

An Improved Consistent Fuzzy Linguistic Preference Relation for Selection Problems

Nor Hanimah Kamis, Daud Mohamad, Nor Hashimah Sulaiman

Department of Mathematics, Faculty of Computer & Mathematical Sciences, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia.

Abstract: In this paper we improvise the work of Wang & Chen (2006) that used fuzzy numbers in the computation of consistent fuzzy preference relation (CFPR). Using the Herrera-Viedma *et al.* (2004) approach, we incorporate the centroid-index formula in constructing the evaluation matrices with fuzzy number elements. This will transform the fuzzy numbers into crisp values. The number of pair-wise comparison of criteria is still in the order $O(n-1)$ for any types of fuzzy number used. Furthermore centroid formula is also used to find the final ranking of alternatives. We found that this improvised technique can remarkably reduce the degree of computation in constructing the evaluation matrices based on consistent fuzzy preference relations besides enhancing Wang & Chen's approach of ranking the alternatives. Using the example employed by Bozdag *et al* (2003), our proposed method involves lesser calculation compared to Wang & Chen (2006) while the result is consistent and practically can solve fuzzy decision making problems, especially for ranking of alternatives.

Key words: Centroid-index, Consistent Fuzzy Preference Relations, Fuzzy Numbers, Decision-making Problems.

INTRODUCTION

The existence and development of fuzzy preference relation method has significantly assist decision makers in solving multi-criteria selection problems. To attain correct solutions in decision process, consistency becomes a very important aspect to consider. Due to this, Herrera- Viedma *et al* (2004) proposed a new characterization of the consistency property defined by the additive transitivity property to ensure better consistency of the fuzzy preference relations. This consistent fuzzy preference relation is broadly used for selection and ranking purposes (Wang, T.C. and T.H. Chang, 2007; Wang, T.C. and Y.H. Chen, 2006; Wang, T.C. and Y.H. Chen, 2007; Wang, T.C. and J.L. Liang, 2006).

Wang & Chen (2006) used triangular fuzzy numbers as input values to handle vague judgments in the selection of computer integrated manufactory (CIM) systems. Though the order of pair-wise comparison based on consistent fuzzy preference relations is only $(n-1)$. Their approach in essence requires more than $(n-1)$ comparisons depending on the type of fuzzy number being used. Because of that, computation becomes more tedious and inconsistent conditions are likely to occur.

To solve this problem, we use centroid- index formula by Chen & Chen (2000) in order to handle fuzzy numbers in a systematic manner. Work done by Chen & Chen overcomes the drawbacks of studies by Yager (1980), Murakami (1983) and Cheng (1998). By considering both x and y values of generalized fuzzy numbers, where α -cut equal to zero, this approach is sufficient and gives more accurate result in determining the center point of fuzzy numbers. The combination of centroid-index formula and the consistent fuzzy preference relations produced a new algorithm in constructing the evaluation matrix in selection process, which is simpler in calculation while the result is accurate and consistent.

MATERIALS AND METHODS

In the following, we briefly explain the concept of fuzzy numbers, α - cut, centroid-index formula by Chen & Chen (2003), consistent fuzzy preference relations and the proposed algorithm for ranking of alternatives.

Fuzzy Numbers and α - Cut:

Fuzzy numbers are fuzzy subset of real numbers that represent the expansion of the idea of confidence

Corresponding Author: Nor Hanimah Kamis, Department of Mathematics, Faculty of Computer & Mathematical Sciences, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia.

intervals (Hsieh, T.Y., 2004). Based on Tanaka (1997), we give the following definition.

Definition 1:

A fuzzy number \tilde{A} is a fuzzy subset on the universe R of real numbers satisfying the following conditions

- i. there exist at least one $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$;
- ii. membership function, $\mu_{\tilde{A}}(x)$ is continuous in an interval.

A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is an example of a type of fuzzy number represented by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & ; \quad a_1 \leq x \leq a_2 \\ 1 & ; \quad a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & ; \quad a_3 \leq x \leq a_4 \\ 0 & ; \quad x < a_1 \text{ or } x > a_4 \end{cases} \quad (1)$$

where a_1, a_2, a_3 and a_4 are real values. When $a_2 = a_3$, \tilde{A} becomes a triangular fuzzy number.

Definition 2:

The α - cut of fuzzy number \tilde{A} is defined as

$$\tilde{A}^\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\} \text{ where } \alpha \in [0, 1]$$

The α - cut of a fuzzy set \tilde{A} is the crisp set \tilde{A}^α that contains elements in the universe R whose membership grades in \tilde{A} are greater than or equal to the specified value of α (Chen, C.T., 2000). Figure 1 shows the graphical representation of a trapezoidal fuzzy number with α - cut.

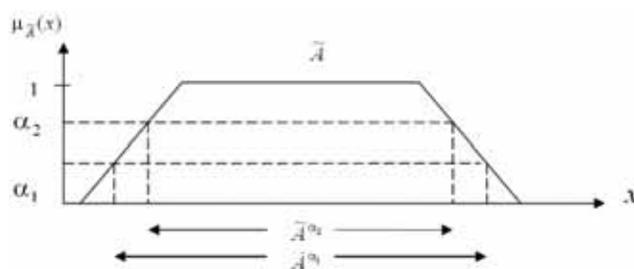


Fig. 1: A trapezoidal fuzzy number.

Centroid-index Formula

The centroid-index formula by Chen & Chen (2003) is used in this paper to determine the center point of each trapezoidal fuzzy number by considering both x and y values. This formula is more suitable to use because it can overcome the drawbacks of the existing ranking methods presented by Cheng (1998), Murakami *et al* (1983) and Yager (1980).

Let $\tilde{N} = (a_1, a_2, a_3, a_4; w_{\tilde{N}})$ be a generalized trapezoidal fuzzy number (Chen, S.H., 1985) in the universe of discourse X shown in Figure 2, where $w_{\tilde{N}} \in [0, 1]$ represents the height of \tilde{N} and a_1, a_2, a_3 and a_4 are real values.

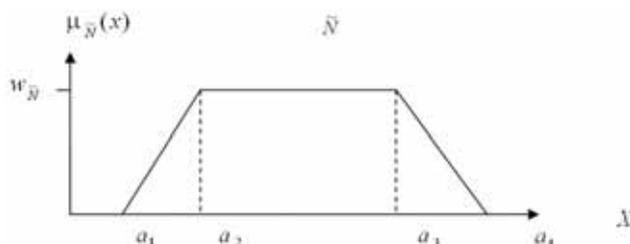


Fig. 2: Generalized trapezoidal fuzzy number (Chen, S.H., 1985).

The centroid-index formula Chen & Chen (2003) for a trapezoidal fuzzy number is given as

$$y_{\tilde{N}} = \begin{cases} \frac{w_{\tilde{N}} \times \left(\frac{a_3 - a_2}{a_4 - a_1} \right) + 2}{6} & ; a_1 \neq a_4 \text{ and } 0 < w_{\tilde{N}} < 1 \\ \frac{w_{\tilde{N}}}{2} & ; a_1 = a_4 \text{ and } 0 < w_{\tilde{N}} < 1 \end{cases} \quad (2)$$

$$x_{\tilde{N}} = \frac{y_{\tilde{N}}(a_3 + a_2) + (a_4 + a_1)(w_{\tilde{N}} - y_{\tilde{N}})}{2w_{\tilde{N}}} \quad (3)$$

The centroid point $(x_{\tilde{N}}, y_{\tilde{N}})$ is then transformed into a single value using the distance formula

$$\text{Distance} = \sqrt{x_{\tilde{N}}^2 + y_{\tilde{N}}^2} \quad (4)$$

This centroid-index formula satisfies the following properties of centroid given by Shieh (2007).

P1: If A and B are fuzzy numbers with their membership functions $\mu_A(x)$ and $\mu_B(x)$ have the relation of $\mu_B(y) = \mu_A(x)$, where $y = x + b$, then $\bar{x}_0(B) = \bar{x}_0(A) + b$, $\bar{y}_0(B) = \bar{y}_0(A)$.

P2: If A and B are fuzzy numbers with their membership functions $\mu_A(x)$ and $\mu_B(x)$ have the relation of $\mu_B(x) = \omega \mu_A(x)$ for all $x \in R$, then $\bar{x}_0(B) = \bar{x}_0(A)$

Consistent Fuzzy Preference Relations:

Definition 3:

A fuzzy preference relation R on the set of alternatives A is a fuzzy set defined on the Cartesian product set $A \times A$ with membership function $\mu_R: A \times A \rightarrow [0, 1]$. The preference relation is expressed by the $n \times n$ matrix $R = (r_{ij})$ where . The preference ratio, r_{ij} of alternative a_i to a_j is determined by

$$r_{ij} = \begin{cases} 0.5 & ; \quad a_i \text{ is different to } a_j \\ (0.5, 1) & ; \quad a_i \text{ is preferred than } a_j \\ 1 & ; \quad a_i \text{ is absolutely preferred than } a_j \end{cases}$$

The preference matrix R is assumed to be additive reciprocal, $p_{ij} + p_{ji} = 1, \forall i, j \in \{1, \dots, n\}$. Herrera-Viedma *et al* (2004) introduced a consistent additive preference relation with several propositions:

Proposition 1:

Consider a set of alternatives, $X = \{x_1, \dots, x_n\}$ and associated with a reciprocal multiplicative preference relation $\tilde{A} = (a_{ij})$ for $a_{ij} \in [1/9, 9]$. Then, the corresponding reciprocal fuzzy preference relation, $\tilde{P} = (p_{ij})$ with $p_{ij} \in [0, 1]$ associated with \tilde{A} is given by the following expression:

$$p_{ij} = g(a_{ij}) = \frac{1}{2}(1 + \log_9 a_{ij})$$

Note that when $a_{ij} \in [1/9, 9]$, $\log_9 a_{ij}$ is considered. Generally, if $a_{ij} \in [1/n, n]$, then $\log_n a_{ij}$ is used.

Proposition 2:

For a reciprocal fuzzy preference relation $\tilde{P} = (p_{ij})$, the following statements are equivalent:

- (i) $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \quad \forall i, j, k$
- (ii) $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \quad \forall i < j < k$

Proposition 3:

For a reciprocal fuzzy preference relation $\tilde{P} = (p_{ij})$, the following statements are equivalent:

- (i) $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \quad \forall i < j < k$
- (ii) $p_{i(i+1)} + p_{(i+1)(i+2)} + \dots + p_{(j-1)j} + p_{ji} = \frac{j-i+1}{2} \quad \forall i < j$.

They further introduced a method of constructing consistent reciprocal fuzzy preference relations \tilde{P} on $X = \{x_1, \dots, x_n, n \geq 2\}$ from (n-1) preference data, $\{P_{12}, P_{23}, \dots, P_{(n-1)n}\}$ given as follows:

Compute the set of preference value S as

$$S = \left\{ p_{ij}, i < j \text{ and } p_{ij} \notin \{P_{12}, P_{23}, \dots, P_{(n-1)n}\} \right\},$$

$$p_{ij} = \frac{j-i+1}{2} - P_{i+1} - P_{(i+1)(i+2)} \dots - P_{(j-1)j} \tag{5}$$

$$\text{Calculate } k = \left| \min \left\{ S \cup \{ P_{12}, P_{23}, \dots, P_{(n-1)n} \} \right\} \right| \quad (6)$$

$$\text{Determine } P = \{ P_{12}, P_{23}, \dots, P_{(n-1)n} \} \cup S \cup \{ 1 - P_{12}, 1 - P_{23}, \dots, 1 - P_{(n-1)n} \} \cup S^c \quad (7)$$

$\dot{P} = f(P)$ can be obtained by

$$f: [-k, 1+k] \rightarrow [0, 1], f(x) = \frac{(x+k)}{(1+2k)} \quad (8)$$

Using consistent fuzzy preference relations, the normalized criteria weights, W_i is calculated as

$$W_i = \frac{A_i}{\sum_{j=1}^n A_j} \quad (9)$$

where the average value of the criteria,

$$A_i = \frac{\sum_{j=1}^n P_{ij}}{n} \quad (10)$$

The Proposed Algorithm:

Even though the order of pair-wise comparison is just $O(n-1)$ when using consistent fuzzy preference relation, the method given by Wang & Chen (2006) in reality involves quite extensive calculations depending on the type of fuzzy numbers used since each component of fuzzy numbers (3 components for a triangular fuzzy number and 4 components for a trapezoidal fuzzy number) has to be calculated. Furthermore, their method gives a final outcome in the form of fuzzy numbers with a very crude estimation using average of the fuzzy number components in order to finalize the ranking.

Rather than calculating each components of fuzzy number as proposed by Wang & Chen (2006), we use the centroid-index formula to transform fuzzy numbers into crisp values. This step would obviously lessen the calculation in constructing the evaluation matrix. Furthermore, the formula applies to any types of fuzzy numbers.

We propose a new algorithm in ranking or selection process by adapting the centroid-index formula Chen & Chen (2003) in a procedure of constructing consistent reciprocal fuzzy preference relations by Herrera-Viedma *et al* (2004). Generally, we improvised Wang & Chen (2006) work and the proposed algorithm is given as follows:

- STEP 1:** Set the input values in terms of fuzzy number.
- STEP 2:** Use (2) and (3) to calculate the centroid point $(x_{\tilde{N}}, y_{\tilde{N}})$ of each fuzzy numbers.
- STEP 3:** Use (4) to calculate the distance of centroid point $(x_{\tilde{N}}, y_{\tilde{N}})$.
- STEP 4:** Normalize the coefficient value in the form of consistent fuzzy preference relations.
- STEP 5:** Construct a consistent reciprocal decision matrix using (5), (6), (7) and (8).
- STEP 6:** Use (9) and (10) to obtain the criteria weights.
- STEP 7:** Determine the weights of each alternative using STEP 1 to STEP 6.
- STEP 8:** Find the final scores by obtaining

$$\sum_{i=1}^n (W_{i \text{ criteria}} \times W_{i \text{ alternative}}) \quad (11)$$

RESULTS AND DISCUSSIONS

To illustrate the application of the proposed method, we consider the original example from Bozdag *et al* (2003) which has also been used by Wang & Chen (2006). A big Turkish Motor Company wishes to purchase computer integrated manufacturing (CIM) systems for machinery parts production. The company decided to select the best CIM systems based on quality (C_1), flexibility (C_2), competition (C_3), experience with new technology (C_4) and expandability (C_5). The alternatives are CIM I (A_1), CIM II (A_2), CIM III (A_3) and CIM IV (A_4). Wang and Chen (2006) converted Bozdag's linguistic scale into consistent fuzzy preference relations scale according to

$$P_{ij} = g(a_{ij}) = \frac{1}{2} \left(1 + \log_{9.5} a_{ij} \right), \quad p_{ij} \in (0,1), \quad a_{ij} \in (1,9.5) \quad , \quad i = 1, \dots, n \quad , \quad j = 1, \dots, n$$

Table 1 displays the linguistic assessment variables representing by fuzzy numbers and its corresponding reciprocal scale.

Table 1: The fuzzy linguistic assessment variables

Linguistic variables	Fuzzy scale	Reciprocal scale
Equally important	(1,1,1)	(1,1,1)
Weakly more important	(5/2,3,7/2)	(2/7,1/3,2/5)
Strongly more important	(9/2,5,11/2)	(2/11,1/5,2/9)
Very strongly more important	(13/2,7,15/2)	(2/15,1/7,2/13)
Absolutely more important	(17/2,9,19/2)	(2/19,1/9,2/17)

Table 2 presents the original pair-wise comparison of criteria. Table 3, 4, 5, 6 and 7 shows the original pair-wise comparisons of alternatives with respect to C_1 , C_2 , C_3 , C_4 and C_5 .

Table 2: Pair-wise comparison matrix for criteria

C	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(3/2,2,5/2)	(5/2,3,7/2)	(3/2,2,5/2)	(11/2,6,13/2)
C_2	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	(1,1,1)	(9/2,5,11/2)
C_3	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(1,1,1)	(2/5,1/2,2/3)	(3/2,2,5/2)
C_4	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	(1,1,1)	(7/2,4,9/2)
C_5	(2/13,1/6,2/11)	(2/11,1/5,2/9)	(2/5,1/2,2/3)	(2/9,1/4,2/7)	(1,1,1)

Table 3: Pair-wise comparison of alternatives with respect to C_1

A	A_1	A_2	A_3	A_4
A_1	(1,1,1)	(5/2,3,7/2)	(7/2,4,9/2)	(3/2,2,5/2)
A_2	(2/7,1/3,2/5)	(1,1,1)	(3/2,2,5/2)	(2/7,1/3,2/5)
A_3	(2/9,1/4,2/7)	(2/5,1/2,2/3)	(1,1,1)	(2/9,1/4,2/7)
A_4	(2/5,1/2,2/3)	(5/2,3,7/2)	(7/2,4,9/2)	(1,1,1)

Table 4: Pair-wise comparison of alternatives with respect to C_2

A	A_1	A_2	A_3	A_4
A_1	(1,1,1)	(3/2,2,5/2)	(5/2,3,7/2)	(7/2,4,9/2)
A_2	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)
A_3	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)
A_4	(2/9,1/4,2/7)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)

Table 5: Pair-wise comparison of alternatives with respect to C_3

A	A_1	A_2	A_3	A_4
A_1	(1,1,1)	(2/5,1/2,2/3)	(2/7,1/3,2/5)	(2/9,1/4,2/7)
A_2	(3/2,2,5/2)	(1,1,1)	(2/5,1/2,2/3)	(2/7,1/3,2/5)
A_3	(5/2,3,7/2)	(3/2,2,5/2)	(1,1,1)	(2/7,1/3,2/5)
A_4	(7/2,4,9/2)	(5/2,3,7/2)	(5/2,3,7/2)	(1,1,1)

Table 6: Pair-wise comparison of alternatives with respect to C_4

A	A_1	A_2	A_3	A_4
A_1	(1,1,1)	(1,1,1)	(5/2,3,7/2)	(3/2,2,5/2)
A_2	(1,1,1)	(1,1,1)	(5/2,3,7/2)	(3/2,2,5/2)
A_3	(2/7,1/3,2/5)	(2/7,1/3,2/5)	(1,1,1)	(2/5,1/2,2/3)
A_4	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(3/2,2,5/2)	(1,1,1)

Table 7: Pair-wise comparison of alternatives with respect to C_1

A	A ₁	A ₂	A ₃	A ₄
A ₁	(1,1,1)	(1,1,1)	(5/2,3,7/2)	(2/5,1/2,2/3)
A ₂	(1,1,1)	(1,1,1)	(3/2,2,5/2)	(2/5,1/2,2/3)
A ₃	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(1,1,1)	(2/9,1/4,2/7)
A ₄	(3/2,2,5/2)	(3/2,2,5/2)	(7/2,4,9/2)	(1,1,1)

Applying the centroid-index formula by Chen and Chen (2003), the centroid points of the corresponding entries in the pair-wise comparison matrix for criteria (Table 2) are obtained as shown in Table 8.

Table 8: The decision matrix in terms of centroid point

C	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	(1, 0.5)	(2, 0.33)	(3, 0.33)	(2, 0.33)	(6, 0.33)
C ₂	(0.52, 0.33)	(1, 0.5)	(2, 0.33)	(1, 0.5)	(5, 0.33)
C ₃	(0.34, 0.33)	(0.52, 0.33)	(1, 0.5)	(0.52, 0.33)	(2, 0.33)
C ₄	(0.52, 0.33)	(1, 0.5)	(2, 0.33)	(1, 0.5)	(4, 0.33)
C ₅	(0.17, 0.33)	(2, 0.33)	(0.52, 0.33)	(0.25, 0.33)	(1, 0.5)

The corresponding distance of centroid point $(x_{\bar{N}}, y_{\bar{N}})$ of each entry is presented in Table 9.

Table 9: The distance of centroid point $(x_{\bar{N}}, y_{\bar{N}})$

Criteria	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	1.1180	2.0276	3.0185	2.0276	6.0093
C ₂	0.6195	1.1180	2.0276	1.1180	5.0111
C ₃	0.4759	0.6195	1.1180	0.6195	2.0276
C ₄	0.6195	1.1180	2.0276	1.1180	4.0139
C ₅	0.3730	0.3894	0.6195	0.4183	1.1180

To present each value of pair-wise comparison in the form of fuzzy preference relations, the normalized coefficient value is shown in Table 10.

Table 10: The normalized coefficient value of fuzzy preference relations

Criteria	C ₁	C ₂	C ₃	C ₄	C ₅
C ₁	1	1.8135	2.6998	1.8135	5.3748
C ₂	0.5541	1	1.8135	1	4.4821
C ₃	0.4257	0.5541	1	0.5541	1.8135
C ₄	0.5541	1	1.8135	1	3.5901
C ₅	0.3336	0.3483	0.5541	0.3741	1

With $n = 5$ criteria, only $(n-1) = 5-1=4$ entry values (p_{12}, p_{23}, p_{34} and p_{45}) are required to construct the decision matrix where

$$p_{12} = \frac{1}{2}(1 + \log_{9.5} 1.8135) = 0.6322$$

$$p_{23} = \frac{1}{2}(1 + \log_{9.5} 1.8135) = 0.6322$$

$$p_{34} = \frac{1}{2}(1 + \log_{9.5} 0.5541) = 0.3689$$

$$p_{45} = \frac{1}{2}(1 + \log_{9.5} 3.5901) = 0.7839$$

The rest of the entries can be determined by applying the Proposition 2 and 3 and are calculated as follows:

$$p_{31} = 1.5 - p_{12} - p_{23} = 0.2356$$

$$p_{41} = 2 - p_{12} - p_{23} - p_{34} = 0.3667$$

$$p_{42} = 1.5 - p_{23} - p_{34} = 0.4989$$

$$p_{51} = 2.5 - p_{12} - p_{23} - p_{34} - p_{45} = 0.0828$$

$$p_{52} = 2 - p_{23} - p_{34} - p_{45} = 0.2150$$

$$p_{53} = 1.5 - p_{34} - p_{45} = 0.3472$$

$$p_{32} = 1 - p_{23} = 0.3678$$

$$p_{43} = 1 - p_{34} = 0.6311$$

$$p_{54} = 1 - p_{45} = 0.2161$$

Table 11 shows entries in the decision matrix based on the consistent fuzzy preference relations. As the entries in the decision matrix are in the interval [0, 1], equation (8) is not applicable.

Table 11: The decision matrix and the criteria weights

Criteria	C ₁	C ₂	C ₃	C ₄	C ₅	A ₁	W ₁
C ₁	0.5	0.6322	0.7644	0.6333	0.9172	0.6894	0.2758
C ₂	0.3678	0.5	0.6322	0.5011	0.7850	0.5572	0.2229
C ₃	0.2356	0.3678	0.5	0.3689	0.6528	0.4250	0.1700
C ₄	0.3667	0.4989	0.6311	0.5	0.7839	0.5561	0.2224
C ₅	0.0828	0.2150	0.3472	0.2161	0.5	0.2722	0.1089

Applying equations (9) and (10), the criteria weights are obtained as 0.2758, 0.2229, 0.17, 0.2224 and 0.1089 respectively. The same procedures are used to calculate the weights of each alternative and the results are displayed in Table 12.

Table 12: Weights for the alternatives

	A ₁	A ₂	A ₃	A ₄
C ₁	0.3385	0.2282	0.1621	0.2713
C ₂	0.3492	0.2831	0.2169	0.1508
C ₃	0.1443	0.2099	0.2755	0.3703
C ₄	0.2888	0.2888	0.1785	0.2440
C ₅	0.2558	0.2558	0.1897	0.2988

Table 13 shows the final scores of alternatives which is obtained by using equation (11).

Table 13: Final scores of the alternatives

	A ₁	A ₂	A ₃	A ₄
C ₁	0.0933	0.0629	0.0447	0.0748
C ₂	0.0778	0.0631	0.0483	0.0336
C ₃	0.0245	0.0357	0.0468	0.0630
C ₄	0.0642	0.0642	0.0397	0.0543
C ₅	0.0279	0.0279	0.0207	0.0325
Final scores	0.29	0.25	0.20	0.26

The comparison of ranking orders by Bozdag et.al^[1], Wang and Chen^[15] and our proposed method is shown in Table 14.

Table 14: The comparison of ranking of alternatives

Method	Final scores				Ranking
	A ₁	A ₂	A ₃	A ₄	
Bozdag et al (2003)	0.635	0.006	0.000	0.305	A ₁ >A ₄ >A ₂ >A ₃
Wang & Chen (2006)	0.32	0.26	0.21	0.29	A ₁ >A ₄ >A ₂ >A ₃
Proposed method	0.29	0.25	0.20	0.26	A ₁ >A ₄ >A ₂ >A ₃

All the three approaches resulted in the same ranking order. It is obvious that our proposed algorithm produced a consistent ranking order with Bozdag et.al^[1] and Wang & Chen (2006).

Conclusion:

In this paper, we presented a new algorithm based on the consistent fuzzy preference relations method in solving multi-criteria selection problems. We improvised Wang & Chen's work by incorporating centroid-index formula. Using the example utilized by Bozdag et al (2003), our proposed method involved lesser calculation

as compared to the previous two methods. This is because our method provides only (n-1) pair-wise comparison in constructing the evaluation matrix for any types of fuzzy number used. This study proved that the proposed method is able to rank alternatives in systematic manner with consistent ranking order.

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