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# Channel Allocation in TETRA Networks Using Channel Reservation

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**Abstract:** In TErrestrial Trunked Radio (TETRA) networks, due to the limited number of available channels in each cell, obtaining the minimum number of allocated channels in order to satisfy Grade of Service (GoS) as well as user priority requirements is a critical task.This paper proposes a channel allocation algorithm for TETRA networks. Considering different levels of users' priority, the proposed algorithm provides the expected GoS for TETRA costumers while the priority criteria are satisfied. Channel allocation procedure for these networks are modeled by a one-dimensional Markov chain and analytical results for some performance metrics such as "call blocking probability" and "channel utilization" are obtained. In the proposed algorithm, the priority of users is guaranteed by channel reservation method. In order to provide the required GoS for users, the minimum number of required channels to be allocated to each priority is calculated. The simulation results of a TETRA network with a realistic traffic scenario for three types of user priorities (high, medium, and low) show that the proposed algorithm uses the minimum number of channels while satisfies all given GoSs.

**Key words:** Channel allocation, priority, reservation, TETRA.

## INTRODUCTION

Nowadays, with the fast growing of cellular networks' applications, the number of mobile users as well as their expectations from mobile telephone services have been changed drastically. The range of these changes goes beyond the public cellular systems. The Private or Professional Mobile Radio (PMR) systems are usually established by a company, organization or a particular group of users, to provide a reliable and internal communication independent of other operators. These systems are usually used in public safety organizations such as police, fire departments, ambulances, mountain rescue teams and so on. TETRA as a European standard for PMR systems was introduced by European Telecommunications Standards Institute (ETSI) and now is widely used, not only in Europe, but also throughout the world (Canales, 2010). One of the characteristics of the PMR systems, including TETRA, is the possibility of serving different users with different levels of priority. This possibility can be interpreted as using different Call Admission Control (CAC) methods or alternatively, defining different minimum levels of acceptable GoS for users with different priority levels (Stavroulakis, 2007).

Frequency spectrum is limited in cellular systems and the goal of different channel allocation methods is to use these scarce and valuable resources, effectively. When many users share a small number of channels (i.e. trunking concept), there is a certain probability called Blocking Probability (BP), that a user will not be able to obtain a connection when he/she needs one, because all channels are busy. BP is one of the most important performance metrics in cellular systems and in many cases it is used as system GoS. The efficient use of available channels plays a critical role in increasing network capacity, improving spectral efficiency, and consequently, results in improving the quality and the efficiency of a communication system. Traditionally, the issue of priority in public cellular networks is considered in handover (HO) calls. One of the common methods to prioritize these HO calls is to reserve some channels for them. In this way, the BP of HO calls decreases while for a fixed number of available channels, the BP of other calls increases. In (Oh, 1992), Cutoff Priority Scheme (CPS) for new and HO calls is introduced; given the number of available channels and the GoS for both types of calls in each cell, it offers an optimal channel allocation that ensures the priority of HO calls. In (Enrico, 1999) in order to reduce the dropping probability of HO calls, it is assumed that those HO requests, who do not obtain any free channel immediately, can be queued. However, the new calls will block as long as there is no free channel. This, results in increasing the delay of HO calls, but the dropping probability of HO calls decreases. In (Chang, 1999) on the contrary to (Enrico, 1999) when there is no free channel, new calls are queued and HO calls are blocked. In (Lau, 1998) there is a possibility for all arrival calls (whether new or

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HO calls) to be queued.

Another way to prioritize calls in cellular systems is the "preemption" method, which means when all channels are busy; those users with preemptive priority are able to drop the non-preemptive users' calls. Therefore, the BP of higher priority users decreases at the cost of increasing the dropping probability of other users with lower priority levels (Zhou, 2009; Tong, 2006; Sheu, 2005).

Similar to public mobile radio systems, in order to provide priority requirements of users, both reservation and preemptive methods can be used in PMR systems as well. Expanding the work in (Oh, 1992) to different levels of priority in PMR systems, and considering different levels of acceptable GoS for TETRA network users, this paper proposes an optimal reservation based channel allocation algorithm to satisfy all users with different levels of priority, while using the minimum number of channels. In the proposed scheme the channels for higher priority users are reserved in such a way that the GoS of all priorities meet the minimum required levels.

The reminder of this paper is organized as follows. In Section 2, the queuing theory based model of channel allocation process in a PMR network is introduced. In Section 3, BP for different priorities is computed and the corresponding properties are described. In the next Section, the proposed algorithm for channel allocation is introduced. In Section 5, the simulation results for suggested algorithm in a TETRA network, with a realistic traffic model and parameters, are expressed. And finally in Section 6, the results are discussed and the paper concludes.

## 2. *Traffic Model and Channel Allocation Process:*

To present the traffic model and then the proposed channel allocation algorithm, a cellular network consisting of $M$ cells is considered, where $C$ channels are allocated to each cell. Without loss of generality and for simplicity, it is assumed that three types of users with low, medium and high priorities are competing to access the channels in each cell. Suppose that low, medium and high priority call attempts are generated in each cell according to Poisson processes with rates $\lambda_L$, $\lambda_M$, and $\lambda_H$, respectively. The average call duration is the same for all calls and it is assumed to be exponentially distributed with mean $1/\mu$. In the proposed channel allocation model, $z$ channels are reserved in each cell for high priority users. These channels are not available for other users. Also, $y$ channels are reserved for high and medium priority users. These channels are not available for the low priority users. Consequently, the rest of channels ($x = C - y - z$) are available for all the users. Therefore, if a user with low priority arrives when more than $x$ channels are busy, he/she will be blocked. Similarly, a user with medium priority will be blocked when $x+y$ channels are busy. And ultimately, a user with the highest priority will be blocked when all the channels are busy (Fig. 1). It should be noted that $x$, $y$ and $z$ are the numbers of reserved channel and there is no specific channel assignment for any of the priorities.

The busy channels at time $t$ are represented by $B(t)$, . Thus $\{B(t),\ t \geq 0\}$is a continues-time Markov chain with $0,1,\ldots,C$ states which can be shown as a birth and death process. The steady state probability in state $n$ ($n = 0,1,\ldots,C$) is represented by $P_n$, whichis defined as $P_n = \lim_{t\to\infty} P\{B(t) = n\}$ (Bhat, 2008). The birth and death model of channel allocation process is shown in Fig. 2.

This model is an $M/M/C/C$ queue which user arrivals are Poisson and service time is exponentially distributed (Bhat, 2008). There are $C$ servers and there is no buffer in the system. When a call arrives while all servers are busy, then the call will be blocked immediately. For computing the BPs, the steady state probabilities should be obtained. The equilibrium equations for the probabilities $P_n$ can be derived from the state diagram shown in Fig. 2 and obtained as:

$$P_n = \begin{cases} \dfrac{a^n}{n!} P_0 & 1 \le n \le x \\ \dfrac{a^x b^{n-x}}{n!} P_0 & x+1 \le n \le x+y \\ \dfrac{a^x b^y c^{n-x-y}}{n!} P_0 & x+y+1 \le n \le C \end{cases} \tag{1}$$

where $a = \dfrac{\lambda_L + \lambda_M + \lambda_H}{\mu}, b = \dfrac{\lambda_M + \lambda_H}{\mu}, \ c = \dfrac{\lambda_H}{\mu}$. Applying normalization condition to (1) we have:

$$\sum_{n=0}^{C} P_n = 1 \Rightarrow P_0 = \left[ 1 + \sum_{n=1}^{x} \frac{a^n}{n!} + a^x \sum_{n=1}^{y} \frac{b^n}{(n+x)!} + a^x b^y \sum_{n=1}^{z} \frac{c^n}{(n+x+y)!} \right]^{-1} \tag{2}$$

**3. *Performance Metrics:***

**3.1. *Blocking Probability (BP)*:**

BP for each priority can be obtained from the $P_n$ probabilities. When $x$ or more channels are busy in a cell, the low priority users are blocked. $B_L$ is the BP for low priority users which equal to:

$$B_L(x,y,z) = \sum_{n=x}^{C} P_n = \left[ \frac{a^x}{x!} + a^x \sum_{n=1}^{y} \frac{b^n}{(n+x)!} + a^x b^y \sum_{n=1}^{z} \frac{c^n}{(n+x+y)!} \right] P_0 \tag{3}$$

If x+$y$ or more channels are busy, then the medium priority users are blocked. The BP of medium priority users which is indicated by $B_M$, is given by:

$$B_M(x,y,z) = \sum_{n=x+y}^{C} P_n = \left[ \frac{a^x b^y}{(x+y)!} + a^x b^y \sum_{n=1}^{z} \frac{c^n}{(n+x+y)!} \right] P_0 \tag{4}$$

High priority users are blocked only when all $C$ channels are busy. $B_H$ is the BP for high priority users and is given by:

$$B_H(x,y,z) = P_C = \frac{a^x b^y c^z}{C!} P_0 \tag{5}$$

***3.2. Channel Utilization:***

The total channel utilization is defined as the ratio of the mean number of occupied channels to the total number of channels, and is calculated as follows:

$$\eta = \frac{1}{C} \sum_{n=0}^{C} n P_n = \frac{1}{C} \left\{ \sum_{n=1}^{x} \frac{a^n}{(n-1)!} + a^x \sum_{n=1}^{y} \frac{b^n}{(n+x-1)!} + a^x b^y \sum_{n=1}^{z} \frac{c^n}{(n+x+y-1)!} \right\} P_0 \tag{6}$$

***3.3. BP Properties:***

$B_L$, $B_M$ and $B_H$ have some useful properties which have been used in the proposed channel allocation algorithm. The most obvious property is that for a fixed number of channels, the BP of the users with high priority is smaller than that of the medium ones, and also the BP of the users with medium priority is smaller than that of the users with low priority. Hence:

$$B_H(x,y,z) \le B_M(x,y,z) \le B_L(x,y,z)$$

The other properties are as follows:

***Property1: for Any Channel Allocation (x,y,z):***

$$B_H(x,y,z+1) < B_H(x,y+1,z) < B_H(x+1,y,z) < B_H(x,y,z)$$

Generally, this property implies that the BP of the high priority users decreases by increasing the channels. In more details, the first inequality on the left shows that by increasing $z$ (comparing with increasing $y$) $B_H$ decreases, because by increasing $z$, only the high priority users have access to additional channel, but by increasing y the users with medium priority also have access to it. Therefore, $B_H$ will decrease more in this case.

Similarly, the second inequality shows that by increasing $y$ (comparing with increasing $x$), $B_H$ decreases more. Using the similar reasoning, we can generalize these inequalities for more than three levels of priority. In this case we suppose that there are $n$ users with different levels of priority, where $x_i$ shows the cutoff priority for $i^{th}$ level and $B_n$ represents BP for the highest priority. Then we have:

$$B_n(x_1, x_2, ..., x_{n-1}, x_n + 1) < B_n(x_1, x_2, ..., x_{n-1} + 1, x_n) < ... < B_n(x_1, x_2 + 1, ..., x_{n-1}, x_n) <$$
$$B_n(x_1 + 1, x_2, ..., x_{n-1}, x_n) < B_n(x_1, x_2, ..., x_{n-1}, x_n)$$

***Property2: for Any Channel Allocation* (x,y,z)**:

$$B_M(x, y+1, z) < B_M(x+1, y, z) < B_M(x, y, z) < B_M(x, y, z+1)$$

The first inequality on the right means that when a channel is added to z, $B_M$ increases. The reason being that, considering the availability of the y channels to the high priority users, the busy channel probability on y reserved channels will increase by increasing z, and $B_M$ which is the BP of medium priority users will increase, consequently. The first inequality on the left shows that adding an additional channel to $y$, will decrease $B_M$ more in comparison with adding this additional channel to $x$. And finally, for the second inequality, it is obvious that adding a channel to $x$ will decrease $B_M$. For more than three levels of priority, BP for medium priorities can be generalized according to the following inequalities, where $B_m$ is the BP for medium priorities and 1> $m$ > $n$:

$$B_m(x_1, ..., x_m + 1, ..., x_n) < B_m(x_1, ..., x_{m-1} + 1, x_m, ..., x_n) < ... < B_n(x_1 + 1, ..., x_m, ..., x_n)$$
$$< B_m(x_1, ..., x_m, ..., x_n) < B_m(x_1, ..., x_{m+1} + 1, ..., x_n) < ... < B_m(x_1, ..., x_m, ..., x_n + 1)$$

***Property3: for Any Channel Allocation (x,y,z):***

$$B_L(x+1, y, z) < B_L(x, y, z) < B_L(x, y, z+1) < B_L(x, y+1, z)$$

The second inequality expresses that by increasing $z$, $B_L$ will increase because it increases the call admission probability for high priority users, and the busy channel probability will increase, consequently. The third inequality from the left shows that adding a channel to $y$ )in comparison to $z$( will increase $B_L$ more. The reason is when $y$ is increased, the call admission probability will increase )in comparison to increasing $z$( and also the users with medium priority have access to the channel. The first inequality indicates the obvious point that by increasing $x$, $B_L$ will decrease. For more than three levels of priority, BP for the lowest priority users can be generalized as follows. Here, $B_l$ is the BP for lowest priority.

$$B_1(x_1 + 1, ..., x_n) < B_1(x_1, ..., x_n) < B_1(x_1, ..., x_n + 1) < B_1(x_1, ..., x_{n-1} + 1, x_n)$$
$$< ... < B_1(x_1, x_2, x_3 + 1, ..., x_n) < B_1(x_1, x_2 + 1, ..., x_n)$$

All of the above inequalities for three levels can be proved mathematically (refer to the appendix).

## 4. *Proposed Algorithm:*

In the proposed algorithm, optimal channel allocation *)x, y, z(*, with minimum number of required channels is found, in order to satisfy the BP criteria for all users. Maximum acceptable BP for low, medium and high priority users are illustrated by $B_{Lmax}$, $B_{Mmax}$, and $B_{Hmax}$ respectively. Mathematically speaking, this algorithm is the solution of the following optimization problem:

$$Minimize \quad x + y + z = C$$

$$Subject\ to \begin{cases} B_L < B_{L\max} & constraint1 \\ B_M < B_{M\max} & constraint2 \\ B_H < B_{H\max} & constraint3 \\ x, y, z \geq 0, integer \end{cases}$$

Using the mentioned properties of BPs, the pseudo code of proposed algorithm can be written as follow:

$Step1:$

$Set\ k = 0\ \ and\ y^k = z^k = 0$

$Find\ the\ smallest\ integer\ x^k\ such\ that\ B_L(x^k, y^k, z^k) < B_{L\max};$

$Step2:$

$If \quad B_H(x^k, y^k, z^k) < B_{H\max}\ \ and\ \ B_M(x^k, y^k, z^k) < B_{M\max}$

$Terminate,\ (x^k, y^k, z^k)\ is\ optimum\ allocation, C = x^k + y^k + z^k;$

$Else$

$Go\,to\,step3;$

$End$

$Step3)\ If \quad B_L(x^k, y^k + 1, z^k) < B_{L\max}$

$If \quad B_M(x^k, y^k, z^k + 1) < B_{M\max}$

$(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k, y^k, z^k + 1);$

$Else$

$(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k, y^k + 1, z^k);$

$End$

$Elseif \quad B_L(x^k, y^k, z^k + 1) < B_{L\max}$

$If \quad B_M(x^k, y^k, z^k + 1) < B_{M\max}$

$(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k, y^k, z^k + 1);$

$Else$

$(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k + 1, y^k, z^k);$

$End$

$Else$

$(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k + 1, y^k, z^k);$

$End$

$k = k + 1;$

$Go\,to\,step2;$

In step1, initial allocation ($x^0$, 0, 0) with minimum number of $x$ that satisfies the constraint1, is found. In step2 when the BPs of all users are smaller than the given thresholds, then algorithm is terminated. Otherwise, according to step3, the number of channels is increased by one. In step3 at first, this additional channel adds to $y$ because it leads to maximum BP of low priorities. If constraint1 is satisfied, then this additional channel adds to $z$ (according to property3 adding a channel to $z$, in comparison to $y$, will decrease $B_L$ more, therefore constraint1 is also satisfied). If constraint2 is not satisfied by this addition, then this added channel adds to $y$ (according to property2, by increasing $y$ constraint2 will be satisfied), then algorithm returns to step2. If by adding $y$, constraint1 is not satisfied, then $z$ will be increased. And if constraint2 is not satisfied, then $x$ will be increased (according to property2, by increasing $x$, constraint2 is satisfied) and then algorithm return to step2. If by increasing $z$, constraint1 is not satisfied, then additional channel adds to $x$ (according to property3), then algorithm returns to step2. Therefore, by applying this algorithm, the BPs of all users would be smaller than the given thresholds, while the number of required channels is minimized.

***5. Simulation Results for a TETRA Network with Realistic Traffic Scenario*:**

In TETRA networks, the number of available channels in each cell is quite limited and obtaining the minimum number of reserved channels to support the priority of users is a critical task.

By applying the proposed algorithm, the minimum number of allocated channels for all priorities can be obtained, while the GoS requirements are satisfied. In this section we test the performance of the proposed algorithm by simulation based on a realistic scenario. According to Scenario 8 (ETSI, 1997) of urban and sub-urban TETRA network on European city, users in this simulation are divided into three groups with different levels of priority; each group has a different GoS as well as a different population distribution (Table 1). It is assume that most users are stationary within a disaster area so handoff is not considered here, but our current work can be easily extended to deal with handoff traffic using the extended version of properties, by simply adding one priority level as a handoff.

According to Table 1 and the proposed algorithm, we consider priorities 1, 2 and 3 as high, medium and low priorities, respectively. In this scenario the individual voice call for Mobile (M) to Mobile, Fixed (F) to Mobile and Mobile to Fixed are considered and also Mobile to Mobile for group call is considered which is represented in Table 2. All of these calls are simplex and require one channel.

By using the proposed algorithm and given GoSs for all priorities, optimum number of allocated channels and reserved channels are obtained for different amounts of users' population (Fig. 3). It is assumed that call duration of calls is 15 seconds.

Fig. 3 shows that for the population of 100 users, the minimum number of allocated channels is 5 )$x$ = 4, $y$ = 1, $z$ = 0(. This means that we do not need to reserve any channel for high priority users. For 1000 users the minimum number of channels reaches to 17 ($x$ = 14, $y$ = 2, $z$ = 1). Based on the obtained number of reserved channels and total number of channels (Fig. 3) for different users' populations, in order to validate the analytical result, the *M/M/C/C* queue model is simulated and the results are compared to the analytical ones. Fig. 4 shows that the results of analytical model and the simulation are the same. Also it is seen that by applying the number of channels obtained from the algorithm, the BPs of all users are smaller than the maximum GoS (Table 1). This means that the proposed algorithm works properly.

In Fig. 5, again a good similarity can be observed between analytical and simulation results. It shows channel utilization for simulation and analytical results for different users' populations per cell.

Table 3 shows other combination of channel allocations for 1000 users and the minimum number of 17 required channels, for example. It can be seen that each suboptimum allocation results in an unsatisfied GoS; this means that the obtained allocation ($x$, $y$, $z$) from this algorithm with minimum required number is optimum.

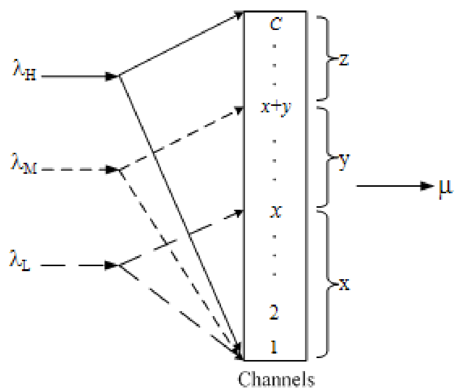


**Fig. 1:** Channel reservation model.

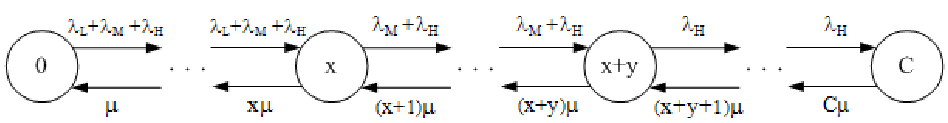

**Fig. 2:** The birth and death model of channel allocation process.

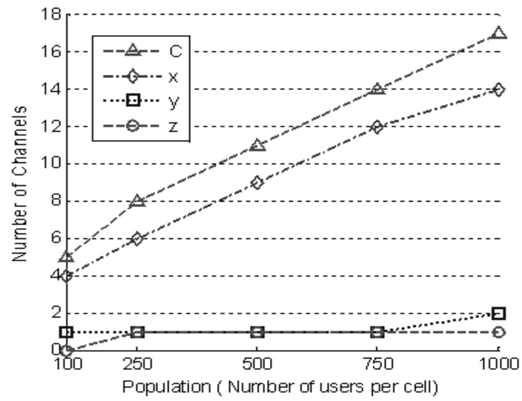


**Fig. 3:** Number of reserved and all channels obtained by the proposed algorithm for a TETRA network with a realistic traffic scenario.

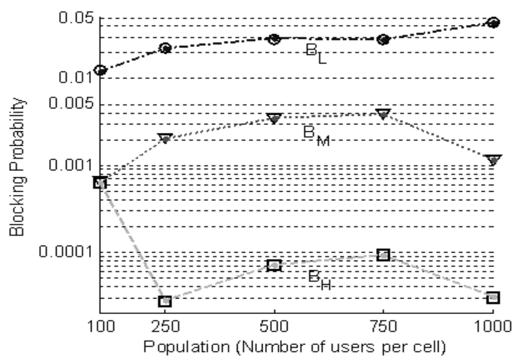


**Fig. 4:** Comparison of simulation and analytical results. Dash lines show analytical results and points show simulation results.

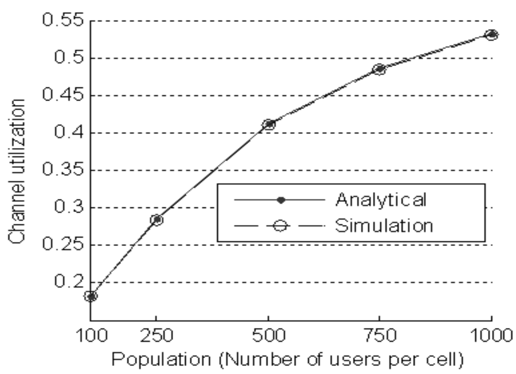


**Fig. 5:** Comparison of channel utilization for analytical and simulation results.

Fig. 6 shows the optimum allocated channel numbers to each priority level, for different users' populations per cell (traffic load). It is illustrated that by increasing the network traffic load, the number of reserved channels ($z$ and y) are fixed or increase very slightly, compared to $x$. This shows the tendency of the algorithm to minimize the number of reserved channels which leads to more channel utilization. Fig. 6 also shows that, for this scenario, due to the small portion of high priority users' population, one reserved channel ($z$ = 1) is enough, not only for scenario 8 with 250 users, but also for 1000 users even when the traffic load for each population is tripled. This means that in normal traffic loads, $z$ channels are not fully utilized and the spectral efficiency may be increased by dividing each of these channels into two half rate sub-channels (sub-rating) (Jain, 2005).

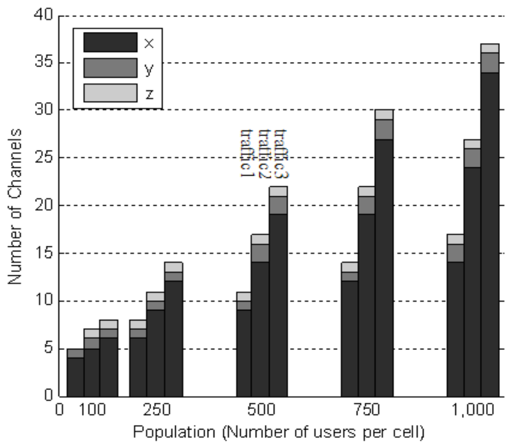


**Fig. 6:** Number of reserved channels for different traffic loads.

Fig. 7 shows the optimum allocated channel numbers to each priority level, for different GoSs. It is seen that, for a given traffic load, any change in GoS for high priority users just affects $z$ and has no impact on $x$ and $y$. Similar experiments show that, for a given traffic, any change in GoS for medium priority users has no impact on $x$.

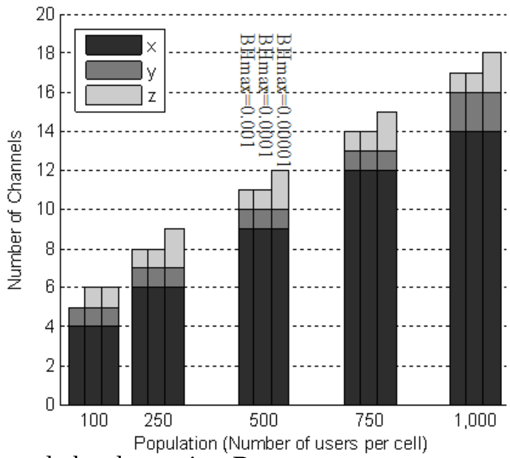


**Fig. 7:** Number of reserved channels by decreasing $B_{Hmax.}$

**Table 1:** GoS and population distribution of users.

| | Priority 1 | Priority 2 | Priority 3 |
|---|---|---|---|
| User population distribution | 5 % | 25 % | 70 % |
| GoS | 0.1 % | 0.5 % | 5 % |

**Table 2:** Reference traffic for Scenario 8 (ETSI ETR 300-2 ed. 1, 1997) of TETRA.

| Type of service | Frequency of request (call/hour) | Call duration (sec) |
|---|---|---|
| M-M Individual call (Voice) | 0.324 (Poisson) | 10-30 (exp) |
| M-F Individual call (Voice) | 0.756 (Poisson) | 10-30 (exp) |
| F-M Individual call (Voice) | 0.756 (Poisson) | 10-30 (exp) |
| M-M Group call (Voice) | 0.36 (Poisson) | 10-30 (exp) |

**Table 3:** Optimum and suboptimum allocations with x + y + z = 17, for 1000 users. The circles show the unsatisfied GoSs

| (x,y,z). | (14,2,1) | (13,3,1) | (13,2,2) | (14,1,2) | (14,3,0) |
|---|---|---|---|---|---|
| $B_L$ (x,y,z) | 0.0444 | 0.0715 | 0.0712 | 0.0435 | 0.0446 |
| $B_M$ (x,y,z) | 0.0012 | 3.6492×10-4 | 0.0021 | 0.0069 | $1.8529 \times 10^{-4}$ |

| | | | | | |
|---|---|---|---|---|---|
| $B_H$ (x,y,z) | $3.0945 \times 10^{-5}$ | $9.5634 \times 10^{-6}$ | $1.5944 \times 10^{-6}$ | $5.1528 \times 10^{-6}$ | $1.8529 \times 10_{-4}$ |

As $z$ channels are only used by high priority users, they have the minimum level of utilization (compared to x and y channels). Therefore, minimizing $z$ increases the spectral efficiency of the system. Fig. 7 shows that for example, if the acceptable GoS for high priority users is modified from 1E-5 to 1E-4, we can decrease $z$ without affecting other performance metrics. This shows the trade-off between the GoS requirements and channel utilization of the system.

***Conclusion:***

We have proposed a channel allocation algorithm for TETRA networks in which some channels are reserved for high priority users. The channel allocation process is modeled by a one-dimensional Markov chain and blocking probability for users with different levels of priority is obtained. Blocking probabilities in reserving method have some interesting property that can be used in channel allocation algorithm. The proposed algorithm is applied on a realistic traffic for TETRA networks and the minimum number of required channels, in order to provide all users' GoSs, is found. Considering the large cells and the scarcity of the available channels in TETRA networks, the proposed algorithm can effectively increase the utilization of channels by minimizing the number of reserved channels.

***Appendix: Proof of BP Properties:***

Since $P_0$ are common in all BPs, we first prove the following inequalities:

$$P_0(x+1, y, z) < P_0(x, y+1, z) < P_0(x, y, z+1) < P_0(x, y, z)$$

***Proof of:***

$$P_0(x, y, z+1) = \left[ \sum_{n=0}^{x} \frac{a^n}{n!} + a^x \sum_{n=1}^{y} \frac{b^n}{(n+x)!} + a^x b^y \sum_{n=1}^{z+1} \frac{c^n}{(n+x+y)!} \right]^{-1}$$

$$= \left[ \sum_{n=0}^{x} \frac{a^n}{n!} + a^x \sum_{n=1}^{y} \frac{b^n}{(n+x)!} + a^x b^y \sum_{n=1}^{z} \frac{c^n}{(n+x+y)!} + \frac{a^x b^y c^{z+1}}{(z+x+y+1)!} \right]^{-1} = \left[ \frac{1}{P_0(x, y, z)} + K \right]^{-1}$$

$$\Rightarrow P_0(x, y, z+1) = P_0(x, y, z) \times \frac{1}{1 + K P_0(x, y, z)} \Rightarrow P_0(x, y, z+1) < P_0(x, y, z) \qquad \text{(A.1)}$$

where $K = \dfrac{a^x b^y c^{z+1}}{(z+x+y+1)!}$ and is positive.

***Proof of:***

$$P_0(x, y+1, z) < P_0(x, y, z+1)$$

$$P_0(x, y+1, z) = \left[ \sum_{n=0}^{x} \frac{a^n}{n!} + a^x \sum_{n=1}^{y+1} \frac{b^n}{(n+x)!} + a^x b^{y+1} \sum_{n=1}^{z} \frac{c^n}{(n+x+y+1)!} \right]^{-1}$$

$$=\left[\sum_{n=0}^{x}\frac{a^n}{n!}+a^x\sum_{n=1}^{y}\frac{b^n}{(n+x)!}+\frac{a^x b^{y+1}}{(x+y+1)!}+a^x b^{y+1}\sum_{n=1}^{z}\frac{c^n}{(n+x+y+1)!}\right]^{-1}$$

$$=\left[\sum_{n=0}^{x}\frac{a^n}{n!}+a^x\sum_{n=1}^{y}\frac{b^n}{(n+x)!}+a^x b^{y+1}\left(\frac{1}{(x+y+1)!}+\frac{c}{(x+y+2)!}+\ldots+\frac{c^z}{(x+y+z+1)!}\right)\right]^{-1}$$

$$=[K_1+bK_2]^{-1} \tag{A.2}$$

where $K_1=\sum_{n=0}^{x}\frac{a^n}{n!}+a^x\sum_{n=1}^{y}\frac{b^n}{(n+x)!}$, $K_2=a^x b^y\left(\frac{1}{(x+y+1)!}+\frac{c}{(x+y+2)!}+\ldots+\frac{c^z}{(x+y+z+1)!}\right)$

$$P_0(x,y,z+1)=\left[\sum_{n=0}^{x}\frac{a^n}{n!}+a^x\sum_{n=1}^{y}\frac{b^n}{(n+x)!}+a^x b^y\sum_{n=1}^{z+1}\frac{c^n}{(n+x+y)!}\right]^{-1}$$

$$=\left[\sum_{n=0}^{x}\frac{a^n}{n!}+a^x\sum_{n=1}^{y}\frac{b^n}{(n+x)!}+a^x b^y\left(\frac{c}{(x+y+1)!}+\frac{c^2}{(x+y+2)!}+\ldots+\frac{c^{z+1}}{(x+y+z+1)!}\right)\right]^{-1}$$

$$=\left[\sum_{n=0}^{x}\frac{a^n}{n!}+a^x\sum_{n=1}^{y}\frac{b^n}{(n+x)!}+a^x b^y c\left(\frac{1}{(x+y+1)!}+\frac{c}{(x+y+2)!}+\ldots+\frac{c^z}{(x+y+z+1)!}\right)\right]^{-1}$$

$$=\left[K_1+cK_2\right]^{-1} \tag{A.3}$$

By using (A.2) and (A.3) we can conclude:

$\frac{P_0(x,y+1,z)}{P_0(x,y,z+1)}=\frac{K_1+cK_2}{K_1+bK_2}<1$. Since $b<c$, therefore the inequality is proved.

Proof of $P_0(x+1,y,z)<P_0(x,y+1,z)$ is similar to .$P_0(x,y+1,z)<P_0(x,y,z+1)$

***Proof of Property 1:***
***Proof of:***

$B_H(x,y,z+1)<B_H(x,y+1,z)$

$$B_H(x,y,z+1)=\frac{a^x b^y c^{z+1}}{(x+y+z+1)!}P_0(x,y,z+1) \tag{A.4}$$

$$B_H(x,y+1,z)=\frac{a^x b^{y+1} c^{z}}{(x+y+z+1)!}P_0(x,y+1,z) \tag{A.5}$$

By using (A.4) and (A.5) the inequality is improved as follow:

$$\frac{B_H(x,y,z+1)}{B_H(x,y+1,z)}=\frac{P_0(x,y,z+1)}{P_0(x,y+1,z)}\times\frac{c}{b}=\frac{K_1+bK_2}{K_1+cK_2}\times\frac{c}{b}=\frac{cK_1+bcK_2}{bK_1+bcK_2}<1 \tag{A.6}$$

Since $b<c$, therefore the inequality is proved.

***Proof of:***

$$B_H(x, y+1, z) < B_H(x+1, y, z)$$

$$B_H(x, y+1, z) = \frac{a^x b^{y+1} c^z}{(x+y+z+1)!} P_0(x, y+1, z) \quad \text{(A.7)}$$

$$B_H(x+1, y, z) = \frac{a^{x+1} b^y c^z}{(x+y+z+1)!} P_0(x+1, y, z) \quad \text{(A.8)}$$

$$\Rightarrow \frac{B_H(x, y+1, z)}{B_H(x+1, y, z)} = \frac{P_0(x, y+1, z)}{P_0(x+1, y, z)} \times \frac{b}{a} = \frac{K_3 + aK_4}{K_3 + bK_4} \times \frac{b}{a} = \frac{bK_3 + abK_4}{aK_3 + abK_4} < 1 \quad \text{(A.9)}$$

By expanding $P_0(x+1, y, z)$ or $P_0(x, y+1, z)$, $K_3$ and $K_4$ are obtained, where: $K_3 = \sum_{n=0}^{x} \frac{a^n}{n!}$

$K_4 = a^x \sum_{n=0}^{y} \frac{b^n}{(n+x+1)} + a^x b^y \sum_{n=1}^{z} \frac{c^n}{(n+x+y+1)}$. Since $a<b$, therefore the inequality is proved.

***Proof of:***

$$B_H(x, y, z) = \frac{a^x b^y c^z}{(x+y+z)!} P_0(x, y, z) \quad \text{(A.10)}$$

$$B_H(x+1, y, z) = \frac{a^{x+1} b^y c^z}{(x+y+z+1)!} P_0(x+1, y, z) \quad \text{(A.11)}$$

$$\frac{B_H(x+1, y, z)}{B_H(x, y, z)} = \frac{P_0(x+1, y, z)}{P_0(x, y, z)} \times \frac{a}{x+y+z+1} < 1 \quad \text{(A.12)}$$

The inequality will be right if $a > x + y + z + 1$. The initial value of $x$ must be larger than $a$ in order for $B_L)x,y,z($ to be smaller than the given GoS.

***Proof of Property 2:***

we prove just $B_M(x, y, z) < B_M(x, y, z+1)$. Other inequalities are similar.

If we suppose, $B_M(x, y, z) = \frac{Num}{Den}$ so we find that:

$$B_M(x, y, z+1) = \frac{Num + \frac{a^x b^y c^{z+1}}{(x+y+z+1)!}}{Den + \frac{a^x b^y c^{z+1}}{(x+y+z+1)!}} > \frac{Num}{Den} = B_M(x, y, z) \quad \text{(A.13)}$$

The above inequality is proved because adding the same positive term $\frac{a^x b^y c^{z+1}}{(x+y+z+1)!}$ to both numerator and denominator leads to a greater amount of fraction.

The proof of property3 is similar to other properties.

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