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On Deviation Degree Methods for Ranking Fuzzy Numbers

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Abstract: Recently, some researchers presented methods for ranking fuzzy numbers based on deviation degree. To avoid more applications that are possible or spread in the future, in this paper, we indicate that the proposed methods have drawbacks. Therefore, they cannot rank fuzzy numbers in all conditions.

Key words: Deviation degree, Triangular fuzzy numbers, Trapezoidal fuzzy numbers, Ranking.

INTRODUCTION

The fuzzy set (Zadeh, L.A., 1965) was extended to develop the intuitionistic fuzzy set (Atanassov, K.T., 1986; 1999) by adding and additional non-membership degree, which may express more abundant and flexible information as compared with the fuzzy set (Li, D.F., 2010). Fuzzy numbers are special case of fuzzy sets and are of importance for fuzzy multiattribute decision-making problems. Many fuzzy ranking indices have been proposed since 1976. Jain (Jain, R., 1976; 1977) proposed the first ranking methods. A canonical way to extend the natural ordering of real numbers to fuzzy numbers was suggested by Bass and Kwakernaak (Bass, S. and H. Kwakernaak, 1977). Dubios and Prade (Dubios, D. and H. Prade, 1978) used maximizing sets to order fuzzy numbers. In 1979, Baldwin and Guild (Baldwin, J.F. and N.C.F. Guild, 1979) indicated that these two methods have some disturbing disadvantages. Yager (Yager, R.R., 1980; 1981) proposed four indices, which may be employed for ordering fuzzy quantities in [0, 1]. Bortolan and Degani (Bortolan, G. and R. Degani, 1985) have been compared and reviewed some of these ranking methods. Chen and Hwang (Chen, S.J. and C.L. Hwang, 1992) thoroughly reviewed the existing approaches, and pointed out some illogical conditions that arise among them. Chen (Chen, S., 1985) Choobineh (Choobineh, F. and H. Li, 1993), Cheng (Cheng, C.H., 1998) have presented some methods, and numerous ranking techniques have been proposed and investigated by Cha and Tsao (Chu, T. and C. Tsao, 2002) and Ma, Kandel and Friedman (Ma, M., A. Kandel and M. Friedman, 2000). Nowadays many researchers have developed methods to compare and to rank fuzzy numbers (Abbasbandy, S. and B. Asady, 2006; Hajjari, T., In Press; 2011; 2010; 2009; 2008; 2007).

Recently, Wang and Lee (Wang, Y.J. and S.H. Lee, 2008), claimed that multiplying the values of $\bar{x}$ and $\bar{y}$, in the method by Chu and Tsao (Chu, T. and C. Tsao, 2002), undermines the importance of $\bar{x}$ In point of their view, larger $\bar{x}$, the larger corresponding fuzzy number is, and if $\bar{x}$ of two fuzzy numbers are the same, then $\bar{y}$ should be used.

To overcome the limitations of some existing studies and simplify the computational procedure Wang *et al*. (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009) proposed an approach to ranking fuzzy number based on left and right deviation degree (L-R variation degree). In their approach the maximal and minimal reference sets are defined to measure L-R deviation degree of fuzzy number and then the transfer coefficient is defined to measure the relative variation of L-R deviation degree and relative variation of fuzzy number as a ranking index.. They also pointed out the method by Wang and Lee (Wang, Y.J. and S.H. Lee, 2008) could not rank fuzzy numbers with the identical centroid points correctly. The presented method overcame the shortcoming of Wang and Lee's method (Wang, Y.J. and S.H. Lee, 2008). In 2009, Asady (Asady, B., 2009) indicated some problems in Wang *et al*.'s method. Therefore, they revised their method. Moreover, Nejad and Mashinchi (Nejad, A.M. and M. Mashinchi, 2011) pointed out that the method proposed by Wang *et al*. (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009) cannot rank fuzzy numbers correctly. They claimed in the mentioned method, whenever either of the left deviation degree, the right deviation degree, the transfer coefficient of the

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fuzzy number is zero or the transfer coefficient of the fuzzy number is one. Therefore, they introduced a novel method for ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy numbers. They declared that the new approach resolves the drawback of Wang *et al.*'s approach (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009). However, it has a shortcoming in some basic properties.

In this paper, we will point out the drawback of those methods, which are based on deviation degree such as

- Wang and Lee (Wang, Y.J. and S.H. Lee, 2008),
- Wang *et al.* (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009),
- Asady (Asady, B., 2009),
- Nejad and Mashinchi's method (Nejad, A.M. and M. Mashinchi, 2011),

moreover, discuss about the problem of the deviation degree methods.

For the sake of clarity, the related concepts of fuzzy theories are presented in Section 2. In Section 3, we review three ranking methods, which are based on deviation degree. Section 4 includes three counter-examples to illustrate the shortcoming of the mentioned methods. The paper is concluded in Section 5.

2. Back ground information:

Definition 2.1.:

First, In general, a generalized fuzzy number A is membership $\mu_A(x)$ can be defined as (Dubios, D. and Prade, H., 1978)

$$\mu_A(x)=\begin{cases} L_A(x) & a\le x\le b \\ \omega & b\le x\le c \\ R_A(x) & c\le x\le d \\ 0 & otherwise, \end{cases} \tag{1}$$

where $0\le\omega\le1$ is a constant, and $L_A:[a,b]\to[0,\omega]$, $R_A:[c,d]\to[0,\omega]$ are two strictly monotonical and continuous mapping from R to closed interval $[0,\omega]$. If ω=1, then A is a normal fuzzy number; otherwise, it is a trapezoidal fuzzy number and is usually denoted by A= (a,b,c,d,ω) or A= (a,b,c,d) if ω=1.

In particular, when b=c, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by A= (a,b,c,d,ω) or A= (a,b,c,d) if ω. Therefore, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since L_A and R_A are both strictly monotonical and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. Let $L_A^{-1}:[a,b]\to[0,\omega]$ and $R_A^{-1}:[a,b]\to[0,\omega]$ be the inverse functions of $L_A(x)$ and $R_A(x)$, respectively. Then $L^{-1}{}_A(r)$ and $R^{-1}{}_A(r)$ should be integrable on the close interval $[0,\omega]$. In other words, both $\int_0^\omega L_A^{-1}(r)dr$ and $\int_0^\omega R_A^{-1}(r)dr$ should exist. In the case of trapezoidal fuzzy number, the inverse functions $L^{-1}{}_A(r)$ and $R^{-1}{}_A(r)$ can be analytically expressed as

$$L_A^{-1}(r)=a+(b-a)r/\omega \quad 0\le\omega\le1 \tag{2}$$

$$R_A^{-1}(r)=d-(d-c)r/\omega \quad 0\le\omega\le1 \tag{3}$$

The set of all elements that have a nonzero degree of membership in A, it is called the support of A, i.e.

$$S(A)=\{x\in X\mid\mu_A(x)\succ0\} \tag{4}$$

The set of elements having the largest degree of membership in A, it is called the core of A, i.e.

$$C(A) = \left\{x \in X \mid \mu_A(x) = \sup_{x \in X} L_A(x)\right\} \tag{5}$$

In the following, we will always assume that A is continuous and bounded support $S(A)$. The strong support of A should be $[a,d]$.

Definition 2.2.:
A function $f:[0,1] \rightarrow [0,1]$ is a reducing function if is s increasing and $f(0)=0$ and $f(1)=1$. We say that s is a regular function if $f(r)=1/2$.

Definition 2.3:
If A is a fuzzy number with r-cut representation, $\left(L_A^{-1}(r), R_A^{-1}(r)\right)$and s is a reducing function, then the value of A (with respect to s); it is defined by

$$Val(A) = \int_0^1 f(r)[L_A^{-1}(r) + R_A^{-1}(r)]dr \tag{6}$$

Definition 2.4.:
If A is a fuzzy number with r-cut representation $\left(L_A^{-1}(r), R_A^{-1}(r)\right)$, and s is a reducing function then the ambiguity of A (with respect to s) is defined by

$$Amb(A) = \int_0^1 f(r)[R_A^{-1}(r) - L_A^{-1}(r)]dr \tag{7}$$

Definition 2.5.:
The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalent represented in (Zadeh, L.A, 1965; Ma, M., A. Kandel, and M. Friedman, 1999; Dubois, D. and H. Prade, 1980) as follows.

For arbitrary $A = \left(L_A^{-1}(r), R_A^{-1}(r)\right)$ and $B = \left(L_B^{-1}(r), R_B^{-1}(r)\right)$ we define addition $(A+B)$ and multiplication by scalar $k \succ 0$ as

$$\begin{aligned} &(\underline{A+B})(r) = \underline{A}(r) + \underline{B}(r) \\ &(\overline{A+B}(r)) = \overline{A}(r) + \overline{B}(r) \\ &(\underline{kA})(r) = k\underline{A}(r), (\overline{kA})(r) = k\overline{A}(r). \end{aligned} \tag{8}$$

To emphasis, the collection of all fuzzy numbers with addition and multiplication as defined by (8) is denoted by E, which is a convex cone. The image (opposite) of $A=(a,b,c,d)$ is $-A=(-d,-c,-b,-a)$ (Zadeh, L.A, 1965; Dubois, D. and H. Prade, 1980).

3. Some Existing Methods Based on Deviation Degree:
In this section, firstly, methods were presented by Wang *et al.* (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009), (Asady, B., 2010) and Nejad and Mashinchi (Nejad, A.M. and M. Mashinchi, 2011) will be reviewed.

Ranking L-R fuzzy numbers based on deviation degree (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009):
Definition 3.1.:
For any groups of fuzzy numbers $A_1, A_2, \ldots, A_n$ in E with support sets $S(A_i), i = 1, \ldots, n.$ Let $S = \bigcap_{i=1}^{n} S(A_i)$ and $x_{min} = \inf S$ and $x_{max} = \sup S.$ Then minimal and maximal reference sets A_{min} and A_{max} are defined as

$$\mu_{A_{\min}}(x) = \begin{cases} \dfrac{x_{\max} - x}{x_{\max} - x_{\min}}, & if\ x \in S \\ 0, & otherwise, \end{cases} \quad (9)$$

$$\mu_{A_{\max}}(x) = \begin{cases} \dfrac{x - x_{\min}}{x_{\max} - x_{\min}}, & if\ x \in S \\ 0, & otherwise. \end{cases} \quad (10)$$

Definition 3.2:

For any groups of fuzzy numbers $A_1, A_2, ..., A_n$ in E, let $A_{\min}$ and $A_{\max}$ be minimal and maximal reference sets of these fuzzy numbers, respectively. Then left and right deviation degree of A_i, i=1,..., n, are defined as follows:

$$d_i^L = \int_{x_{\min}}^{t_i} \left(\mu_{A_{\min}}(x) - L_A^{-1}(x) \right) dx \quad (11)$$

$$d_i^R = \int_{u_i}^{x_{\max}} \left(\mu_{A_{\max}}(x) - R_A^{-1}(x) \right) dx \quad (12)$$

where t_i and u_i, i=1,2,...,n are the abscissas of the crossover points of L_{Ai} and μ_{Amin} and R_{Ai} and μ_{Amin} respectively.

Definition 3.3.:

For any groups of fuzzy numbers $A = (a_i, b_i, c_i, d_i, \omega)$ in E, its expectation value of centroid is defined as follows:

$$M_i = \frac{\int_{a_i}^{d_i} x \mu_A(x) dx}{\int_{a_i}^{d_i} \mu_A(x) dx} \quad (13)$$

$$\lambda_i = \frac{M_i - M_{\min}}{M_{\max} - M_{\min}} \quad (14)$$

where M_{max} = max $\{M_1, M_2, ..., M_n\}$ and M_{max} = min $\{M_1, M_2, ..., M_n\}$.

Based on (11), (12) and (14), the ranking index value of fuzzy numbers A_i, i = 1,...,n, is given by

$$d_i = \begin{cases} \dfrac{d_i^L \lambda_i}{1 + d_i^R (1 - \lambda_i)}, & M_{\max} \neq M_{\min}, \ i = 1, 2, ..., n, \\ \dfrac{d_i^L}{1 + d_i^R} & M_{\max} = M_{\min}, \ i = 1, 2, ..., n. \end{cases} \quad (15)$$

Now, by using (15), for any two fuzzy numbers A_i and A_j the ranking order is based on the following rules.

1. $A_i \succ A_j$ if and only if $d_i \succ d_j$,
2. $A_i \prec A_j$ if and only if $d_i \prec d_j$,
3. $A_i \approx A_j$ if and only if $d_i = d_j$.

The revised method of ranking L-R fizzy number based on deviation degree (Asady, B., 2010):

Asady (Asady, B., 2010) revised Wang *et al*. (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009) method and suggested $D(.)$ operator for ranking of fuzzy numbers as follows:

Consider two fuzzy numbers A and B the ranking order is based on the following situations:

If $D(A) \prec D(B)$, then $A \prec B$.

If $D(A) \succ D(B)$, then $A \succ B$.

If $D(A) = D(B)$, then

if $\gamma_A \neq \gamma_B$, $D^*(A) \prec D^*(B)$ then $A \prec B$,

if $\gamma_A \neq \gamma_B$, $D^*(A) \succ D^*(B)$ then $A \succ B$,

else $A \approx B$.

where

$$D(A) = \frac{D_A^L}{1 + D_A^R} \tag{16}$$

$$D^*(A) = \frac{D_A^L \gamma}{1 + D_A^R \gamma} \tag{17}$$

where

$$D_A^L = \int_0^1 \left(R_A^{-1}(x) + L_A^{-1}(x) - 2x_{\min} \right) dx \tag{18}$$

$$D_A^R = \int_0^1 \left(2x_{\max} - R_A^{-1}(x) - L_A^{-1}(x) \right) dx \tag{19}$$

Ranking fuzzy numbers based on the left and the right sides of fuzzy numbers (Nejad, A.M. and M. Mashinchi, 2011):

Recently Nejad and Mashinchi (Nejad, A.M. and M. Mashinchi, 2011) pointed out the drawback of Wang *et al*. (Wang, Z.X., Y.J. Liu, Z.P. Fan. and B. Feng, 2009) hen they presented a novel ranking method as follows.

Definition 3.4.:

Let $A = (a_i, b_i, c_i, d_i, \omega)$, $I = 1,2,...,n$, are fuzzy numbers in E, $a_{min} = \min\{a_1, a_2, ..., a_n\}$ and $d_{max} = \max\{d_1, d_2, .., d_n\}$

The areas s_i^L and s_i^R of the left and right sides of the fuzzy number A_i are defined as

$$s_i^L = \int_0^{\omega} \left(L_A^{-1}(r) - a_{\min} \right) dr \tag{20}$$

$$s_i^R = \int_0^{\omega} \left(d_{\max} - R_A^{-1}(r) \right) dr. \tag{21}$$

Based on (14), (16) and (17) the proposed ranking index is

$$s_i = \frac{s_i^L \lambda_i}{1 + s_i^R (1 - \lambda_i)}, \; i = 1, 2, ..., n.$$

Then the ranking order follows next rules.

$A_i \succ A_j$ if and only if $s_i \succ s_j$,

$A_i \prec A_j$ if and only if $s_i \prec s_j$,

$A_i \approx A_j$ if and only if $s_i = s_j$.

To obtain the reasonable they added two triangular fuzzy numbers A_0 and A_{n+1}, where

$$A_0 = (a_0, b_0, d_0),$$
$$a_0 = 2b_0 - d_0,\ b_0 = \min\{a_i,\ i = 1,2,\ldots,n\},$$
$$d_0 = (d + b_0)/2,\ d = \min\{d_i,\ i = 1,2,\ldots,n\}$$

and

$$A_{n+1} = (a_{n+1}, b_{n+1}, d_{n+1}),$$
$$a_{n+1} = (b_{n+1} + a)/2,\ b_{n+1} = \max\{d_i,\ i = 1,2,\ldots,n\},$$
$$d_{n+1} = 2b_{n+1} - a_{n+1}),\ a = \max\{a_i,\ i = 1,2,\ldots,n\}.$$

Then they ranked fuzzy numbers $A_1, A_2, \ldots, A_n$ based on the ranking area values $s_1, s_2, \ldots, s_n$

Nevertheless, the new ranking method has drawback.

In the next section, we discuss on those methods that based on deviation degree by a number numerical counter examples.

4. Disscution and Counter Examples:

The idea of ranking fuzzy numbers by deviation degree is useful, but a significant approaches should be reserved the important properties such that

$$A \le B \Leftrightarrow -B \le -A$$

$$A \le B \Leftrightarrow A + C \le B + C$$

$$A \le B \wedge B \le C \Rightarrow A \le C$$

Now we give some numerical example to show the drawback of the aforementioned methods.

***Counter-examples*:**

Example 4.1.:

Given two triangular fuzzy number A=(0.2,0.5,0.8) and B=(0.4,0.5,0.6) (Nejad, A.M. and M. Mashinchi, 2011), which are indicated in Fig. 1.

The ranking order by Nejad and Mashinchi is $A \prec B$. The images of two numbers A and B are A = (-0.8,-0.5-0.2), B = (-0.6,-0.5,-0.4) respectively, then the ranking order is $-B \prec -A$

On the other hand, ranking order for A and B and their images by Wang *et al.*'s method and Asady's

revised are $A \approx B, -A \approx -B$ respectively.

This example could be indicated that all methods are reasonable. However, we will show that functions of all three methods are not the same in different conditions.

Example 4.2.:

Consider the three triangular fuzzy numbers A = (1,2,6), B = (2.5,2.75,3) and C= (2,3,4), which are taken from Asady's revised (Asady, B., 2010)(See Fig. 2).

Utilizing Nejad and Mashinchi's method the ranking order is $A \prec B \prec C$ and the ranking order of their images will be obtained $-C \prec -A \prec -B$, which is illogical.

By using Wang *et al.*'s method the ranking order is $B \prec A \prec C$ and for their images is $-A \approx -C \prec -B$ which is unreasonable too.

From point of revised deviation degree method (Asady, B., 2010) the ranking orders are $B \prec A \prec C$, $-C \prec -A \prec -B$, respectively.

From this example, it seems the revised method can rank correctly.

In the next example, we will indicate that none of the methods based on deviation degree can rank correctly in all situations.

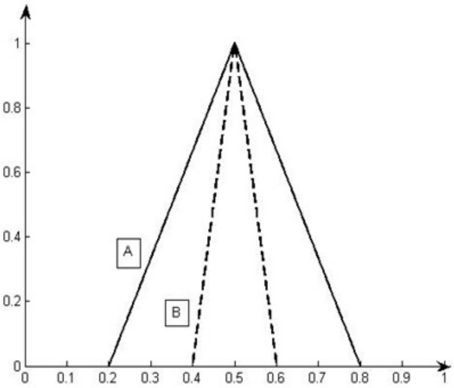

Fig. 1:

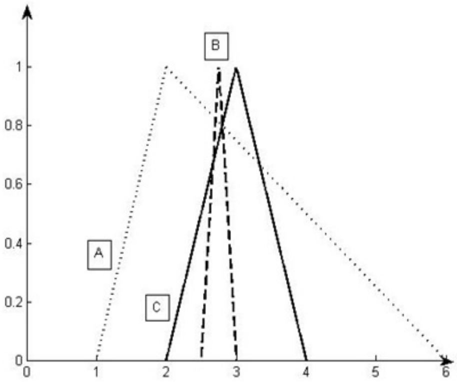

Fig. 2:

Example 4.3.:

Let the triangular fuzzy number A = (1,2,3) and the fuzzy number B=(1,2,4) with the membership function (See Fig. 3).

$$\mu_B(x) = \begin{cases} \left[1-(x-2)^2\right]^{1/2} & 1 \le x \le 2, \\ \left[1-\frac{1}{4}(x-2)^2\right]^{1/2} & 2 \le x \le 4, \\ 0 & otherwise. \end{cases}$$

Using Asady's method the ranking order is obtained $A \prec B$ However, the ranking order of their images is $-A \prec -B$ which is unreasonable.

From mentioned examples, we can theorize that ranking fuzzy numbers based on deviation degree cannot rank fuzzy numbers in all situations.

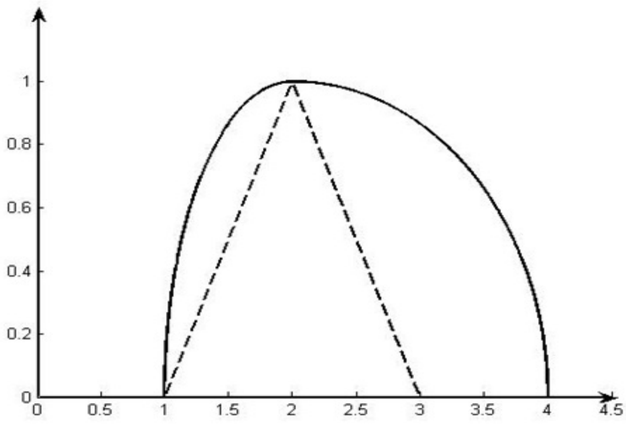

Fig. 3:

5. Conclusion:

With the increasing development of fuzzy set theory in various scientific fields and the need to compare fuzzy numbers in different areas. It is important that the main properties should be reserved. However, we found that the methods, which were based in deviation degree, could not rank fuzzy numbers correctly in all situations. Therefore, to avoid more possible misapplications or spread in the future, in this paper we showed the deviation degree methods have drawback.

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