

Investigation the Effects of Air Hole Deformation on the Total Dispersion of Photonic Crystal Fiber

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Abstract: Finite difference time domain method has been used to analyze and simulate optical response of the photonic crystal fibers. The simulation method has taken account the material dispersion in a fiber with solid core. The effect of air hole deformation on total dispersion diagram has been also studied. The result of theoretical simulation was employed to propose a holy fiber with a flattened dispersion behavior.

Key words: Finite-Difference Time Domain method (FDTD), waveguide and material dispersion, Photonic Crystal Fiber (PCF).

INTRODUCTION

Photonic crystal fibers are fibers with internal alternating structure made of holes filled with air and ending up with a hexagonal network. Light could be emitted along defects in fiber crystal structure. The defect is made by elimination of one or more central holes. A combination of optical fiber and photonic crystals properties make unique properties which are not available in conventional fibers. Freedom in design creates fibers which are single mode in all optical ranges and there is no definite cut off wavelength for them (John D. Joannopoulos, 2008). By manipulating the structure, it is possible to create optimal dispersion properties in fibers, and PCFs which have zero or insignificant dispersion along the visible wavelength could be designed and made. If the structure defect is made by eliminating central hole, the conduction of an electromagnetic wave in photonic crystal fiber could be made based on modified total internal reflection mechanism. The modification caused by air holes network that provide the influence of higher modes until just one main mode is moved (John D. Joannopoulos, 2008). There are varieties of numerical methods in photonic crystal analysis, each with some advantages and disadvantages. In general, numerical methods could be divided into two general groups, numerical methods in time domain and frequency domain. Among the methods in time domain, we could mention finite difference method in time domain (Taflove, 2005) and finite elements in time. If we are going to find dispersion or variance, and diffraction in a special wavelength, we could apply frequency domain methods designed and expanded for this purpose, some of which are as follows: expansion of plane wave (PWB) (Ferrando, 1999), finite differences in frequency domain (FDFD) (Qiu, 2001), Wannier function method (WFM), finite elements (FEM) method in frequency domain (Brecht, 2000), multi-pole method (MMP) and approximate methods such as beam propagation method (BPM) (Mitchell, 1980). In the analysis made in this article, FDTD method has been used, which is one of the best and well-known methods and is applied in most writings as a reference for comparison and reliance on other methods. In calculations, material dispersion has been also added to relations using sellmeier relation (Jiang, 2006). The applied boundary condition is CPML which is a convolutional form of perfect match layer absorption (Taflove, 2005). In the first chapter, an abstract of calculation methods is presented, in the second chapter a general structure with changes in air hole and lattice constant and also suggested some structures for reaching the dispersion of almost zero and flat dispersion are analyzed, and finally the conclusion of the obtained results is presented.

Numerical Method and Relations:

One of the FDTD problems, except it need so long time, is that the wavelength dependence of refractive index don't coming in our calculation. On the other hand, in most cases Maxwell equations are excluded from J parameter that is polarization current density.

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$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \tag{1}$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J} \tag{2}$$

The total dispersion consists of material and waveguide dispersion by this type of analysis. Hence, waveguide dispersion is obtained at first, and finally in order to show total dispersion, material dispersion is calculated separately and add up the dispersion obtained from the analysis (Shuqin, 2005).

Material dispersion could be accounted in different wavelength using a relation called sellmeire (Malitson, 1965). For PCFs which are fused silica type and in the range of wavelength desirable for them (0.5~2µm), this relation is simplified to two resonant frequencies without losing any accuracy (Jiang, 2006).

$$\epsilon_r(\omega) = n(\omega)^2 = 1 + \sum_{k=1}^2 \frac{b_k \omega_k^2}{\omega_k^2 - \omega^2} \tag{3}$$

With using J parameter and considering it dependence to refractive index and electrical field, sellmeier relation could be inserted into Maxwell equation, so that the defect suggested in FDTD analysis would be obviated (Jiang, 2006).

$$J(\omega) = i\omega P(\omega) = i\omega \epsilon_0 \chi(\omega) E(\omega) \tag{4}$$

$$\chi(\omega) = \epsilon_r(\omega) - 1 \tag{5}$$

Using the above mentioned points and also applying Yee-mesh, the relations could be discretized in time and place (Taflove, 2005). Suppose that the light emission direction is parallel with z axel, since there are no changes in this direction in Maxwell equations, z derivatives which are dependent to $exp(-i\beta z)$ (β is propagation constant) could be replaced by $-i\beta$. 3-D hybrid fields related to fibers conduction mode could be just calculated by 2-D method (Asi, 1992). For example, electric field parallel with x could be written as follows:

$$E_x \Big|_{m,n}^{l+1} = E_x \Big|_{m,n}^l + \frac{\Delta t}{\epsilon_0} \left[\frac{H_z \Big|_{m,n}^{l+1/2} - H_z \Big|_{m,n-1}^{l+1/2}}{\Delta y} + i\beta H_y \Big|_{m,n}^{l+1/2} - \sum_{k=1}^2 (J_{k,x}^+ \Big|_{m,n}^{l+1/2} + J_{k,x}^- \Big|_{m,n}^{l+1/2}) \right] \tag{6}$$

Here, index l shows the discrete time step, and m, n indexes show discrete points series in x - y plates. t is time increment and x and y are space increment respectively along x, y directions.

The remaining equations for other magnetic and electric field components can be accounted in the same way (Jiang, 2006). J components should be also made discrete in the same manner noting that they are dependent to electric fields based on relation (4).

$$J(\omega) = \sum_{k=1}^2 \left[J_k^+(\omega) + J_k^-(\omega) \right] \tag{7}$$

$$J_k^\pm(\omega) = -\frac{i\varepsilon_0}{2} b_k \omega_k^2 \frac{1}{\omega \pm \omega_k} E(\omega) \tag{8}$$

For example for x direction, we would have:

$$J_{k,x}^+ \Big|_{m,n}^{l+1/2} = \frac{1-i\Delta t\omega_k/2}{1+i\Delta t\omega_k/2} J_{k,x}^+ \Big|_{m,n}^{l-1/2} + \frac{\varepsilon_0 b_{k,x} \Big|_{m,n} \Delta t\omega_k^2/2}{1+i\Delta t\omega_k/2} E_x \Big|_{m,n}^l \tag{9}$$

$$J_{k,x}^- \Big|_{m,n}^{l+1/2} = \frac{1+i\Delta t\omega_k/2}{1-i\Delta t\omega_k/2} J_{k,x}^- \Big|_{m,n}^{l-1/2} + \frac{\varepsilon_0 b_{k,x} \Big|_{m,n} \Delta t\omega_k^2/2}{1-i\Delta t\omega_k/2} E_x \Big|_{m,n}^l \tag{10}$$

Other components in the other directions can be accounted with the same method (Jiang, 2006). Another fundamental problem of the nature FDTD algorithm which is recursive is that the dispersed or radiated fields are reflected into response region when reaching environment bounded. In order to remove this problem, the best solution is to use an absorption layer or Perfect Match Layer (PML) (Taflove, 2005), to absorb waves reaches the bounded and do not reflect them. PML relations usually applied failed to absorb evanescent waves (Taflove, 2005). Therefore, PML should be enough away from obstacle made by these evanescent waves until the effects of these waves disappeared. This method also faces problem in simulations with long time duration or when the considered structure is long and thin (Teixeira, 1999). Since we intend to analyze a PCF and calculate the material dispersion, we need a bounded method in which the long time calculations and the thin structure would not have any effect on its operation. Therefore, in simulation, a border method which is a convolutional form of PML method has been applied. This method is called CPML (Convolutional Perfect Match Layer), and the basis of which is the same PML, that its indexes has been convoluted and shows higher precision against evanescent waves and is suitable for applications the purpose of which is to obtain ranges by propagated materials. The details of relations and how to use this method has been brought in reference (Taflove, 2005).

Numerical Results:

At the beginning a PCF is considered based on Fig.(1) and has been studied and analyzed through the FDTD method which was previously described. As is shown in the structure, the number of five rings was selected as the air-hole rings. In our analysis, we have considered the increasing space in the direction of x equal to $\Delta x=A/20$, and presume that always $\Delta x=\Delta y$. The value of time step of Δt is also calculated from the following relation in order to make the algorithm stable:

$$\Delta t \leq 0.1/(c\sqrt{1/(\Delta x)^2 + 1/(\Delta y)^2 + (\beta^2/4)}) \tag{11}$$

To show the dispersion changes with the diameter changes of air-holes, the analysis of lattice constants of 1 μ m, 2 μ m and 3 μ m has been done with the proportions of diameter of d/A equal to 0.7 and 0.9, with the

transit of 30000 temporal steps and regarding fifty field sample points in different times and was carried to the frequency area by Fourier transform. The peaks of the spectral intensity correspond to the locations of the eigen frequencies of PCF modes. The first peak corresponds to the fundamental mode, which we are most concerned about. By starting up the analysis in the different dispersions and using the equation of $n_{eff} = \beta/k_0$, the refractive index of the fundamental mode is calculated. The value of dispersion would be simply calculated through this parameter based on the following relation:

$$D = -\frac{\lambda}{c} \frac{\partial^2 \text{Re}(n_{eff})}{\partial \lambda^2} \tag{12}$$

The results are shown in Figs. (2) and (3). As is shown in these figures, the air-holes getting smaller and the diagram is getting closer to the material dispersion which is resulted from the Sellmeier relation. This fact is quite natural with respect to the huge volume occupied by the fused silica. The results are in accordance with the (Koshiba, 2004) taken from FEM method.

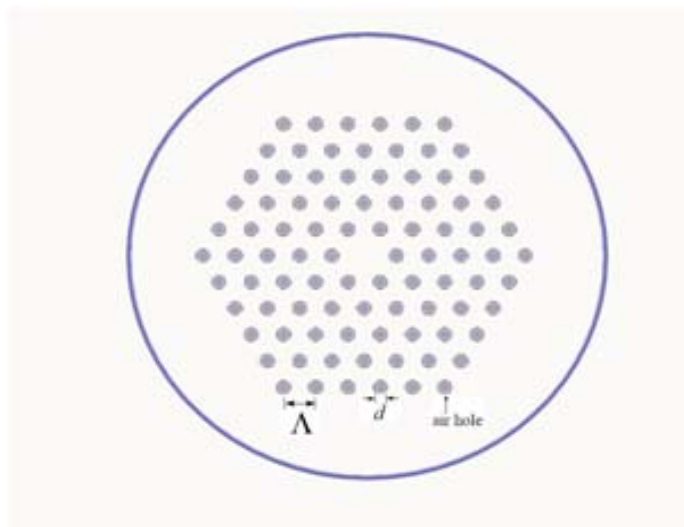


Fig. 1: PCF structure with solid core.

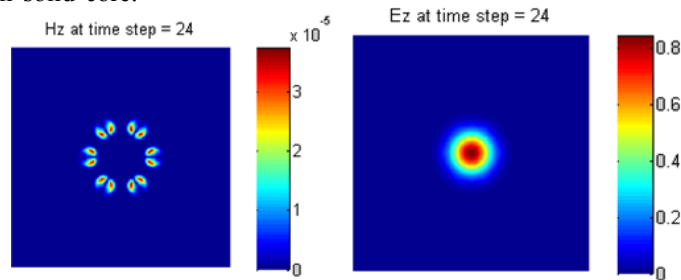


Fig. 2: Electric and magnetic field in z direction.

The dimension changes of even one hole can affect the fiber particularities which lead to the problems in the making process. According to the researches, changes in the internal holes cause to the more serious changes in our respective output. In order to fully analyze, a photonic crystal fiber with five rings, a lattice constants of $\Lambda = 2.3 \mu m$ with the diameter of holes equal to 0.3Λ and the refractive index of the fundamental mode was calculated in different wavelength. Changes in one hole in away that we considered as an oval with the maximum diameter of 0.3Λ and the minimum of 0.1Λ , was occurred. For one time, this change was replaced for a first-row hole, the other time for a second-row holes and finally for a third-row one (Fig. 4). As is shown in Fig. 5, the most of the changes is related to the status in which the form of the hole is in the first ring and the least change is related to the spot that the change of form has been occurred in the third ring. The form change occurred is often due to the non-uniform distribution of heat throughout the making process

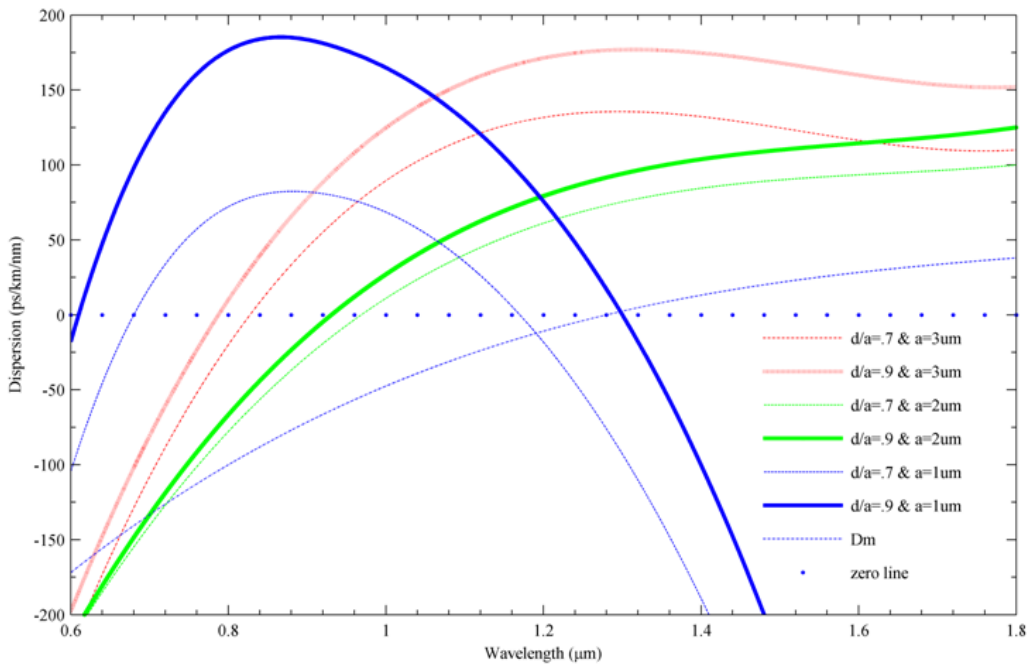


Fig. 3: Dispersion diagram for several structure.

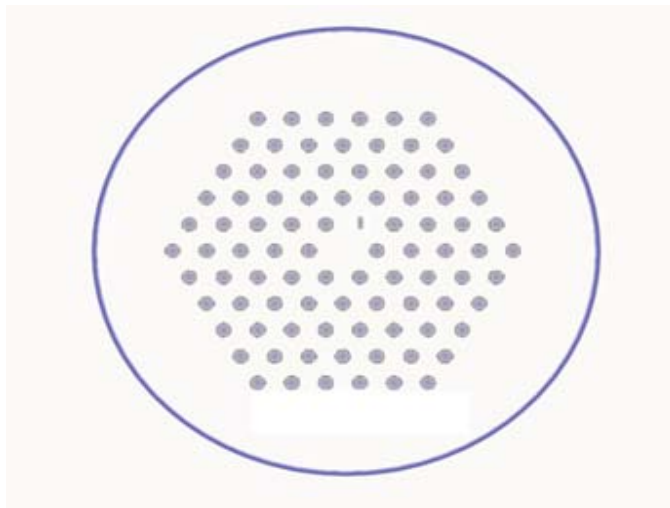


Fig. 4: Air hole deformation in first ring.

which usually happens in the outer holes. With the least change in the diagonal of holes and the lattice constants in different rings of a photon crystal fiber, we can make lots of changes in the dispersion diagram and the other particularities of fiber. Reaching to a flat and almost zero dispersion, is one of the important aims which can not be occurred with the changes in all of the holes.

The structures investigated so far, air holes diameter were the same in all of the different rings. Now consider a structure in which the length of the holes in each row is different from the other rows such as Fig. (6-1). A structure is presented here which a lattice constants is equal to $\Lambda=1.56\mu\text{m}$, and the air hole diameters of the first, second, third and four rings are as follows: $d_1/\Lambda=0.32$, $d_2/\Lambda=0.45$, $d_3/\Lambda=0.67$ and $d_4/\Lambda=0.95$. Also, in Fig (6-2), a photonic crystal fiber is shown with five rings. In this structure, the lattice constant is $\Lambda = 1.58\mu\text{m}$, and the air hole diameters of first, second, third, fourth and fifth rings are as follows: $d_1/\Lambda=0.31$, $d_2/\Lambda=0.45$, $d_3/\Lambda=0.55$, $d_4/\Lambda=0.63$ and $d_5/\Lambda=0.95$. The dispersion diagram of these two structures based on the wavelength is shown in Fig (7). The dispersion diagram is very close to zero in the area of wavelength of 1.3 to $1.75\mu\text{m}$. Fig (6-2) which has more rings possesses a dispersion diagram close to zero.

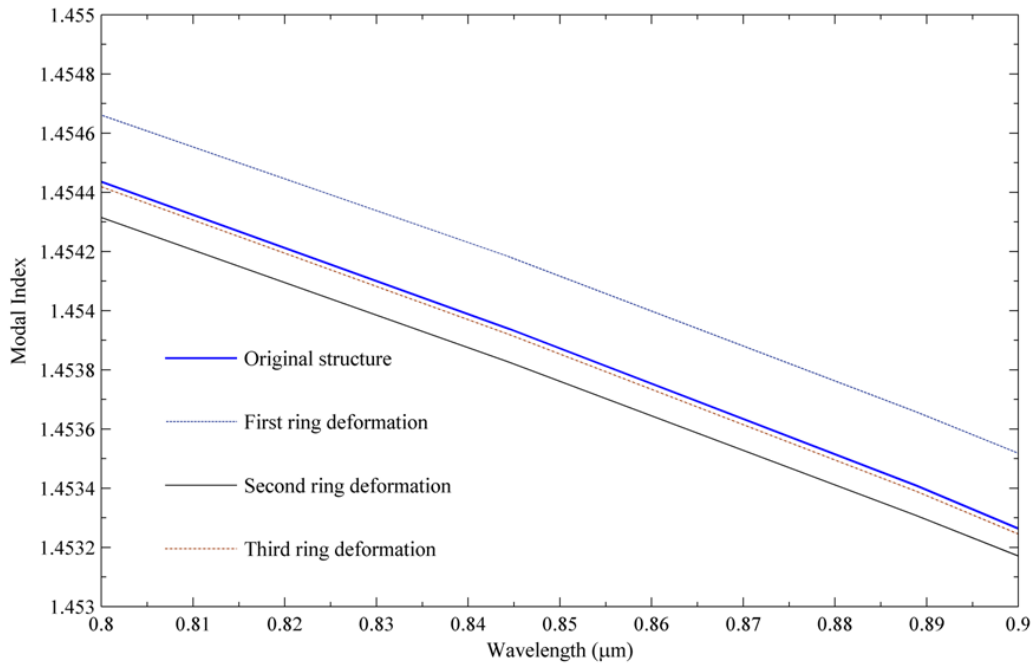


Fig. 5: Modal index diagram for air hole deformation in different ring.

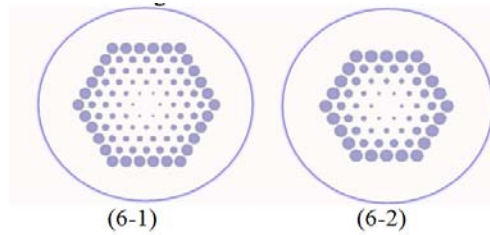


Fig. 6: Photonic crystal fiber with different air hole diameters in each ring, (a)with four ring, (b)with five ring

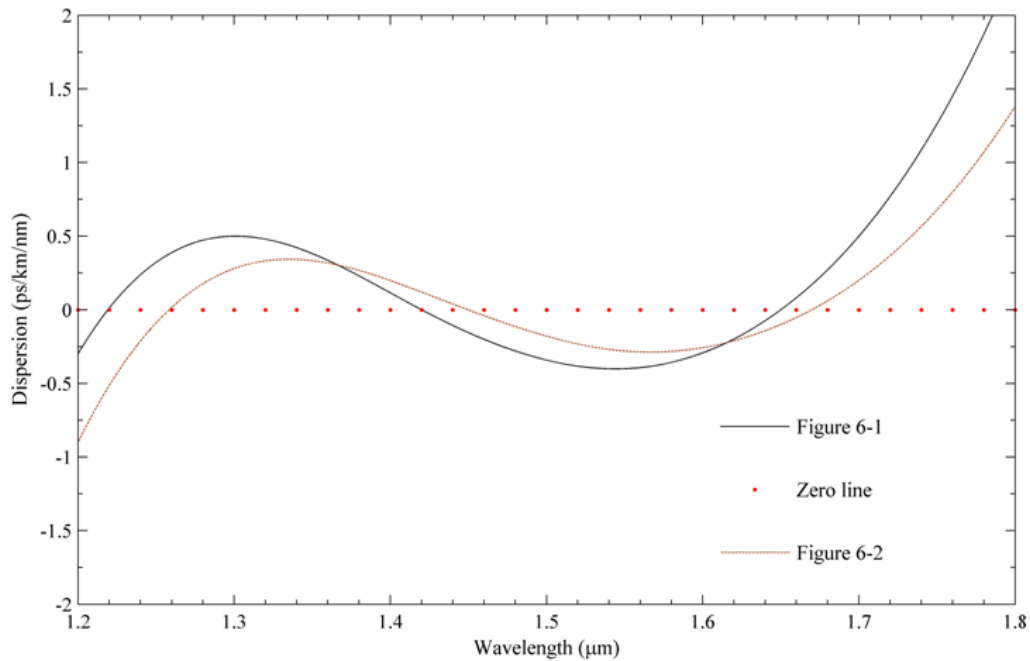


Fig. 7: Dispersion diagram, solid line for (6-1), dash line for (6-2).

In the structure we studied so far, all the holes were in the form of circle and the variable parameter was the diameter of these circles. Now we suggest the structures in which the elliptical has been used and we simulate these structures. This structure with the lattice constant of $\Lambda=2.3\mu\text{m}$ and the diameter of $d=1.8\mu\text{m}$ are for the first ring $a=0.55\mu\text{m}$ and $b=0.7\mu\text{m}$ and for the second and third ring $a=1.049\mu\text{m}$ and $b=1.335\mu\text{m}$. As is shown in figure (8-2), in order to investigate the effect of the rings, a structure similar to the Fig (8-1) with two additional rings has been considered. In this structure, in the interval of wavelength, the dispersion is reported to be flat and near to zero. Also, structure with more number of rings has shown a better result.

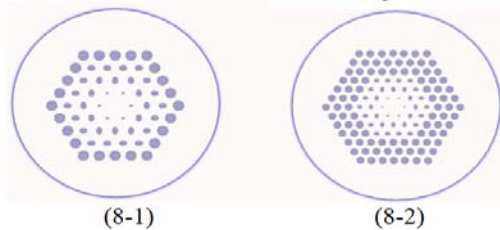


Fig. 8: Photonic crystal fiber with different air hole diameters and elliptical holes in each ring, (1)with four ring, (2)with six ring

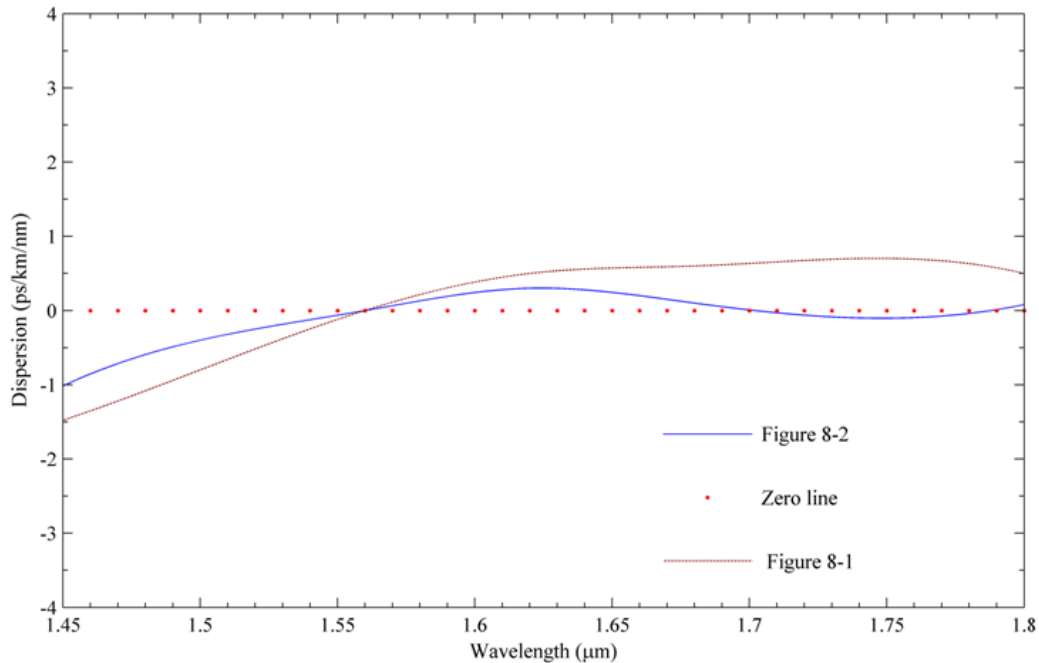


Fig. 9: Dispersion diagram, dash line for (8-1), solid line for (8-2)

Conclusion:

In this article, we have seen the stimulation of photonic crystal fiber and the total dispersion diagram are shown by the valid and precise numerical method of FDTD with respect to the article dispersion, and the border condition of CPML. Then we analyzed the dispersion diagram by changing the lattice constants and the diagonal of changes. By using the structures with different diagonals in each ring and also the oval holes we would come to a flat and near to zero diagrams in definite wave lengths. As the number of rings is increased, better results are achieved.

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