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A Novel Method for Power Allocation in MIMO-OFDM Systems with the Purpose of Increasing the Whole Capacity

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Abstract: Along with the growing increase of the demand for establishing a safe and fast connection between different users, for the purpose of transferring large audio-visual data using a wireless technology, a great number of studies have been carried out on the Multiple-Input Multiple-Output Orthogonal Frequency-Division Multiplexing Systems. In the algorithm presented in this article, first a simple, but efficient, model for multiple-input multiple-output orthogonal frequency-division multiplexing systems is offered. Besides, where the Channel state matrix in the Transmitter is known to us, we have tried to find a desirable capacity for such a system, putting the discussions of the information theory into practice as well as applying the method of singular value decomposition in channel matrixes. The simulation results prove such a claim.

Key words: Multiple-input Multiple-output Systems, Water-filling Algorithm, Orthogonal Frequency Multiplexing, Diversity, Beam-forming, formatting.

INTRODUCTION

Increasing the network capacity and the link throughput rate is one of the main objectives of developing future wireless communication systems in order to support high-rate functions. Not requiring a bandwidth or any excessive power consumption, MIMO techniques make use of diversity and multiplexing methods and are considered as a significant progress in the field of throughput and reducing the error probability as well as in raising the data pass rate in the communication systems. Besides, they increase the capacity and the spectral efficiency of systems to a great extent. System capacity increases linearly proportionate to the minimum number of transmitter and receiver antennas. Most often, there is also a compromise between multiplexing and diversity (rate increase and error probability reduction) depending on the conditions. According to the literature, such a claim has been proved through the wireless communication links in the flat fading and frequency selective fading channels. Therefore, MIMO techniques have found their way into the modern communication. On the other hand, a high spectral efficiency, reasonable performance in frequency selective fading channels and a good inherent resistance against narrow band interferers have resulted in a growing tendency toward the use of OFDM.

Recently, OFDM has been used for a variety of purposes. The application of OFDM in mobile communication channels is largely effective in solving the problems resulted from multipath fading. Therefore, it has been considered as the main mobile standard candidate in the fourth generation. The present article is aimed at providing a comprehensive analysis of the information theory, in an attempt to find the capacity of the MIMO-OFDM systems. To this end, having the singular values for the channel state matrix and using beam-forming filter and matched filter in transmitter and receiver, respectively, proposed iterative algorithm will result in an appropriate allocation of the power among the sub-channels.

1. *The Channel Model for MIMO-OFDM Systems:*

A MIMO-OFDM system may be illustrated as in figure 1. M_T and M_R are the number of transmitter and receiver antennas, respectively.

This model could be expressed mathematically as follows:

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$$v_1 = H_1 u_1 + \overline{n_1}$$

$$v_N = H_N u_N + \overline{n_N} \tag{1}$$

In the proposed model, first we assigned a transmit and a receive filter for each H_n, based on the matrixes resulted from the decomposition of the singular values for the channel state matrix, as illustrated in figure 1. Our final objective is to maximize the achievable rate

2. *Describing the Proposed Power Allocation Scheme, Through Transmitting of the Principal Eigen State of the Channel State Matrix:*

For each H_n in figure2, we assume v_n to be the transmit filter and u_n* as receive filter. Therefore, the output of the receive filter will be:

$$z_n = u_n^* H_n v_n a_n + u_n^* n_n \rightarrow n = 1,2,....,N \tag{2}$$

In which, a_n is the data signal and $\overline{n}_n$ indicates the noise vector. Taking v_n and u_n* as the principal Eigenvectors (corresponding to the maximum Eigen values of $H_n H_n^*$ and $H_n^* H_n$ the value of the received signal to noise ratio will be maximized, because in each sub-channel, the maximum Eigen value of $H_n H_n^*$ is multiplied by the data signal. This process is further explained in the following sections.

In this case the equation (2) can be re-expressed as:

$$z_n = \sqrt{s_n^{(1)}} a_n + w_n \rightarrow n = 1, 2..., N \tag{3}$$

In which, $w_n = u_n^* \overline{n}_n$ and $s_n^{(1)}$ is the maximum Eigen value of $H_n H_n^*$.

However, since we have only one scalar channel, the special multiplexing gain of MIMO will be lost and the principal Eigen state may lead into a significant decrease of the rate.

3. Describing the Proposed Power Allocation Scheme, Through Transmitting the Eigen States of the Channel State Matrix:

In the previous section, we were dealing with the principal Eigen state, in such a way that, only the maximum Eigen value of $H_n H_n^*$ was used to transmit the data. In this section, the optimum beam-forming is applied, while all the Eigen values are used in the process of transmitting. Therefore, the achievable MIMO channel rate will be maximized.

Here, rather than the weight vectors, we use matrixes for transmit and receive filters. In MIMO-OFDM, the optimum beam-forming state (maximizing the capacity) is based on the decomposition of the singular value H.

Assuming $H_n = U_n D_n V_n^*$ as a decomposition of the singular value of H_n. then, M nonzero Eigen values of $H_n H_n^*$ are $\left\{ s_n^{(m)} \right\}$. Having the channel state information in the transmitter, same as the receiver, for each OFDM tone, we choose V_n as a beam-forming filter in the transmitter and U_n^* as a corresponding filter in the receiver. As illustrated in figure 3, the MIMO-OFDM channel is converted into a bank of $M \times N$ memoryless scalar channels around the space (m) and the frequency (n).

$$z_1^{(1)} = \sqrt{s_1^{(1)}} a_1^{(1)} + w_1^{(1)}$$

$$z_1^{(M)} = \sqrt{s_1^{(M)}} a_1^{(M)} + w_1^{(M)}$$

$$z_N^{(1)} = \sqrt{s_N^{(1)}} a_N^{(1)} + w_N^{(1)} \tag{4}$$

$$z_N^{(M)} = \sqrt{s_N^{(M)}} a_N^{(M)} + w_N^{(M)}$$

Where $z_1^{(1)}$ to $z_1^{(M)}$ are related to the first power, and $z_N^{(1)}$ to $z_N^{(M)}$ to the N_{th} tone.

As illustrated in figure 3, $\left\{a_n^{(m)}\right\}$ consists the input signals of the Eigen beam-forming filters in the transmitter, and $\left\{z_n^{(m)}\right\}$ indicates the output signals of the corresponding filters in the receiver. Since V_n and U_n are both unitary, $\left\{w_n^{(m)}\right\}$ noise has equal statistical characteristics as the noise vector in equation (1). That is, $\left\{w_n^{(m)}\right\}$are the Gaussian i.i.d random variables with a zero mean and where $E\left[\left|w_n^{(m)}\right|^2\right] = N_0$. For each tone, the Eigen beam-forming produces M special channels (Eigen values). In the Eigen-mode beam-forming, a channel with the number of {a} input and {b} output is corresponding to a channel with the number of {b} input and {a} output channel, where both have equal Eigen values equal to $\left\{s_n^{(m)}\right\}$

4. *Mean and Outage Capacity:*

In a wireless communication system, the transmitter is of a low power. If we consider the MIMO-OFDM model shown in figure1, in which $v_N = H_N u_N + \overline{n}_N$, for $n = 1,2,..., N$ assuming, $\hat{H} = diag\left[H_1,...,H_N\right]$ then the covariance matrix of the U_N input signal vectors, in the nth memoryless H_N will be $Q_n = E\left[u_n u_n^* \middle| \hat{H}\right]$

There are two constrains in terms of the energy: the whole energy should satisfy the long-term constraint, also known as the average energy constraint :

$$E\left[\frac{1}{N}\sum_{n=1}^{N} tr\left(Q_n\right)\right] = \overline{E} \tag{5}$$

Moreover, the whole energy should satisfy the short-term constraint, also known as the temporary energy constraint :

$$\frac{1}{N}\sum_{n-1}^{N} tr\left(Q_n\right) = \overline{E} \tag{6}$$

In both cases, signal-to-noise ratio for each antenna is defined as:

$$\rho = \frac{\overline{E}}{N_o} \tag{7}$$

With a long-term constraint, the transmitter can control the power according to the current channel state, which is known as the power control for single-user communication.

For a SIMO channel (MT=$_1$, $M_R \geq 1$), the immediate SNR requirement is:

$$\rho(s) = \begin{Bmatrix} \dfrac{2^{R_{-1}}}{s}, \dfrac{2^{R_{-1}}}{s} \geq \rho_{TH} \\ 0, \dfrac{2^{R_{-1}}}{s} \leq \rho_{TH} \end{Bmatrix}$$

Where ρ_{TH} is a threshold. The outage probability will be:

$$P_{OUT} = \Pr ob\left[\frac{2^{R_{-1}}}{s} > \rho_{TH}\right] = 1 - \frac{\Gamma(M_R, \rho_{TH})}{\Gamma(M_R)}$$

When SNR is:

$$\rho = E\left[\rho\ (s)\right] = \left(2^R - 1\right).\ \frac{\Gamma(M_R - 1, \rho_{TH})}{\Gamma(M_R)}$$

P_{OUT} and ρ are connected by ρTH.

5. *Power Allocation for MIMI-OFDM Systems in the Proposed Scheme:*

For the parallel channels in 5, first we consider the mutual information. For the nth tone, we assume equals the decomposition of the singular value H_n.

As explained in figure 3, the input vector for each H_n is $U_n = V_n\ a_n$, and $a_n = \left[a_n^{(1)} \ldots\ldots, a_n^{(m)}\right]^T$.

Assuming $H = diag\left[H_1, \ldots., H_N\right]$, the covariance matrix x_n can be re-expressed as the following:

$$n = E\left[u_n\, u_n^*\, H\right] = V_n E_n V_n^*$$

Where,

$$E_n = diag\left\{e_n^{(1)}, \ldots., e_n^{(M)}\right\} = E\left[a_n\, a_n^*\, H\right].$$

$$I_{TRCSI} = \frac{1}{N}\sum_{n=1}^{N} \log_2 \det\left(I_{M_R} + \frac{H_n Q_n H_n^*}{N_0}\right) \quad = \frac{1}{N}\sum_{n=1}^{N} \log_2 \det\left(U_n\left(I_{M_R} + \frac{D_n E_n D_n}{N_0}\right)U_n^*\right) = \frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{M}\left(1 + \frac{s_n^{(m)} e_n^{(m)}}{N_0}\right)$$

Which is a function of $\left\{e_n^{(m)}\right\}$ and $\left\{s_n^{(m)}\right\}$. In contrast to the equal distribution of energy for the state where the channel state information is unknown, having the channel state information, the transmitter does an extra job known as power allocation. Under the long- or short-term energy constraint, the transmitter should decide about $\left\{e_n^{(m)}\right\}$, in order to achieve a particular objective, such as maximizing the mean capacity $E\left[I_{TRCSI}\right]$.

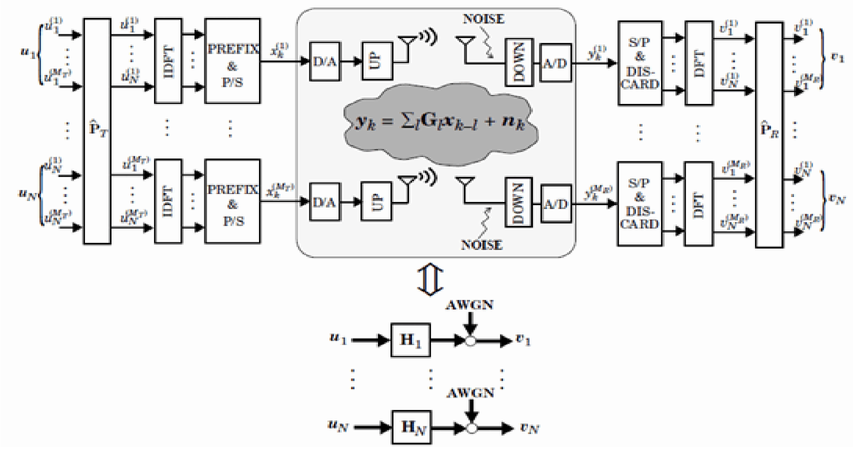

Fig. 1: The diagram block of MIMO-OFDM system and the corresponding channel model.

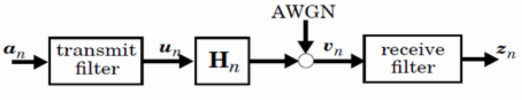

Fig. 2: The Diagram Block for the Transmit Beam-forming.

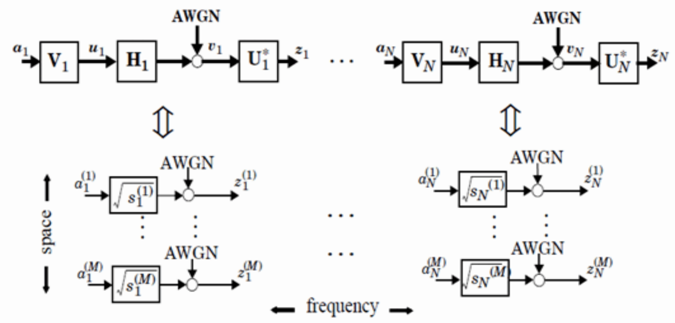

Fig. 3: The eigen beam-forming converts the MIMO-OFDM channel into a bank of scalar channels around the space and frequency.

6. *The Simulation Results:*

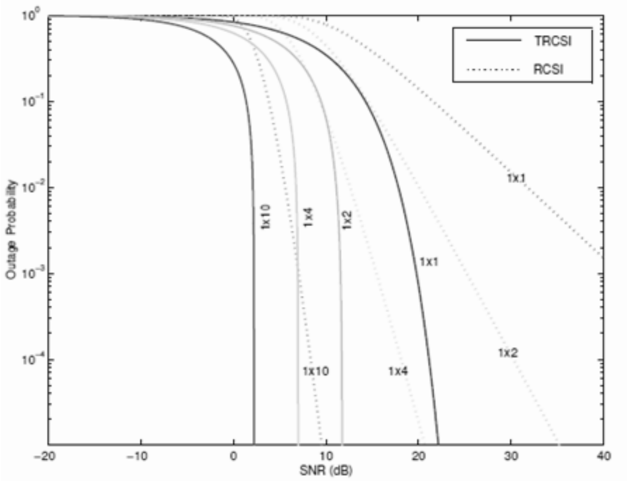

Fig. 4: Outage probability for $M_T = 1$ and $M_R \geq 1$ uncorrelated Rayleigh fading channel when channel state information is known at the transmitter at R=3 bits per signaling interval. Outage probability when CSI is known at the transmitter is plotted as a benchmark.

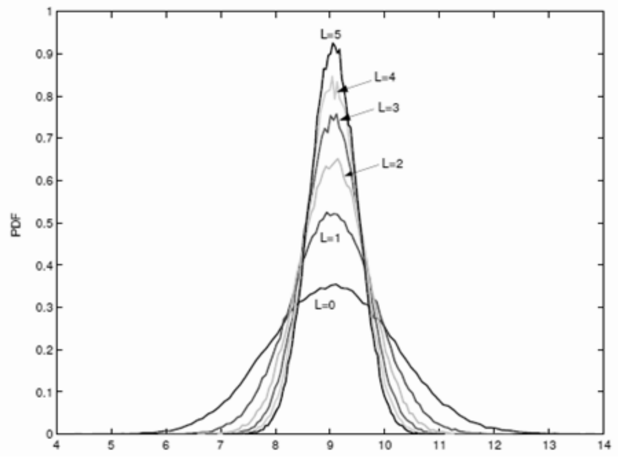

Fig. 5: PDF of mutual information of a 4×4 uncorrelated Rayleigh fading channel with Lε {0,1,2,3,4,5} at $\rho = 7$

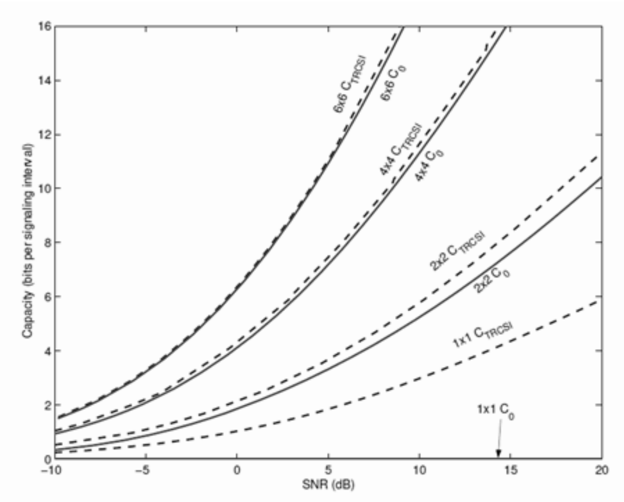

Fig. 6: A comparison of C_{TRCSI} and C_o a on a spatial noncoherent Rayleigh fading channel, where Mε {1,2,4,6}.

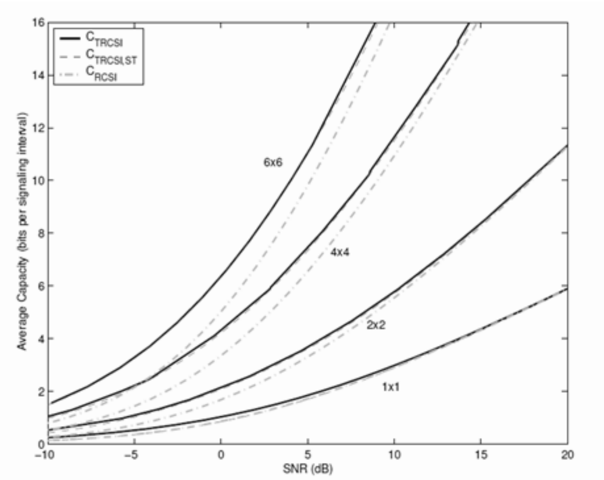

Fig. 7: A comparison of C_{TRCSI} and C_{RCSI} on a spatial noncoherent $M \times M$ Rayleigh fading channel, where Mε {1,2,4,6}.

7. *The Numerical Results:*

In this section, the numerical results will be presented as an evidence for the previously offered theories. In simulations, the multiple access Gaussian channels are considered parallel to K (= βN) less than β, that is, 500 carriers, having the equal number of users, where varies between 0.1 and 1 and the PSK binary modulation is used.

In figure 4, P_{out} in R=3 (bits per signaling) when M_R $\varepsilon\{1,2,4,10\}$ is illustrated. The issue that P_{out} for M_R $\geq$ 1 when ρ_{TH} =0 is ρ =(2^R-1). $\frac{\Gamma(M_R-1)}{\Gamma(M_R)}$ can be proved. P_{out} is also plotted as a benchmark when channel state information is not available at the transmitter. It is obvious that having CSI at the transmitter is a substantial point, especially for small M_R . however, when M_R is large, the advantage is not noticeable, but exists. For example, the SNR advantage to obtain P_{OUT} =10^{-3} for M_R = 2 is approximately 4dB.

In figure 5, PDF of mutual information of a 4×4 uncorrelated Rayleigh fading channel with Lε $\{0,1,2,3,4,5\}$(L is the memory of available channel), at $\rho \rightarrow 7$. It can be seen that as the number of paths grow, the PDF get higher values.

In figure 6, the outage capacities and the capacity when channel information is available in transmitter are plotted for different signal-to-noise ratios, and for the states where the channel state information is available in the transmitter, in different signal-to-noise ratios for the MIMI-OFDM system, when the channel has a flat fading and Rayleigh distribution and the proposed power allocation scheme is applied for the sub-channels of the system. Note that modulation process has been used and the PSK is binary. As it is shown, for the small signal-to-noise ratios, the obtained capacity, when the channel state information is available in the transmitter, is more than the zero-outage capacity, particularly when the number of the transmitter and receiver antennas is low. However, with the increase of the number of antennas, the values of these two capacities approach one another, even for different signal-to-noise ratios. Another point to be noted is the fact that when there is only one antenna in the transmitter and one in the receiver, for different ratios, the zero-outage capacity equals to zero. In other words, the zero-outage capacity will be impossible to obtain.

From figure 7, It can be seen that when the signal-to-noise is low ($\rho \rightarrow 0$) then, $C_{TRCST} \geq C_{TRCST,\mathrm{ST}} \geq \mathrm{C}_{\mathrm{RCST}}$ is true,

However with the increase of this ratio such a possibility will be diminished. Nevertheless, the difference between C_{TRCSi} and C_{RCST} is not a significant one. For large signal-to-noise ratios, in case, $M_T \geq M_R$ the difference tends to zero.

Figure 7 illustrates the C_{TRCSI} and C_{RCSI} the obtained from the power allocation scheme proposed here, for a spatial non-coherent $M \times M$ MIMO channel, where Mε $\{1,2,3,4,6\}$. The capacity difference between C_{TRCSI} and C_{TRCSI} increases as M enlarges. Such a difference for small signal-to-noise ratios is trivial.

However, for the large ratios, C_{RCSI} converges to C_{TRCSI}. Moreover, $C_{TRCSI,ST}$, which is the system capacity with the assumption of a channel state matrix in the transmitter and the receiver, is plotted having applied the short-term constraint.

According to figure 7, $C_{TRCSI,ST}$ is almost as large as C_{TRCSI} especially when M is large, which indicates that, as M grows, the water-filling theory for the components of the channel (for C_{TRCSI}) becomes unnecessary.

Conclusion:

In the present article, we considered the power allocation, simplifying the existing constraints and offering a simple strategy for the MIMO- OFDM closed-loop systems, that is, for a state where the channel state matrix is available in the transmitter. The conclusion is that with the application of the beam-forming method proposed here, the adaptive power allocation is no more necessary for the transmitter to reach an appropriate capacity; rather a combination of the represented method and a uniform allocation would suffice.

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