A be a general fuzzy number and $(\underline{A}(r),\overline{A}(r))$ be its parametric form. To obtain the parametric symmetric triangular fuzzy number $S[x_0,\sigma]$, which is the nearest to A, the researchers used the weighted distance (10) and minimized

$$d_p(A,S[x_{0p},\sigma_p])=\left([I(A)-I(S[x_{0p},\sigma_p])]^2+[D(A)-D(S[x_{0p},\sigma_p])]^2\right)^{\frac{1}{2}} \tag{11}$$

with respect to x_{0p} and σ_p , where

$$I(S[x_{0p},\sigma_p]) = \frac{1}{2}\int_0^1 [x_{0p} - \sigma_p(1-r) + x_{0p} + \sigma_p(1-r)]dr = x_{0p}$$

$$D(S[x_{0p},\sigma_p]) = \int_0^1 [x_{0p} + \sigma_p(1-r) - x_{0p} + \sigma_p(1-r)]f(r)dr$$

$$= \int_0^1 2\sigma_p(1-r)f(r)dr.$$

In order to minimize the previous function, it suffices to minimize

$$\overline{D}_p(A,S[x_{0p},\sigma_p]) = d_p^2(A,S[x_{0p},\sigma_p])$$

$$= \left[\frac{1}{2}\int_0^1(\overline{A}(r) + \underline{A}(r) - 2x_{0p})dr\right]^2 + [\int_0^1 \left(\overline{A}(r) - \underline{A}(r) - 2\sigma_p(1-r)\right) f(r(dr]^2$$

If $S[x_{0p},\sigma_p]$ minimizes $\overline{D}_p(A,S[x_{0p},\sigma_p])$, then $S[x_{0p},\sigma_p]$ provides a defuzzification of *A* with the defuzzifier x_{0p} and fuzziness σ_p .

So to minimize $\overline{D}_p(A,S[x_{0p},\sigma_p])$, this article proposes,

$$\frac{\partial \overline{D}_p(A,S[x_{0p},\sigma_p])}{\partial \sigma_p} = -4\int_0^1\left(\overline{A}(r) - \underline{A}(r)\right)(1-r)f(r)dr + 8\sigma_p\int_0^1(1-r)^2 f(r)dr$$

and

$$\frac{\partial \overline{D}_p(A,S[x_{0p},\sigma_p])}{\partial x_{0p}} = -2\int_0^1(\underline{A}(r) + \overline{A}(r) - 2x_{0p})dr.$$

To solve system the following sets of equations:

$$\frac{\partial \overline{D}_p(A,S[x_{0p},\sigma_p])}{\partial \sigma_p} = 0 \qquad \frac{\partial \overline{D}_p(A,S[x_{0p},\sigma_p])}{\partial x_{0p}} = 0.$$

The results are

$$\sigma_p = \frac{\int_0^1[\overline{A}(r) - \underline{A}(r)](1-r)f(r)dr}{2\int_0^1(1-r)^2 f(r)dr}, \tag{12}$$

$$x_{0p} = \frac{1}{2}\int_0^1\left(\underline{A}(r) + \overline{A}(r)\right)dr. \tag{13}$$

Remark 1:

Considering *f(r)* = *r*, the nearest parametric symmetric triangular defuzzification of *A* is given by the defuzzifier

$$x_{0p} = \frac{1}{2}\int_0^1\left(\underline{A}(r) + \overline{A}(r)\right)dr.$$

and fuzziness

$\sigma_p = 6\int_0^1 [\overline{A}(r) - \underline{A}(r)](1-r)rdr$

The above defuzzification approach can be applied to two fuzzy numbers whenever a single fuzzy quantity is desirable. Let *A* and *B* be a fuzzy numbers with parametric forms $A = (\underline{A}(r), \overline{A}(r))$ and $B = (\underline{B}(r), \overline{B}(r))$. To find a parametric symmetric triangular fuzzy number $S[x_{0p}, \sigma_p]$ near both *A* and *B*, this effort minimizes

$$\overline{\overline{D}}(x_{0p}, \sigma_p) = \overline{D}_p(A, S[x_{0p}, \sigma_p]) + \overline{D}_p(B, S[x_{0p}, \sigma_p])$$

$$= \left[\frac{1}{2}\int_0^1 [(\underline{A}(r) - \underline{S}[x_{0p}, \sigma_p](r)) + (\overline{A}(r) - \overline{S}[x_{0p}, \sigma_p](r))]dr\right]^2$$

$$+ \left[\int_0^1 [(\overline{A}(r) - \overline{S}[x_{0p}, \sigma_p](r)) - (\underline{A}(r) - \underline{S}[x_{0p}, \sigma_p](r))]f(r)dr\right]^2$$

$$+ \left[\frac{1}{2}\int_0^1 [(\underline{B}(r) - \underline{S}[x_{0p}, \sigma_p](r)) + (\overline{B}(r) - \overline{S}[x_{0p}, \sigma_p](r))]dr\right]^2$$

$$+ \left[\int_0^1 [(\overline{B}(r) - \overline{S}[x_{0p}, \sigma_p](r)) - (\underline{B}(r) - \underline{S}[x_{0p}, \sigma_p](r))]f(r)dr\right]^2$$

Thus, this study attempts to find a lodger point, (x_{0p}, σ_p) for which

$$\frac{\partial \overline{\overline{D}}(x_{0p}, \sigma_p)}{\partial \sigma_p} = 0 \quad , \quad \frac{\partial \overline{\overline{D}}(x_{0p}, \sigma_p)}{\partial x_{0p}} = 0 \quad . (*)$$

Then

$$\frac{\partial \overline{\overline{D}}(x_{0p}, \sigma_p)}{\partial \sigma_p} = \frac{\partial \overline{D}_p(A, S[x_{0p}, \sigma_p])}{\partial \sigma_p} + \frac{\overline{D}_p(B, S[x_{0p}, \sigma_p])}{\partial \sigma_p}$$

$$= -4\int_0^1 \left(\overline{A}(r) - \underline{A}(r)\right)(1-r)f(r)dr + 8\sigma_p\int_0^1 (1-r)^2 f(r)dr$$

$$-4\int_0^1 \left(\overline{B}(r) - \underline{B}(r)\right)(1-r)f(r)dr + 8\sigma_p\int_0^1 (1-r)^2 f(r)dr = 0$$

and

$$\frac{\partial \overline{\overline{D}}(x_{0p}, \sigma_p)}{\partial x_{0p}} = \frac{\partial \overline{D}_p(A, S[x_{0p}, \sigma_p])}{\partial x_{0p}} + \frac{\overline{D}_p(B, S[x_{0p}, \sigma_p])}{\partial x_{0p}}$$

$$= -2\int_0^1 \left(\underline{A}(r) + \overline{A}(r) - 2x_{0p}\right)dr -= -2\int_0^1 \left(\underline{B}(r) + \overline{B}(r) - 2x_{0p}\right)dr = 0.$$

Hence, solving the sets of equations (*), results in

$$\sigma_p = \frac{\int_0^1 [\overline{A}(r)+\overline{B}(r)-\underline{A}(r)-\underline{B}(r)](1-r)f(r)dr}{4\int_0^1 (1-r)^2 f(r)dr} \tag{14}$$

and

$$x_{0p} = \frac{1}{4}\int_0^1 [\underline{A}(r) + \underline{B}(r) + \overline{A}(r) + \overline{B}(r)]dr. \tag{15}$$

It is assumed that $f(r) = r$, then

$$\sigma_p = 3\int_0^1 [\overline{A}(r) + \overline{B}(r) - \underline{A}(r) - \underline{B}(r)](1-r)rdr,$$

and

$$x_{0p} = \frac{1}{4}\int_0^1 [\underline{A}(r) + \underline{B}(r) + \overline{A}(r) + \overline{B}(r)]dr.$$

4. Numerical Examples and Application:

In this section the authors present numerical examples to illustrate the difference between the proposed method in this paper and the given method in Ming, M., *et al* 2000.

Example 1:

Consider a plateau

$\underline{A}(r) = a + (b-a)r$, $\overline{A}(r) = d - (d-c)r,$

where $a \le b \le c \le d$. The nearest parametric symmetric triangular defuzzification procedure yields

$$x_{0p} = \frac{1}{2}\int_0^1 [\underline{A}(r) + \overline{A}(r)]dr = \frac{1}{2}\int_0^1 [(a+d) + (b-a)r - (d-c)r]dr$$

$$= \frac{a+b+c+d}{4}$$

and

$$\sigma_p = \frac{\int_0^1 [\overline{A}(r) - \underline{A}(r)](1-r)f(r)dr}{2\int_0^1 (1-r)^2 f(r)dr}$$

$$= \frac{1}{24}\int_0^1 [d-(d-c)r - a - (b-a)r]r(1-r)dr = \frac{c+d\_a\_b}{2}$$

In the specific case where b = c , therefore,

$$x_{0p} = \frac{1}{4}(a+2b+d)\ , \quad \sigma_p = \frac{1}{2}(d-a)$$

Example 2:

Consider the Gaussian membership function $A(x) = e^{\frac{-(x-\mu_0)^2}{\sigma_0^2}}$ which in its parametric form is

$$\underline{A}(r) = \mu_0 + \sigma_0\sqrt{-Lnr} \ , \ \overline{A}(r) = \mu_0 - \sigma_0\sqrt{-Lnr}$$

then

$$x_{0p} = \frac{1}{2}\int_0^1 [\underline{A}(r) + \overline{A}(r)]dr = \mu_0,$$

and

$$\sigma_p = \frac{\int_0^1 [\overline{A}(r) - \underline{A}(r)](1-r)f(r)dr}{2\int_0^1 (1-r)^2 f(r)dr} = \frac{\sigma_0\sqrt{\pi}}{6}(9\sqrt{2} - 4\sqrt{3})$$

Furthermore, $\sigma = \frac{3}{8}\sigma_0\sqrt{\pi}(4-\sqrt{2})$ (Ming, M., *et al*, 2000), therefore, it is obvious that $\sigma_p \leq \sigma$.

Example 3:

Let A be a plateau and B a triangular fuzzy number denoted by

$$\underline{A}(r) = r \ , \ \overline{A}(r) = 3 - r,$$

$$\underline{B}(r) = 2 + r \ , \ \overline{B}(r) = 4 - r$$

The defuzzification procedure yields

$$x_{0p} = \frac{1}{4}\int_0^1 [\underline{A}(r) + \underline{B}(r) + \overline{A}(r) + \overline{B}(r)]dr = \frac{1}{4}\int_0^1 9rdr = \frac{9}{4}$$

and

$$\sigma_p = \frac{\int_0^1 [\overline{A}(r) + \overline{B}(r) - \underline{A}(r) - \underline{B}(r)](1-r)f(r)dr}{4\int_0^1 (1-r)^2 f(r)dr}$$

$$= \int_0^1 (5-4r)(1-r)rdr = \frac{3}{2}.$$

Example 4:

This effort the parametric symmetric triangular defuzzification procedure to obtain a fuzzy partition from two extreme values. Thus, given the extreme values 0 and 1, the fuzzy number "medium" $A^{(1)}$ is defined as

$$\underline{A}^{(1)}(r) = \frac{r}{2} \ , \ \overline{A}^{(1)}(r) = 1 - \frac{r}{2}$$

Then, 0 and $A^{(1)}$, were defuzzified to obtain "lower medium" $A^{(2,1)}$ for which

$$x_{0p} = \frac{1}{4}\int_0^1 \left[\underline{A}^{(1)}(r) + 0 + \overline{A}^{(1)}(r) + 0\right]dr = \frac{1}{4},$$

$$\sigma_p = 3\int_0^1 \left[\overline{A}^{(1)}(r) + 0 - \underline{A}^{(1)}(r) + 0\right](1-r)rdr = \frac{1}{4},$$

thus $\underline{A}^{(2,1)}(r) = 0 + \frac{1}{4}r$ and $\overline{A}^{(2,1)}(r) = \frac{1}{2} - \frac{1}{4}r$.

Therefore, $A^{(1)}$ and 1 are defuzzified, to get the "upper medium" $\overline{A}^{(2,3)}$ with $x_{0p} = \frac{3}{4}$ and $\sigma_p = \frac{1}{4}$, i.e.

$$\underline{A}^{(2,3)}(r) = \frac{1}{2} + \frac{1}{4}r \ , \ \overline{A}^{(2,3)}(r) = 1 - \frac{1}{4}r$$

Updating the "medium" by defuzzifying the "lower medium" $A^{(2,1)}$ and "upper medium" $A^{(2,3)}$, results in "medium" $A^{(2,2)}$ centered at

$$x_{0p} = \frac{1}{4}\int_0^1 \left(\frac{r}{4} + \frac{1}{2} - \frac{r}{4} + \frac{1}{2} + \frac{r}{4} + 1 - \frac{r}{4}\right)dr = \frac{1}{2},$$

with

$$\sigma_p = 3\int_0^1 (1-r)^2 rdr = \frac{1}{4},$$

[i.e. $\underline{A}^{(2,2)}(r) = 0.25 + 0.25r \ , \ \overline{A}^{(2,2)}(r) = 0.75 - 0.25r$]

Thus, a fuzzy partition is obtained with five elements $P = \{0, A^{(2,1)}, A^{(2,2)}, A^{(2,3)}, 1\}$.
As the fuzzy partition becomes smaller, therefore, the fuzziness of its elements decreases.

Example 5:
Consider the Gaussian membership function in Example (2) with $\mu_0 = 1$ and $\sigma_0 = \frac{1}{2}$. The parametric symmetric triangular defuzzification procedure is applied to obtain a fuzzy partition from two extreme values, 0 and 2. Then

$$\underline{A}^{(1)}(r) = 1 + \frac{\sqrt{-Lnr}}{2} \ , \ \overline{A}^{(1)}(r) = 1 - \frac{\sqrt{-Lnr}}{2}$$

Further, 0 and $A^{(1)}$ are defuzzified, to obtain $A^{(2,1)}$ for which

$$x_{0p} = \frac{1}{4}\int_0^1 \left[\underline{A}^{(1)}(r) + 0 + \overline{A}^{(1)}(r) + 0\right]dr = \frac{1}{2}$$

$$\sigma_p = 3\int_0^1 \left[\overline{A}^{(1)}(r) + 0 - \underline{A}^{(1)}(r) + 0\right](1-r)rdr = \frac{1}{2}.$$

Thus $\underline{A}^{(2,1)}(r) = 0 + \frac{r}{2}$ and $\overline{A}^{(2,1)}(r) = 1 - \frac{r}{2}$.

Therefore, defuzzifying $A^{(1)}$ and 2, results in $A^{(23)}$ with $x_{0p} = \frac{3}{2}$ and $\sigma_p = \frac{1}{2}$, [i.e. ,]

$\underline{A}^{(2,3)}(r) = 1 + \frac{r}{2} \quad \overline{A}^{(2,3)}(r) = 2 - \frac{r}{2}$

The "medium" is updated by defuzzifying the "lower medium" $A^{(21)}$ and "upper medium" $A^{(23)}$ which results in the "medium" $A^{(22)}$ centered at

$x_{0p} = 1$, with x0p = 1; and $\sigma_p = \frac{1}{2}$,

[i.e. $\underline{A}^{(2,2)}(r) = \frac{1}{2} + \frac{r}{2} \quad \overline{A}^{(2,2)}(r) = \frac{3}{2} - \frac{r}{2}$]

Finally, this endeavor obtains a fuzzy partition with five elements $P = \{0, A^{(2,1)}, A^{(2,2)}, A^{(2,3)}, 1\}$.

Obviously, as the fuzzy partition becomes smaller the fuzziness of its elements decreases.

5 Conclusion:

Fuzzy systems have gained more and more attention from researchers and practitioners in various fields. In such systems, the output represented by fuzzy sets into a scalar value known as the defuzzification process. Several analytic methods have been proposed for this problem, but in this paper, the authors proposed a new approach to the problem of defuzzification using the weighted metric between two fuzzy numbers. In this study, some preliminary results on properties of such defuzzification.

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