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Stability of Ternary Cubic Derivations on Ternary Frèchet Algebras

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Abstract: In this paper, we prove the generalized Hyers–Ulam–Rassias stability and superstability of ternary cubic derivations on ternary Frèchet algebras.

Key words: Generalized Hyers-Ulam-Rassias stability; Cubic functional; Frèchet algebras; Ternary derivation.

INTRODUCTION

We say a functional equation (ξ) is stable if any function g satisfying the equation (ξ) approximately is near to true solution of (ξ). Also, we say that a functional equation is superstable if every approximately solution is an exact solution of it. The stability problem of functional equations originated from a question of Ulam [48] in 1940, concerning the stability of group homomorphisms. We are looking for situations when the homomorphisms are stable, i.e., if a mapping is almost a homomorphism, then there exists a true homomorphism near it. The case of approximately additive mappings was solved by Hyers [36] under the assumption that G_1 and G_2 are Banach spaces. In 1978, a generalized version of the theorem of Hyers for approximately linear mappings was given by Rassias (1978).

In 1991, Z. Gajda (1991) answered the question for the case $p > 1$, which was raised by Rassias. This new concept is known as Hyers–Ulam–Rassias stability of functional equations.

Jun and Kim (2002) introduced the following functional equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+12f(x) \qquad (1.1)$$

and they established the general solution and the generalized Hyers–Ulam–Rassias stability for the functional equation (1.1). The function $f(x) = x^3$ satisfies the functional equation (1.1), which is thus called a cubic functional equation. Every solution of the cubic functional equation is said to be a cubic function. Jun and Kim proved that a function f between real vector spaces X and Y is a solution of (1.1) if and only if there exists a unique function $C: X \times X \times X \times X \to Y$ such that $f(x) = C(x, x, x)$ for all $x \in X$, and C is symmetric for each fixed one variable and is additive for fixed two variables.

Abbas Najati and Choonkil Park (2008) established the general solution and the generalized Hyers-Ulam-Rassias stability problem for the functional equation

$$2f(x+2y)+f(x-2y)=5f(x+y)+5f(x-y)+15f(y) \qquad (1.2)$$

For more detailed definitions of such terminologies, we can refer to (Abbaszadeh, 2010; Ebadian, 2010; Eshaghi 2009; Eshaghi 2009; Eshaghi 2010; Eshaghi 2009; Eshaghi 2009; Eshaghi 2009; Eshaghi 2008; Eshaghi 2010; Eshaghi 2010; Eshaghi 2010; Eshaghi 2010; Eshaghi 2009; Eshaghi 2009; Eshaghi 2010; Eshaghi 2099; Eshaghi 2009; Eshaghi 2009; Eshaghi 2010; Eshaghi 2010; Eshaghi 2010; Eshaghi 2009; Eshaghi 2010; Farokhzadand 2010; Park 2010; Park 2010; Shakeri, 2010).

Ternary algebraic operations were considered in the 19 th century by several mathematicians such as A. Cayley (1981) who introduced the notion of cubic matrix which in turn was generalized by Kapranov, Gelfand and Zelevinskii (1994) in 1990, (Bazunova, 2004; Park 2010; Zettl, 1983).

Let A be a ternary algebra. A is called a ternary Frechet algebra if A is a complete metrizable locally convex topological vector space (with metric d) which, satisfies

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$$d([a,b,c],0) \le d(a,0)d(b,0)d(c,0)$$

for all $a, b, c \in A$.

We recall that a mapping $D: A \to A$ is called a ternary cubic derivation on ternary Frechet algebra A if D is a cubic function for all $x, y, z \in A$.

$$D([x,y,z]) = [D(x),y^3,z^3] + [x^3,D(y),z^3] + [x^3,y^3,D(z)]$$

Recently, M. Bavand Savadkouhi, M. Eshaghi Gordji, J.M. Rassias and N. Ghobaipour (2009), investigated approximate ternary Jordan derivationson Banach ternary algebras. For more detailed definitions of such terminologies, we refer to (Eshaghi Gordji, 2010; Eshaghi Gordji, 2010; Eshaghi Gordji, 2010; Eshaghi Gordji, 2010; Eshaghi Gordji, 2009; Park, 2009).

In this paper, we investigate the stability and superstability of ternary cubic derivations on ternary Frechet algebras.

2. Main Results:

We start our work with the following theorem, which solve the generalized Hyers–Ulam–Rassias stability of ternary cubic derivations on ternary Frechet algebras.

Theorem 2.1:

Let A be a Frechet algebra by metric d. Suppose $f: A \to A$ is a mapping for which there existsa function $\varphi: A\times A\times A \to [0,\infty)$ such that

$$\sum_{j=0}^{\infty} \frac{1}{2^{3j}} \varphi(2^j x, 2^j y, 2^j z) < \infty \qquad (2.1)$$

$$d(2f(x+2y) + f(2x-y), 5f(x+y) + 5f(x-y) + 15f(y)) \le \varphi(x,y,0), \qquad (2.2)$$

$$d(f[x,y,z]), [f(x),y^3,z^3] + [x^3,f(y),z^3] + [x^3,y^3,f(z)]) \le \varphi(x,y,z) \qquad (2.3)$$

for all $x, y, z, \in A$. Then there exists a unique ternary cubic derivation $D: A \to A$ such that for all $x \in A$

$$d(f(x), D(x)) \le \frac{1}{8} \tilde{\varphi}(x,0,0) \qquad (2.4)$$

for all $x, y, z, \in A$.

$$\tilde{\varphi} = \sum_{j=0}^{\infty} \frac{\varphi(2^j x, 2^j y, 2^j z)}{2^{3j}}, \qquad (2.5)$$

Proof:

By putting $x = y = 0$ in (2.2), we get $f(0) = 0$. Again, if we put $y = 0$ in (2.2), we have

$$d(f(2x), 8f(x)) \le \varphi(x,0,0), \qquad (2.6)$$

for all $x \in A$. It follows from (2.6) that

$$d\left(\frac{f(2x)}{2^3}, f(x)\right) \le \frac{\varphi(x,0,0)}{2^3}, \qquad (2.7)$$

for all $x \in A$. By Rassias' method on inequality (2.7) ([32]), one can use induction on n to show that

$$d\left(\frac{f(2^n x)}{2^{3n}}, f(x)\right) \leq \frac{1}{8}\sum_{j=0}^{n-1} \frac{\varphi(2^j x, 0,0)}{2^{3j}}, \qquad (2.8)$$

for all $x \in$ A and all non-negative integers n. Hence,

$$d\left(\frac{f(2^{n+m} x)}{2^{3(n+m)}}, \frac{f(2^m x)}{2^{3m}}\right) \leq \frac{1}{8}\sum_{j=m}^{n+m-1} \frac{\varphi(2^j x, 0,0)}{2^{3j}},$$

for all non-negative integers n and m with $n \geq m$ and all $x \in A$. By (2.1), it follows that the sequence $\{\frac{f(2^n x)}{2^{3n}}\}$ is Cauchy. On the other hand, A is complete, then $\frac{f(2^n x)}{2^{3n}}$ is convergent.
So we can define the mapping $D: A \to A$ as follows:

$$D(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^{3n}} \qquad (2.9)$$

for all $x \in A$. Replacing x, y by $2^n x$ and $2^n y$, respectively, in (2.2) and multiplying both sides of (2.2) by $\frac{1}{2^{3n}}$, we get

$$d(2D(x+2y) + D(2x-y), 5D(x+y) + 5D(x-y) + 15D(y))$$
$$= \lim_{n\to\infty} \frac{1}{2^{3n}} d(f(2^n(x+2y)) + f(2^n(2x-y)), 5f(2^n(x+y))$$
$$+ 5f(2^n(x-y)) + 15f(2^n y))$$
$$\leq \lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 0)}{2^{3n}}$$

for all $x,y \in A$ and all non-negative integers n. Taking the limit, as $n \to \infty$, we have

$$2D(x+2y) + D(2x-y) = 5D(x+y) + 5D(x-y) + 15D(y),$$

for all $x, y \in A$ This means that D is cubic. Moreover, it follows from (2.8) and (2.9) that

$$d(f(x), D(x)) \leq \frac{1}{8}\tilde{\varphi}(x,0,0)$$

for all $x \in A$. By (2.3), we get

$$d(D[x,y,z], [D(x), y^3, z^3] + [x^3, D(y), z^3] + [x^3, y^3, D(z)])$$
$$= \lim_{n\to\infty} \frac{1}{2^{9n}} d(f[2^n x, 2^n y, 2^n z]), [f(2^n x), (2^n y)^3, (2^n z)^3]$$

$$+[(2^n x)^3, f(2^n y), (2^n z)^3] + [(2^n x)^3, (2^n y)^3, f(2^n z)])$$

$$\leq \lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 2^n z)}{2^{9n}}$$

for all *x, y, z* ∈ *A* and all non-negative integers *n*. Taking the limit, as $n \to \infty$, we obtain $D([x,y,z]) = [D(x), y^3, z^3] + [x^3, D(y), z^3] + [x^3, y^3, D(z)]$,

for all *x, y, z* ∈ *A*. Hence, *D* is a cubic derivation.

We claim that *D* is unique. To see, let D' : A → A be another ternary cubic derivation satisfying (2.4). Then we have

$$d(D(x), D'(x)) = \frac{1}{2^{3n}} d\big(D(2^n x), D'(2^n x)\big)$$

$$\leq \frac{1}{2^{3n}}\Big(d\big(D(2^n x), f(2^n x)\big) + d(f(2^n x), D'(2^n x))\Big)$$

$$\leq \frac{1}{4.2^{3n}} \tilde{\varphi}(2^n x, 0, 0)$$

which tends to zero as $n \to \infty$ for all *x* ∈ *A*. So, the claim is proved. Thus, the mapping D: A →A is a unique ternary cubic derivation satisfying (2.4).

Theorem 2.2:

Let *A* be a Frechet algebra by metric *d*. Let *f: A →A* be a mapping for which there exists a function φ: A×A×A → [0,∞) satisfying (2.2), (2.3) and

$$\sum_{j=0}^{\infty} 2^{9j} \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, \frac{z}{2^j}\right) < \infty \qquad (2.10)$$

for all *x, y, z* ∈ *A*. Then there exists a unique ternary cubic derivation D: A →A such that

$$d\big(f(x), D(x)\big) \leq \tilde{\varphi}\left(\frac{x}{2}, 0, 0\right) \qquad (2.11)$$

for all *x*∈*A*. Here,

$$\tilde{\varphi}(x,y,z) = \sum_{j=0}^{\infty} 2^{3j} \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, \frac{z}{2^j}\right) \qquad (2.12)$$

for all $x, y, z \in A$.

Proof. Replacing *x* by $\frac{x}{2}$ in (2.6), we get

$$d(2^3 f(\frac{x}{2}), f(x)) \leq \varphi\left(\frac{x}{2}, 0,0\right), \qquad (2.13)$$

for all $x \in A$ and all non-negative integers n. Hence,

$$d\left(2^{3n} f(\frac{x}{2^n}), f(x)\right) \leq \sum_{j=0}^{n-1} 2^{3j} \varphi\left(\frac{x}{2^{j+1}}, 0,0\right) \qquad (2.14)$$

$$d\left(2^{3(n+m)} f(\frac{x}{2^{n+m}}), 2^{3m} f(\frac{x}{2^m})\right) \leq \sum_{j=m}^{n+m-1} 2^{3j} \varphi\left(\frac{x}{2^{j+1}}, 0,0\right)$$

for all non-negative integers n and m with $n \geq m$ and all $x \in A$. It follows from the convergence of (2.10) that the sequence $\{2^{3n} f(\frac{x}{2^n})\}$ is Cauchy. Due to the completeness of A, this sequence is convergent. So, one can define the mapping $D: A \to A$ by

$$D(x) = \lim_{n\to\infty} 2^{3n} \frac{f(x)}{2^n} \qquad (2.15)$$

for all $x \in A$. Replacing x, y by $\frac{x}{2^n}, \frac{y}{2^n}$, respectively, in (2.2) and multiplying both sides of (2.2) by 2^{3n} we get

$$d\big(D(x+2y) + D(2x-y), 5D(x+y) + 5D(x-y) + 15D(y)\big)$$

$$= \lim_{n\to\infty} 2^{3n} d(f(\frac{x+2y}{2^n}) + f(\frac{2x-y}{2^n}), 5f\left(\frac{x+y}{2^n}\right) + 5f\left(\frac{x-y}{2^n}\right) + 15f(\frac{y}{2^n}))$$

$$\leq \lim_{n\to\infty} 2^{3n} \varphi(\frac{x}{2^n}, \frac{y}{2^n}, 0)$$

for all $x, y \in A$ and all non-negative integers n. Taking the limit, as $n \to \infty$, we obtain

$$d(2\big(D(x+2y) + D(2x-y), 5D(x+y) + 5D(x-y) + 15D(y)\big),$$

for all $x, y \in A$. Then D is cubic. By the same method as the above theorem, one may show that the mapping $D: A \to A$ is a unique ternary cubic derivation satisfying (2.11).

By Theorem 2.1, we solve the following generalized Hyers–Ulam–Rassais stability problem of ternary cubic derivations in ternary Banach algebras.

Corollary 2.3:

Let A be a ternary Banach algebras. Let $f: A \to A$ be a mapping for which there exists a function $\varphi: A \times A \times A \to [0, \infty)$ satisfying (2.1),

$$\| 2f(x+2y) + f(2x-y) - 5f(x+y) - 5f(x-y) - 15f(y) \| \leq \varphi(x,y,0), \qquad (2.16)$$

$$\| f([x,y,z)]) - [f(x), y^3, z^3] - [x^3, f(y), z^3] - [x^3, y^3, f(z)] \| \le \varphi(x,y,z), \qquad (2.17)$$

for all *x, y, z* $\in$ *A*. Then there exists a unique ternary cubic derivation $D: A \to A$ such that

$$\| f(x) - D(x) \| \le \frac{1}{8} \tilde{\varphi}(x,0,0) \qquad (2.18)$$

for all *x* $\in$ *A*. Here,

$$\tilde{\varphi}(x,y,z) = \sum_{j=0}^{\infty} \frac{1}{2^{3j}} \varphi(2^j x, 2^j y, 2^j z)$$

for all *x, y, z* $\in$ *A*.

Proof:

By putting $d(x,y) = \| x - y \|$ for all $x, y \in A$, it follows from Theorem 2.1.

By Theorem 2.2, we solve the following generalized Hyers–Ulam–Rassais stability problem of ternary cubic derivations in ternary Banach algebras.

Corollary 2.4:

Let *A* be a ternary Banach algebra. Let *f*: *A*→*A* be a mapping for which there exists a function $\varphi: A \times A \times A \to [0, \infty)$ satisfying (2.10), (2.16) and (2.17) for all *x, y, z* $\in$ *A*. Then there exists a unique ternary cubic derivation $D: A \to A$ such that

$$\| f(x) - D(x) \| \le \tilde{\varphi}\left(\frac{x}{2}, 0, 0\right), \qquad (2.19)$$

for all *x*$\in$*A*. Here,

$$\tilde{\varphi}(x,y,z) = \sum_{j=0}^{\infty} 2^{3j} \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, \frac{z}{2^j}\right)$$

for all *x, y, z* $\in$ *A*.

Proof:

By putting $d(x,y) = \| x - y \|$ for all $x, y \in A$, it follows from Theorem 2.2.

By Theorems 2.1 and 2.2, we solve the following Hyers–Ulam–Rassais stability problem of ternary cubic derivations in Fr?chet algebras.

Theorem 2.5:

Let *A* be a Frechet algebra by metric *d*. Let *p*≥*0* be given with p≠3. Assume θ is a non- negative real number and let $f: A \to A$ be a mapping such that

$$d\big(2f(x+2y)+f(2x-y),5f(x+y)+5f(x-y)+15f(y)\big)$$

$$\leq \theta\left(d(x,0)^p+d(y,0)^p\right),(2.20)$$

$$d(f([x,y,z]),[f(x),y^3,z^3]+[x^3,f(y),z^3]+[x^3,y^3,f(z)]$$

$$\leq \theta\left(d(x,0)^p+d(y,0)^p+d(z,0)^p\right)(2.21)$$

for all $x,y,z \in A$. Then, there exists a unique ternary cubic derivation $D: A \to A$ such that

$$d\big(D(x),f(x)\big) \leq \frac{\theta.d(x,0)^p}{(8-2^p)}, \qquad (2.22)$$

holds for all $x \in A$, where $p < 3$, or the inequality

$$d\big(D(x),f(x)\big) \leq \frac{\theta.d(x,0)^p}{(2^p-8)}, \qquad (2.23)$$

holds for all $x \in A$, where $p < 3$.

Proof:

Assume that $p < 3$. By putting $x = y = 0$ in (2.20), we get $f(0)= 0$. If we put $y = 0$ in (2.37) and multiply both sides of (2.37) by $\frac{1}{6}$, we get

$$d\left(\frac{f(2x)}{2^3},f(x)\right) \leq \frac{\theta.d(x,0)^p}{8},$$

for all $x \in A$. One can use induction on n to show that

$$d\left(\frac{f(2^n x)}{2^{3n}},f(x)\right) \leq \frac{\theta}{8}\sum_{j=0}^{n-1} 2^{j(p-3)} d(x,0)^p \qquad (2.24)$$

for all $x \in A$ and all non-negative integers n. Hence,

$$d\left(f\left(\frac{2^{(n+m)}x}{2^{3(n+m)}}\right),f\left(\frac{2^m x}{2^{3m}}\right)\right) \leq \frac{\theta}{8}\sum_{j=m}^{n+m-1} 2^{j(p-3)} d(x,0)^p,$$

for all non-negative integers n and m with $n \geq m$ and all $x \in A$. It follows from $p < 3$ that the sequence $\{\frac{f(2^n x)}{2^{3n}}\}$ is Cauchy. Due to the completeness of A, this sequence is convergent. So, one can define the mapping D: A →A by

$$D(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^{3n}} \qquad (2.25)$$

for all $x \in A$. Replacing x, y by $2^n x$, $2^n y$, respectively, in (2.20) and multiplying both sides of (2.20) by $\frac{1}{2^{3n}}$, we get

$$d\big(2D(x+2y) + D(2x-y), 5D(x+y) + 5D(x-y) + 15D(y)\big)$$

$$= \lim_{n\to\infty} d(f(\frac{2^n(x+2y)}{2^{3n}}) + f(\frac{2^n(2x-y)}{2^{3n}}), 5f\left(\frac{2^n(x+y)}{2^{3n}}\right) + 5f\left(\frac{2^n(x-y)}{2^{3n}}\right) + 15f(\frac{2^n(y)}{2^{3n}}))$$

$$\leq \lim_{n\to\infty} 2^{n(p-3)}\,\theta(d(x,0)^p + d(y,0)^p)$$

$$\to 0$$

for all $x, y \in A$ and all non-negative integers n. So, we have

$$2D(x+2y) + D(2x-y) = 5D(x+y) + 5D(x-y) + 15D(x),$$

and

$$d(D(x), f(x)) \leq \frac{\theta . d(x,0)^p}{(8-2^p)},$$

for all $x \in A$. It follows from (2.21) that

$$d(D[x,y,z]), [D(x), y^3, z^3] + [x^3, D(y), z^3] + [x^3, y^3, D(z)])$$

$$= \lim_{n\to\infty} \frac{1}{2^{9n}} d(f[2^n x, 2^n y, 2^n z]), [f(2^n x), (2^n y)^3, (2^n z)^3]$$

$$+ [(2^n x)^3, f(2^n y), (2^n z)^3] + [(2^n x)^3, (2^n y)^3, f(2^n z)])$$

$$\leq \lim_{n\to\infty} 2^{n(p-9)}\theta(d(x,0)^p + d(y,0)^p + d(z,0)^p)$$

$$\to 0$$

for all $x, y, z \in A$. So we have

$$D([x,y,z]) = [D(x), y^3, z^3] + [x^3, D(y), z^3] + [x^3, y^3, D(z)],$$

for all $x, y, z \in A$.

Now, let D`: A→A be another ternary cubic derivation satisfying (2.22). Then we have

$$d(D(x)\; D'(x)) = \frac{1}{2^{3n}} d(D(2^n x), D'(2^n x))$$

$$\leq \frac{1}{2^{3n}}\left(d(D(2^n x), f(2^n x)) + d(f(2^n x), D'(2^n x))\right)$$

$$\leq \frac{2\theta . 2^{n(p-3)}}{(8-2^p)} d(x,0)^p$$

which tends to zero as $n \to \infty$ for all $x \in A$. So we have D(x) = D`(x) for all x∈A. This proves the uniqueness of *D*. Thus, the mapping D: A→A is a unique ternary cubic derivation satisfying (2.22). Similarly, one obtains the result for the case $p > 3$.

By Theorems 2.1 and 2.2, we solve the following Hyers–Ulam–Rassais stability problem of cubic derivations in ternary Banach algebras.

Corollary 2.6:

Let *A* be a ternary Banach algebra. Let $p \geq 0$ be given with $p \neq 3$. Let θ be a non-negative real number, and let *f*: A→A be a mapping such that

$$\|2f(x+2y) + f(2x-y) - 5f(x+y) - 5f(x-y) - 15f(y)\|$$

$$\leq \theta(\|x\|^p + \|y\|^p), \qquad (2.26)$$

$$\|f([x,y,z]) - [f(x),y^3,z^3] - [x^3,f(y),z^3] - [x^3,y^3,f(z)]\|$$

$$\leq \theta(\|x\|^p + \|y\|^p + \|z\|^p), \qquad (2.27)$$

for all *x, y, z*∈A. Then there exists a unique ternary cubic derivation D: A→A such that

$$\| (D(x) - f(x) \| \leq \frac{\theta}{(8-2^p)} \| x \|^p, \qquad (2.28)$$

holds for all *x*∈A, where $p < 3$, or the inequality

$$\| (D(x) - f(x) \| \leq \frac{\theta}{(2^p-8)} \| x \|^p \qquad (2.29)$$

holds for all *x*∈A, where $p > 3$.

Proof:

In Theorem 2.5, by putting $d(x,y) = \|x-y\|$ for all *x, y,* ∈A, we obtain the conclusion.

The following corollary is the Hyers–Ulam stability problem of ternary cubic derivations in ternary Banach algebras.

Corollary 2.7:

Let *A* be a ternary Banach algebra. Let ε be a non-negative real number, and let D: A→A be amapping such that

$$\|2f(x+2y) + f(2x-y) - 5f(x+y) - 5f(x-y) - 15f(y)\| \leq \varepsilon, \qquad (2.30)$$

$$\|f([x,y,z]) - [f(x),y^3,z^3] - [x^3,f(y),z^3] - [x^3,y^3,f(z)]\| \leq \varepsilon, \qquad (2.31)$$

for all $x, y, z \in A$. Then there exists a unique ternary cubic derivation D: A→A such that

$$\| (D(x) - f(x) \| \leq \frac{\varepsilon}{7}, (2.32)$$

holds for all $x \in A$.

Proof:

In Corollary 2.6, by putting $p = 0$ and $\theta = \varepsilon$, we obtain the conclusion of the corollary.

Now, we establish the superstability of ternary cubic derivations on ternary Frechet algebras as follows:

Corollary 2.8:

Let p_1, p_2, p_3 be real numbers such that $p_1 + p_2 + p_3 \neq 1$. Let A be a Frechet algebra by metric d. Suppose f: A →A is a cubic mapping such that

$$d(f([x,y,z]),[f(x),y^3,z^3] + [x^3,f(y),z^3] + [x^3,y^3,f(z)]) \leq d(x,0)^{p_1} d(y,0)^{p_2} d(z,0)^{p_3},$$

for all $x, y, z \in A$. Then f is a ternary cubic derivation.

Proof: It follows from Theorems 2.1 and 2.2, by putting

$$\varphi(x,y,z) = d(x,0)^{p_1} d(y,0)^{p_2} d(z,0)^{p_3}$$

for all $x, y, z \in A$.

Moreover, we have the following superstability of ternary cubic derivations on ternary Banach algebras:

Corollary 2.9. Let p_1, p_2, p_3 be real numbers such that $p_1 + p_2 + p_3 \neq 1$. Let A be a ternary Banach algebra. Suppose
f: A → A is a cubic mapping such that

$$\|f([x,y,z]) - [f(x),y^3,z^3] - [x^3,f(y),z^3] - [x^3,y^3,f(z)]\| \leq \|x\|^{p_1} \|y\|^{p_2} \|z\|^{p_3}$$

for all $x, y, z \in A$. Then f is a ternary cubic derivation.

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