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A New Approach for Solving System of Fully Fuzzy Polynomial Equations

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Abstract: In this paper, a new approach for solving system of fully fuzzy polynomial equations based on nonlinear programing (shown as NLP) with equality constrain is presented. It is easy to apply the proposed method. This method can also lead to improve numerical methods. In this work, an architecture of NLP is also proposed to find a fuzzy positive root of a system of fuzzy polynomial equations (if exists). Finally, we illustrate our approach by numerical examples.

Keywords: Fuzzy numbers; Fully fuzzy polinomial equations.

INTRODUCTION

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh (1975), Dubois and Prade (1978). We refer the reader to (Kaufmann, 1985; Colak, 2009) for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy numbers are used to study a variety of problems ranging from fuzzy topological spaces (Caldas, 2005) fuzzy differential equations (Abbasbandy, 2005), fuzzy linear and nonlinear systems (Abbasbandy, 2005; Abbasbandy, 2006), fuzzy linear programming (Kumar, 2011) and particle physics (Elnaschie, 2005).

System of fuzzy polynomial equations play a major role in various areas such as pure and applied mathematics, engineering and social sciences. Previous papers (Abbasbandy, 2005; Abbasbandy, 2008), tried to find the numerical solution x ε R (if exists) of a fuzzy polynomial equation such as $A_1x+A_2x^2+...+A_nx^n=A_0$ where A_0, A_1,...,A_n are fuzzy numbers and system F, where F denotes a system of s fuzzy polynomial equations such as:

$$\begin{aligned} f_1(x_1,x_2,\dots,x_n) &= \tilde{c}_{10}, \\ &\vdots \\ f_l(x_1,x_2,\dots,x_n) &= \tilde{c}_{l0}, \\ &\vdots \\ f_s(x_1,x_2,\dots,x_n) &= \tilde{c}_{s0}, \end{aligned} \tag{1}$$

where x_1 ,x_2, x_n the elements $\tilde{c}_{10},\tilde{c}_{20},\dots,\tilde{c}_{s0}$ in the right-hand side vector are arbitrary fuzzy numbers and all coefficients are fuzzy numbers.

Now, consider Eq.(1) where all coefficients and x_1 ,x_2, x_n are non-negative fuzzy numbers. In this paper we are interested in finding a non-negative fuzzy root of such as system of fully fuzzy polynomial equations (SFFPE).

2. Preliminaries:

In this section the basic notations used in fuzzy calculus are introduced. We start by defining the fuzzy number.

Definition 1:

(Kaufmann, 1985). The characteristic function μ_A of a crisp set $u \subseteq X$ assigns a value either 0 or 1 to

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each member in *X*. This function can be generalized to a function $\mu_{\tilde{u}}$ such that the value assigned to the element of the universal set *X* fall within a specified range i.e. $\mu_{\tilde{u}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set *u*.

The function $\mu_{\tilde{u}}$ is called the membership function and the set $\tilde{u} = (x, \mu_{\tilde{u}}); x \in X$ defined by $\mu_{\tilde{u}}(x)$ for each ϰ ε X is called a fuzzy set.

Definition 2:

(Kaufmann, 1985). A popular fuzzy number is the triangular fuzzy number $\tilde{u} = (a,b,c)$ where *b* denotes the modal value. The membership function of a triangular fuzzy number is defined by:

$$\mu_{\tilde{u}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b, \text{*3mm} \\ \dfrac{x-c}{b-c}, & b \le x \le c, \text{*3mm} \\ 0, & \text{1cm } otherwise. \end{cases}$$

Definition 3:

(Kaufmann, 1985). A triangular fuzzy number (*α, b, c*) is said to be non-negative fuzzy number iff $\alpha \ge 0$.

Definition 4:

(Kaufmann, 1985). Two triangular fuzzy number $\tilde{u} = (a,b,c)$ and $\tilde{v} = (e,f,g)$ are said to be equal if and only if $\alpha = e$,$b = f$, $c = g$.

Definition 5:

(Dubois, 1980). A vector $\tilde{x} = (\tilde{x}_j)$ is called a fuzzy vector, if each element of $\tilde{x}$ is a fuzzy number.

$\tilde{x}$ will be positive (negative) fuzzy vector and denoted by $\tilde{x} \succ 0\ (\tilde{x} \prec 0)$ if each element of $\tilde{A}$ be positive (negative). Similarly, non-negative and non-positive fuzzy vectors may be defined.

Definition 6:

Let $\tilde{u} = (a,b,c)$ and $\tilde{v} = (e,f,g)$ be two triangular fuzzy numbers then (Kaufmann, 1985):

1. $\tilde{u} \oplus \tilde{v} = (a+e, b+f, c+g),$
2. $-\tilde{u} = (-c,-b,-a),$
3. $\tilde{u} - \tilde{v} = (a-g, b-f, c-e),$
4. $\tilde{u} \otimes \tilde{v}; \begin{cases} (ae,bf,cg), & a \ge 0, \\ (ag,bf,cg), & a < 0, c \ge 0, \\ (ag,bf,ce), & c < 0, \end{cases}$

where $\tilde{u}$ be any triangular fuzzy number and $\tilde{v}$ be a non-negative triangular fuzzy number.

System of Fully Fuzzy Polynomial Equations:

Consider the system of polynomial equations

$$
\begin{aligned}
&f_1(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)=\tilde{c}_{10},\\
&\qquad\vdots\\
&f_l(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)=\tilde{c}_{l0},\\
&\qquad\vdots\\
&f_s(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)=\tilde{c}_{s0},
\end{aligned}
\tag{2}
$$

where

$$f_l(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)=\tilde{a}_{l0}=\sum_{i=1}^{n}\tilde{c}_{li}\otimes\tilde{x}_i\oplus\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{lij}\otimes\tilde{x}_i\otimes\tilde{x}_j\oplus$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\tilde{c}_{lijk}\otimes\tilde{x}_i\otimes\tilde{x}_j\otimes\bar{x}_k\oplus\ldots,$$

all coefficients are arbitrary triangular fuzzy numbers, the elements $\tilde{c}_{10},\tilde{c}_{20},\ldots,\tilde{c}_{s0}$ in the right-hand side vector are arbitrary fuzzy numbers and the unknown elements $x_1,x_2,\ldots,x_n$ are non-negative fuzzy numbers, is called a system of fully fuzzy polynomial equations (SFFPE).

A fuzzy number vector

$\tilde{x}=(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n),$

given by $\tilde{x}_j=(x'_j,x_j,x''_j)^T, 1\le j\le n,$ is called a solution of the SFFPE (2) if

$$f_l(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)=\tilde{c}_{l0},\quad 1\le l\le s. \tag{3}$$

Using arithmetic operations, defined in Section 2, we have $\tilde{x}_j^k=(x'^k_j,x^k_j,x''^k_j)$,

$\tilde{x}_j\otimes\tilde{x}_i=(x'_jx'_i,x_jx_i,x''_jx''_i)$ for $k,i,j=1,2,\ldots,n.$

3.1 Shortcoming of the Existing Methods:

In this section, the shortcoming of the existing methods (Abbasbandy, 2006; Abbasbandy, 2008) for solving fuzzy polynomial equation and system of fuzzy polynomial equations, respectively are pointed out.

- Abbasbandy and Otadi (2006) considered fuzzy polynomial equation of the form $\tilde{a}_1x+\tilde{a}_2x^2+\ldots+\tilde{a}_nx^n=\tilde{a}_0$ where $\tilde{a}_i, i=0,1,\ldots,n$ are fuzzy numbers and x is a unknown real number. The existing method (Abbasbandy, 2006) is applicable only if the unknown x is a real number, e.g., it is not possible to find the solution of fully fuzzy polynomial equation, chosen in Example 1, by using the existing method (Abbasbandy, 2006) where $\tilde{x}$ is a non-negative fuzzy number.

Example 1:

$$\tilde{x}^3\oplus(-10,-9,-8)\otimes\tilde{x}^2\oplus(24,26,27)\otimes\tilde{x}=(-135,24,164),$$

$\tilde{x}$ is a non-negative triangular fuzzy number.

- Abbasbandy *et al.* (2008) investigated the solution of system of s fuzzy polynomial equations such as:

$$f_1(x_1, x_2, \ldots, x_n) = \tilde{c}_{10},$$
$$\vdots$$
$$f_l(x_1, x_2, \ldots, x_n) = \tilde{c}_{l0},$$
$$\vdots$$
$$f_s(x_1, x_2, \ldots, x_n) = \tilde{c}_{s0},$$

where $x_1, x_2, \ldots, x_n \in R$ and all coefficients are fuzzy numbers. The existing method (Abbasbandy, 2008) is applicable only if the unknown vector x is a real number , e.g., it is not possible to find the solution of SFFPE, chosen in Example 2, by using the existing method (Abbasbandy, 2008) where $\tilde{x}$ is a non-negative fuzzy number vector.

Example 2:

$$\begin{cases} \tilde{x}_1 \oplus \tilde{x}_1 \otimes \tilde{x}_2 = (3,10,18) \\ (-2,-1,1) \otimes \tilde{x}_1 \oplus \tilde{x}_1^2 \oplus (1,2,3) \otimes \tilde{x}_2 = (-3,10,27), \end{cases}$$

$\tilde{x}_1, \tilde{x}_2$ are non-negative triangular fuzzy numbers.

3.2 the Model:

In this subsection, a new method is proposed to find the fuzzy solution of SFFPE.

Consider the SFFME (2) where all the parameters $\tilde{c}_{li}, \tilde{c}_{lij}, \ldots,$ and $\tilde{x}_i$ are represented by triangular fuzzy numbers $(c'_{li}, c_{li}, c''_{li}), (c'_{lij}, c_{lij}, c''_{lij}), \ldots$ and (x'_i, x_i, x''_i) respectively then the SFFPE may be written as:

$$\sum_{i=1}^{n} (c'_{li}, c_{li}, c''_{li}) \otimes (x'_i, x_i, x''_i) \oplus \sum_{i=1}^{n} \sum_{j=1}^{n} (c'_{lij}, c_{lij}, c''_{lij}) \otimes (x'_i, x_i, x''_i) \otimes (x'_j, x_j, x''_j) \oplus \ldots = (c''_{l0}, c_{l0}, c''_{l0}), \qquad 1 \le l \le s, \tag{4}$$

Assuming $(c'_{li}, c_{li}, c''_{li}) \otimes (x'_i, x_i, x''_i) = (m'_{li}, m_{li}, m''_{li})$ and

$(c'_{lij}, c_{lij}, c''_{lij}) \otimes (x'_i, x_i, x''_i) \otimes (x'_j, x_j, x''_j) = (n'_{lij}, n_{lij}, n''_{lij})$ the SFFPE (4) may be written as:

$$\sum_{i=1}^{n} (m'_{li}, m_{li}, m''_{li}) \oplus \sum_{i=1}^{n} \sum_{j=1}^{n} (n'_{lij}, n_{lij}, n''_{lij}) \oplus \ldots = (c'_{l0}, c_{l0}, c''_{l0}), \quad 1 \le l \le s, \tag{5}$$

and $(x'_i, x_i, x''_i), 1 \le i \le n$ are non-negative triangular fuzzy numbers.

Using arithmetic operations, defined in Section 2, the NLP problem for solving FFME (4) is as follows:

$$Minimize \quad r_1 + r_2 + \ldots + r_{3s} \tag{6}$$

$$subject\ to \begin{cases} \sum_{i=1}^{n} m'_{1i} + \sum_{i=1}^{n}\sum_{j=1}^{n} n'_{1ij} + \ldots + r_1 = c'_{10}, \\ \sum_{i=1}^{n} m'_{2i} + \sum_{i=1}^{n}\sum_{j=1}^{n} n'_{2ij} + \ldots + r_2 = c'_{20}, \\ \vdots \\ \sum_{i=1}^{n} m'_{si} + \sum_{i=1}^{n}\sum_{j=1}^{n} n'_{sij} + \ldots + r_s = c'_{s0}, \\ \sum_{i=1}^{n} m_{1i} + \sum_{i=1}^{n}\sum_{j=1}^{n} n_{1ij} + \ldots + r_{s+1} = c_{10}, \\ \vdots \\ \sum_{i=1}^{n} m''_{si} + \sum_{i=1}^{n}\sum_{j=1}^{n} n''_{sij} + \ldots + r_{3s} = c''_{s0}, \\ x'_i \geq 0, x_i - x'_i \geq 0, \\ x''_i - x_i \geq 0, r_p \geq 0, 1 \leq i \leq n, 1 \leq p \leq 3s, \end{cases}$$

where r_p ` is a artificial variable.

Let us consider a FFPE with arbitrary coefficients, chosen in Example 1 and using the proposed method:

$$\tilde{x}^3 \oplus (-10,-9,-8) \otimes \tilde{x}^2 \oplus (24,26,27) \otimes \tilde{x} = (-135,24,164),$$

$\tilde{x}$ is a non-negative triangular fuzzy number.

Solution: Let $\tilde{x} = (x', x, x'')$, then given FFPE may be written as:

$$(x',x,x'')^3 \oplus (-10,-9,-8) \otimes (x',x,x'')^2 \oplus (24,26,27) \otimes (x',x,x'') = (-135,24,164),$$

(x', x, x'') is a non-negative triangular fuzzy number.

Then above SFFPS may be written as:

$$\left(x'^3 - 10x''^2 + 24x', x^3 - 9x^2 + 26x, x''^3 - 8x'^2 + 27x''\right) = (-135,24,164),$$

(x', x, x'') is a non-negative triangular fuzzy number.

Now by using the proposed method, the above FFPE may be converted into the following crisp system

$$\begin{cases} x'^3 - 10x''^2 + 24x' = -135, \\ x^3 - 9x^2 + 26x = 24, \\ x''^3 - 8x'^2 + 27x'' = 164. \end{cases}$$

Now the above linear system can be solved by using NLP:

$$Minimize\ \ r_1 + r_2 + r_3$$

$$subject\ to \begin{cases} x'^3 - 10x''^2 + 24x' + r_1 = -135, \\ x^3 - 9x^2 + 26x + r_2 = 24, \\ x''^3 - 8x'^2 + 27x'' + r_3 = 164, \end{cases}$$

where $r_1, r_2, r_3, x', x - x', x'' - x \geq 0$. The optimal solution of NLP problem is $x' = 1, x = 2, x'' = 4$. Therefore, the fuzzy solution is given by $\tilde{x} = (1, 2, 4)$.

Let us consider a SFFPE with arbitrary coefficients, chosen in Example 2 and using the proposed method:

$$\begin{cases} \tilde{x}_1 \oplus \tilde{x}_1 \otimes \tilde{x}_2 = (3,10,18) \\ (-2,-1,1) \otimes \tilde{x}_1 \oplus \tilde{x}_1^2 \oplus (1,2,3) \otimes \tilde{x}_2 = (-3,10,27), \end{cases}$$

$\tilde{x}_1, \tilde{x}_2$ are non-negative triangular fuzzy numbers.

Solution: Let $\tilde{x}_1 = (x'_1, x_1, x''_1)$, and $\tilde{x}_2 = (x'_2, x_2, x''_2)$, then given SFFPE may be written as:

$$\begin{cases} (x'_1, x_1, x''_1) \oplus (x'_1, x_1, x''_1) \otimes (x'_2, x_2, x''_2) = (3,10,18) \\ (-2,-1,1) \otimes (x'_1, x_1, x''_1) \oplus (x'_1, x_1, x''_1)^2 \oplus (1,2,3) \otimes (x'_2, x_2, x''_2) = \\ (-3,10,27), \end{cases}$$

(x'_1, x_1, x''_1) and (x'_2, x_2, x''_2) are non-negative triangular fuzzy numbers.

Then above SFFPE may be written as:

$$\begin{cases} (x'_1 + x'_1 x'_2, x_1 + x_1 x_2, x''_1 + x''_1 x''_2) = (3,10,18), \\ (-2x''_1 + x'^2_1 + x'_2, -x_1 + x_1^2 + 2x_2, x''_1 + x''^2_1 + 3x''_2) = (-3,10,27), \end{cases}$$

(x'_1, x_1, x''_1) and (x'_2, x_2, x''_2) are non-negative triangular fuzzy numbers.

Now by using the proposed method, the above SFFPE may be converted into the following crisp system

$$\begin{cases} x'_1 + x'_1 x'_2, x_1 + x_1 x_2 = 3, \\ x_1 + x_1 x_2 = 10, \\ x''_1 + x''_1 x''_2 = 18, \\ -2x''_1 + x'^2_1 + x'_2 = -3, \\ -x_1 + x_1^2 + 2x_2 = 10, \\ x''_1 + x''^2_1 + 3x''_2 = 27. \end{cases}$$

Now the above linear system can be solved by using NLP:

$$Minimize\ \ r_1 + r_2 + \ldots + r_6$$

$$subject\ to \begin{cases} x'_1 + x'_1 x'_2, x_1 + x_1 x_2 + r_1 = 3, \\ x_1 + x_1 x_2 + r_2 = 10, \\ x''_1 + x''_1 x''_2 + r_3 = 18, \\ -2x''_1 + x'^2_1 + x'_2 + r_4 = -3, \\ -x_1 + x_1^2 + 2x_2 + r_5 = 10, \\ x''_1 + x''^2_1 + 3x''_2 + r_6 = 27, \\ \end{cases}$$

where $r_1, r_2, \ldots, r_6, x_1 - x'_1, x_2 - x'_2, x''_1 - x_1, x''_2 - x_2 \geq 0$. The optimal solution of NLP problem is $x'_1 = 1, x'_2 = 2, x_1 = 2, x_2 = 4, x''_1 = 3, x''_2 = 5$. Therefore, the fuzzy solution is given by $\tilde{x}_1 = (1,2,3), \tilde{x}_2 = (2,4,5)$.

4 Summary and Conclusions:

In this work we proposed a new model to solve a system of fully fuzzy polynomial equations where all coefficients are arbitrary triangular fuzzy numbers, the elements $\tilde{c}_{10}, \tilde{c}_{20}, \ldots, \tilde{c}_{s0}$ in the right-hand side vector are arbitrary fuzzy numbers and the unknown elements $x_1, x_2, \ldots, x_n$ are non-negative fuzzy numbers. To illustrate the proposed method numerical examples are solved.

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