Australian Journal of Basic and Applied Sciences, 5(5): 35-40, 2011
ISSN 1991-8178

Mathematical Analysis of Fluid Flow through Channels with Slip Boundary

[1]Suharsono, [2]Y. H. Wu, [3]B. Wiwatanapataphee

[1]Dept. of Mathematics & Statistics Curtin University of Technology Perth WA 6845, Australia
Dept. of Mathematics, Lampung University, Indonesia
[2]Dept. of Mathematics & Statistics Curtin University of Technology Perth WA 6845, Australia
[3]Dept. of Mathematics Faculty of Science Mahidol University Bangkok 10400 Thailand

Abstract: In this paper, we present analytical solutions for transient flow of Newtonian fluid through micro channels with Navier slip boundary. The derivation of the solutions is based on Fourier series expansion in space. We then investigate the influence of the slip parameters on the phenomena of the transient flow and show the transient pressure field and velocity profile at various instants of time.

Key words: component; transient flow; slip boundary; rectangular micro channels.

INTRODUCTION

Over the last ten years, one of the important research focuses worldwide has been on the study of materials at micro and nanoscales (Wiwatanapataphee, *et al.,* 2009; Wu, *et al.,* 2008). The advances in this area has led to the development of many engineering devices and systems in microscale and nanoscale Kuo *et al.,* (2003). These devices and systems usually involve fluid flow through microchannels, referred to as microflows (Bourlon *et al.,* 2007; Ho and Tai, 1998; Herwig, O. Hausner, 2003; Huang *et al.,* 2007; Wiwatanapataphee, 2009; Wu, 2008). Application examples include drug delivery systems Su and Lin, (2004) biological sensing and energy conversion devices Nakane *et al.,* (2005). As the functional characteristics of the system depends on the flow behaviour of fluid in the system, the study of microflows is attracting more and more attention from the research communities in order to derive a better understanding of the mechanism of microflows and develop better models (Gad-el-Hak, 1999; Wiwatanapataphee *et al.,* 2009; Wu and Wiwatanapataphee, 2008).

The governing field equations for the flow of incompressible Newtonian fluids are the incompressible continuity equation and the Navier–Stokes equations. In addition, a boundary condition has to be imposed on the field equations. The no-slip condition is usually used. However it is a hypothesis rather than a condition deduced from physics principle, and thus its validity has been continuously debated in the scientific literature Wu and Wiwatanapataphee, (2008). Varies evidences of slip flow of a fluid on a solid surface have been reported. For example Chauveteau (1982), Tuinier and Taniguchi (2005), and Vargas and Manero, (1989) studied the flow of polymer solutions in porous media and showed that the apparent viscosity of the fluids near the wall is lower than that in the bulk and consequently the fluids can exhibit the phenomenon of apparent slip on the wall.

Various investigations have been made to study flow problems of Newtonian and non-Newtonian fluids with Navier slip boundary condition (Deshmukh, D.G. Vlachos, 2005; Lee *et al.,* 2007; Saidi, 2006; Sahu, *et al.,* 2007; Pascal, 2006; Donghyun and Parviz, 2007; Yousif and Melka, 1997). Some attempts have also been made to derive alternative formulae for the determination of the slip length Yang and Zhu, (2006). Although exact and numerical solutions to various flow problems of Newtonian fluids under the no-slip assumption have been obtained and are available in literature (Slattery, 1999; Wu, B. Wiwatanapataphee, 2007; Wiwatanapataphee, *et al.,* 2004; Wiwatanapataphee, *et al.,* 2004), very few exact solutions for the slip case are available in literature. Recently, some steady state and transient slip solutions for the flows through a pipe, a channel and an annulus have been obtained (Yang, K.Q. Zhu, 2006; Matthews and Hill, 2007). Recently, Wu et. al studied pressure gradient driven transient flows of incompressible Newtonian liquids in micro-annuals under a Navier slip boundary condition. They use Fourier series in time and Bessel functions in space to find out exact solutions Wu and Wiwatanapataphee, (2008). In this paper, we present a new result for the transient flow of Newtonian fluids in rectangular microtubes with a slip boundary condition. The rest of the paper is

Corresponding Author: Suharsono, Dept. of Mathematics & Statistics Curtin University of Technology Perth WA 6845, Australia Dept. of Mathematics, Lampung University, Indonesia
Email: suharsono.s@postgrad.curtin.edu.au

organized as follows. In the following section, we first define the problem and then present its mathematical formulation. In Section 3, we present the new solution for the velocity field and demonstrate its variation with time and its profile across the channel cross-section.

Ii. Problem Description and Mathematical Formulation:

We consider the flow of an incompressible Newtonian fluid through a rectangular micro tube with the z-axis being in the axial direction as shown in Figure 1. The differential equations governing the flow include the continuity equation and the Navier–Stokes equations as follows

$$\frac{\partial U_j}{\partial x_j} = 0\,, \tag{1}$$

$$\rho\left(\frac{\partial U_j}{\partial t} + U_i\frac{\partial U_j}{\partial x_i}\right) = -\frac{\partial p}{\partial x_j} + \mu\frac{\partial^2 U_i}{\partial x_i \partial x_i} + \rho g_j\,, \left(i=1,2,3;\; j=1,2,3\right) \tag{2}$$

where p and U_j are respectively the fluid pressure and velocity vector, g_j is the gravitational acceleration, ρ and μ are respectively the fluid density and viscosity and x_i denotes coordinates.

As the flow is axially symmetric, the velocity components in the x and y directions vanish, namely $U_1=U_x=0$ and $U_2=U_y=0$. Thus the continuity equation (1) becomes

$$\frac{\partial U_3}{\partial x_3} = \frac{\partial U_z}{\partial z} = 0,$$

which gives rise to $U_3 = v = v(x, y, t)$.

As the flow is horizontal, $g_3 = g_z = 0$, and hence Eq. (2) becomes

$$\rho\left(\frac{\partial v}{\partial t}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial z}$$

In this work, we consider the fluid flow driven by the pressure field with a pressure gradient *q(t)* which can be expressed by a Fourier series, namely

$$\frac{\partial p}{\partial z} = q(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right] \tag{3}$$

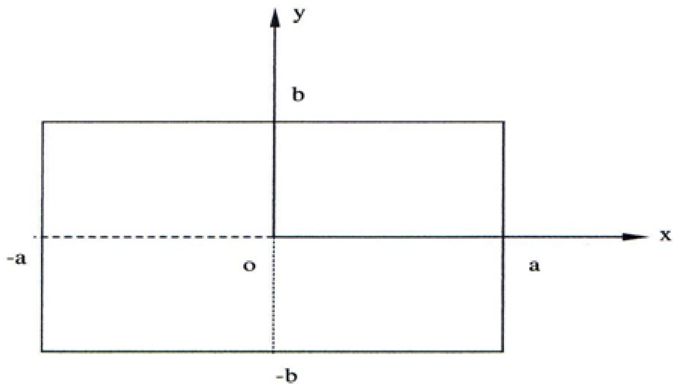

Fig. 1: The cross section of the channel and coordinate system

As the problem is axially symmetric, we only need to consider a quadrant of the cross-section in the computation.

By applying the Navier slip conditions in the first quadrant of the rectangular cross section, as in the paper by Wu et al., (2008) and Duan & Murychka (2007), for every time t, we have

$$\frac{\partial v}{\partial y}(x,0)=0; \quad 0\le x\le a$$

$$\frac{\partial v}{\partial x}(0,y)=0; \quad 0\le y\le b$$

$$v(x,b)+l\frac{\partial v}{\partial y}(x,b)=0; \quad 0\le x\le a$$

$$v(a,y)+l\frac{\partial v}{\partial x}(a,y)=0; \quad 0\le y\le b \tag{4}$$

Iii. Exact Solution for the Transient Velocity Field:

Consider the unsteady Navier Stokes equation

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\frac{\rho}{\mu}\frac{\partial v}{\partial t}=\frac{1}{\mu}\frac{\partial p}{\partial z} \tag{5}$$

If v_n is the solution of (5) for $\frac{\partial p}{\partial z}=c_n e^{in\omega t}$, then the complete solution of (5) for $\frac{\partial p}{\partial z}=\mathrm{Re}\sum_{n=1} c_n e^{in\omega t}=\sum_{n=1} a_n\cos(n\omega t)+b_n\sin(n\omega t)$

is $v=\sum_{n=1}\mathrm{Re}(v_n)$,

Through lengthy derivation, we obtain the following exact solution for Eq. (5) subject to conditions (4):

$$v(x,y,t)=\sum_{n=1}^{\infty}\mathrm{Re}[e^{in\omega t}(A_n\cosh(\gamma_n x)\cos(k_n y)+B_n\cos(\alpha_n x)\cosh(\beta_n y)-\frac{c_n}{in\omega\rho})]$$

where

$$\gamma_n=\sqrt{k_n^{\ 2}+iq_n},\ q_n=\frac{n\omega\rho}{\mu}$$

$$\beta_n=\sqrt{\alpha_n^{\ 2}+iq_n}$$

$$A_n=\frac{2}{b}\int_0^b \mathrm{Re}\left[\frac{c_U\cos(k_n y)}{\cosh(\gamma_n a)+l\,\gamma_n\sinh(\gamma_n a)}\right]dy$$

$$B_n=\frac{2}{a}\int_0^a \mathrm{Re}\left[\frac{c_V\cos(\alpha_n x)}{\cosh(\beta_n b)+l\,\beta_n\sinh(\beta_n b)}\right]dx$$

$$c_U=c_V=\sum_{n=1}^{\infty}\mathrm{Re}\left(\frac{c_n}{in\omega\rho}\right)$$

To demonstrate the velocity field, we present results for the case $\frac{\partial p}{\partial z}=b_1\sin(\omega t)$ with $b_1=5$, $a=b=1$ and $l=0.1$, ω=0.2 for μ=1.1 and ρ= 30.

Fig.2 shows the variation of the driving pressure gradient in one cycle. Fig.3 shows the variation of axial velocity along the x-axis at varies instants of times.

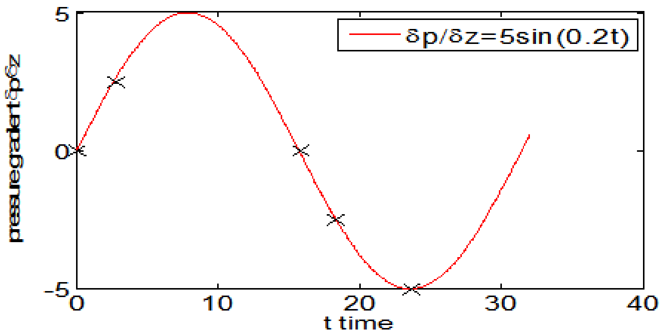

Fig. 2: Pressure gradient driving the flow of the fluid

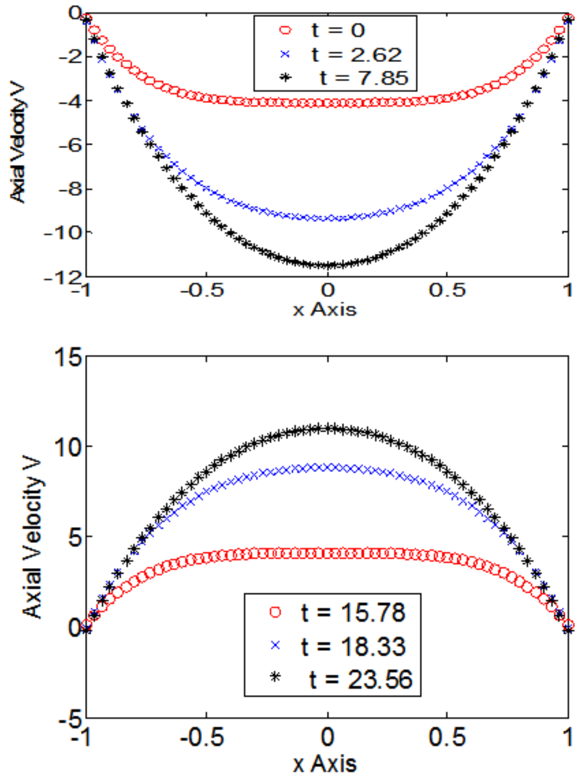

Fig. 3: Variation of velocity profile along the x-axis at varies instants of times during one cycle.

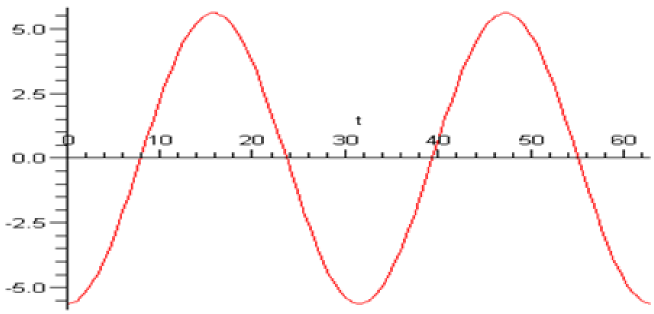

Fig. 4: Variation of axial velocity with time at the point (x,y) = (0,0)

Conclusion:

In this paper we present an exact solution for transient flow of incompressible Newtonian fluid in rectangular microtubes with a Navier slip condition on the boundary. We also show the velocity profile in the cross section for the case of $\frac{\partial p}{\partial z} = 5\sin(0.2t)$.

REFERENCES

Bourlon, B., J. Wong, C. Miko, 2007. Nanoscale probe for fluidic and ionic transport, Nature Nanotechnology, 2(2): 104.

Chauveteau, G., 1982. Rodlike polymer solution flow through fine pores: influence of pore size on rheological behavior, Journal of Rhelogy, 26(2): 111.

Deshmukh, S.R., D.G. Vlachos, 2005. CFD simulations of coupled, countercurrent combustor/reformer microdevices for hydrogen production, Industrial & Engineering Chemistry Research, 44(14): 4982.

Donghyun, Y., M. Parviz, 2007. Effects of hydrophobic surfaces on the drag and lift of a circular cylinder, Physics of Fluids, 19: 081701.

De, L., Vargas, O. Manero, 1989. On the slip phenomenon of polymeric solutions through capillaries, Polymer Engineering and Science, 29(18): 1232.

Duan, Z., Y.S. Muzyschka, 2007. Slip flow in non-circular microchannels. Microfluid Nanofluid, 3: 473-484.

Gad-el-Hak, M., 1999. The fluid mechanics of microdevices—The Freeman scholar lecture, Journal of Fluids Engineering-Transactions of the ASME, 121(1): 5.

Herwig, H., O. Hausner, 2003. Critical view on new results in micro-fluid mechanics: An example, International Journal of Heat and Mass Transfer., 46(5): 935.

Ho, C.M., Y.C. Tai, 19983. Micro-electro-mechanical systems (MEMS) and fluid flows, Annual Review of Fluid Mechanics, 30: 579.

Huang, H., T.S. Lee, C. Shu, 2007. Lattice Boltzmann method simulation gas slip flow in long microtubes, International Journal of Numerical Methods for Heat & Fluid Flow., 17(5-6): 587.

Kuo, T.C., D.M. Cannon, M.A. Shannon, P.W. Bohn, J.V. Sweedler, 2003. Hybrid three-dimensional nanofluidic/microfluidic devices using molecular gates, Sensors and Actuators, A 102: 223.

Lee, H.B., I.W. Yeo, K.K. Lee, 2007. Water flow and slip on NAPL-wetted surfaces of a parallel-walled fracture - art. no. L19401, Geophysical Research Letters, 34(19): 19401.

Matthews, M.T., J.M. Hill, 2007. Newtonian flow with nonlinear Navier boundary condition, Acta Mechanica, 191(3–4): 195.

Nakane, J.J., M. Akeson, A. Marziali, 2005. Nanopores sensors for nucleic acid analysis, Journal of Physics Condensed Matter, 15: R1365.

Pascal, J.P., 2006. Instability of power-law fluid flow down a porous incline, Journal of Non-Newtonian Fluid Mechanics, 133(2–3): 109.

Sahu, K.C., P. Valluri, P.D.M. Spelt, O.K. Matar, 2007. Linear instability of pressure-driven channel flow of a Newtonian and a Herschel–Bulkley fluid, Physics of Fluids, 19: 122101.

Saidi, F., 2006. Non-Newtonian flow in a thin film with boundary conditions of Coulomb's type, Zamm-Zeitschrift Fur Angewandte Mathematik Und Mechanik, 86(9): 702.

Slattery, J.C., 1999. Advanced Transport Phenomena, Cambridge University Press.

Su, Y.C., L.W. Lin, 2004. A water-powered micro drug delivery system, Journal of Microelectromechanical Systems, 13(1): 75.

Thompson, P.A., S.M. Troian, 1997. A general boundary condition for liquid flow at solid surfaces, Nature, 389: 360.

Tuinier, R., T. Taniguchi, 2005. Polymer depletion-induced slip near an interface, Journal of Physics Condensed Matter, 17: L9.

Wiwatanapataphee, B., Y.H. Wu, J. Archapitak, P.F. Siew, B. Unyong, 2004. A numerical study of the turbulent flow of molten steel in a domain with a phase change boundary, Journal of Computational and Applied Mathematics, 166: 307.

Wiwatanapataphee, B., D. Poltem, Y.H. Wu, Y. Lenbury, 2006. Simulation of pulsatile flow of blood in stenosed coronary artery bypass with graft, Mathematical Biosciences and Engineering, 3(2): 371.

Wiwatanapataphee, B., Y.H. Wu, K. Maobin Hu, A. Chayantrakom, 2009. study of transient flows of Newtonian fluids through micro-annuals with slip boundary. J. Phys. A: Math. Theor., 42: 065206.

Wu, Y.H., B. Wiwatanapataphee, 2007. Modelling of turbulent flow and multi-phase heat transfer under electromagnetic force, Discrete and Continuous Dynamical Systems-Series B 8(3): 695.

Wu, Y.H., B. Wiwatanapataphee, 2008. Maobin Hu, Pressure-driven transient flows of Newtonian fluids through microtubes with slip boundary, Physica A. 387: 5979-5990.

Yang, S.P., K.Q. Zhu, 2006. Analytical solutions for squeeze flow of Bingham fluid with Navier slip condition, Journal of Non-Newtonian Fluid Mechanics, 138(2–3): 173.

Yousif, H.A., R. Melka, 1997. Bessel function of the first kind with complex argument, Computer Physics Communications, 106: 199.