Corona Losses Calculation in HVAC Transmission Lines Using FEM and Comparison with HVDC Corona Losses

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Abstract: This paper describes a new method of the corona loss in bipolar HVDC and HVAC transmission lines using Finite Element Method (FEM). The present method implements the potentials and field at conductor surface as boundary conditions. In present method programming calls for only one loop and a new approach for updating of space charge densities around the conductor is implemented using Rung-Kutta integration method. Fast convergence and reduction iteration numbers is characterized of the proposed method. In addition to in this paper in the one case study the corona loss calculation in HVAC lines implemented with this methods and another case study was performed with the RMS value of applied voltage and simulation was performed similar to in HVDC unipolar lines.

Key words: Corona, Corona Power Loss, ion diffusion coefficient, Finite Element method (FEM)

INTRODUCTION

one of problem associated with HVDC and HVAC transmission lines is corona power loss. Many attempts were made to solve ionized field using Charge Simulation Method (CSM), Boundary Element Method, and Finite Element Method.

But none of them has been taken in account the effect of the diffusion coefficient as function of electric field and climate temperature and air density, etc.

The present method implements the potentials and electric field at conductor surface as boundary conditions, however; in previous method deal only with the potentials in conductor and ground plane and check the field on conductor surface later.

The latest method for corona power loss calculation is FEM method that is used in this paper, but some innovations, such as using new updating space charge densities along electric field lines, instead of using flux-tube and writing continuity current equation along it. In previous method programming calls for two loops to convergence, one for convergence of potentials and another for convergence of electric field at conductor surface. in this method only one loop is needed for convergence of space charge density, this of course reduces the complexity if computation and leads to reduction of the number of iterations, for updating space charge densities around the conductor, the rung-kutta integration method is used to calculate charge densities along electric field lines, whereas previous method, using flux-tubes along electric field lines.

To show accurate of the proposed method two conductors to ground plane configuration in laboratory and full scale model were considered to analyze.

Bipolar Corona Field Equations:

The main system of equations describing bipolar DC corona is as follows:

\[ \nabla \cdot \vec{E} = \frac{\rho_+ - \rho_-}{\varepsilon_0} \]  

(1)

\[ \nabla \cdot \vec{J}_e = \tau \frac{\rho_+ \rho_-}{\varepsilon} \]  

(2)

\[ \vec{j}_e = k_s \rho_e \vec{E} \]  

(3)

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\[ \vec{J} = \vec{J}_+ + \vec{J}_- \]  \hfill (4) \\
\[ \vec{E} = -\nabla \Phi \]  \hfill (5)

Equations (1-5) are respectively Poisson’s equation, positive and negative current density vector, \( \vec{J}_+ \) continuity condition of \( \vec{J}_+ \), total current density vector \( \vec{J} \) and the continuity condition of \( \vec{J}_- \). \( k_+ , k_- \) are the motilities of positive and negative ions, \( \rho_+ , \rho_- \) are the positive and negative space-charge densities values, \( R_i \) is the ion recombination coefficient in air and \( e \) is the electron charge.

**Simplifications in calculation process:**

All attempts reported before were based on some simplifying, the most common ones are:

1. The entire electrode spacing is filled with unipolar space charge of the same polarity as the coronating conductor.
2. The space charge affects only the magnitude and not the direction of the electric field. This assumption was suggested at first by Deutsch and take referred to as "Deutsch's assumption".
3. The positive and negative ion motilities \( k_+ , k_- \) are constant.
4. The ion diffusion coefficient is neglected.
5. The surface field of the coronating conductor remains at onset value \( E_{on} \), which is known as Kaptzov's assumption (Sarma and W. Janischewsky). For conductor to plane configuration \( E_{on} \) is expressed in kV/cm as:

\[ E_0 = 30\eta[1 + \sqrt{(0.0906 / R)}] \quad \text{kV/cm} \]  \hfill (6)

In another assumption (App, 1969), the electric field \( E_c \) at coronating conductor surface is assumed to be a function of applied voltage, i.e.

\[ E_{on} = E_{on} f_i(V/V_0) \]  \hfill (7)

Where \( f_i \) is assumed to have the following forms:

\[ f_i(V/V_0) = 1.1339 - 0.1678(V/V_0) + 0.03(V/V_0)^2 \]  \hfill (8)

With \( f_i \) assumed to be unity at \( V=V_0 \), where \( V_0 \) is the corona onset voltage.

In this paper the magnitude of electric field at conductor surface for updating space charges is evaluated in each step with FEM method by ANSYS software and used in next step for integration of space charge for updating along electric field lines.

**Boundary conditions:**

Solution of the equations describing the space-charge modified field requires all three boundary conditions as follows:

1. The potential on the ground plane is zero.
2. The potential on the coronating conductors are equal to the applied voltage \( +V, -V \) respectively.
3. The magnitude of the electric field in each step of simulation was calculated by ANSYS and it used in integration process of updating space charges.

The boundary conditions (a) and (b) were applied in model construction in ANSYS and the third boundary condition was implemented in integration process.

**Proposed Method Of Analysis:**

1-Bipolar HVDC:
configuration of model:
For the conductor to plane configuration, two conductors of radius R is located at a height H above the grounded plane, Fig.1.

Fig. 1: the conductor to plane configuration in bipolar HVDC transmission lines

The proposed method is described follow:

Mesh Generation:
Whereas the calculation of electric field and potential was performed using FEM method, therefore the model must to be meshed. The meshing process was made with triangular element, Fig.2.

It is well known that FEM calls for bounded regions in which the mesh to be generated. Hence as it has been shown in Fig.2, fictitious boundaries X1-X2 and Rout are assumed around the discharging conductor. This choice was found to be satisfactory in light of the fact the computed results did not change for larger value of X1-X2 and, Rout.

Because Rout is not at infinity, the potential of the nodes at fictitious boundary, Rout, is not zero, but the potential at X1-X2 is equal to potential of ground plane and is equal to zero.

First estimation of space charges:
The configuration considered in the present method consists of a single cylinder of radius R of a height H above the grounded plane.

The bipolar co-ordinates \( \alpha \) and \( \beta \), shown in Fig.3, are employed, which are related to the Cartesian co-ordinates x and y by following relations:
\[ y = \frac{a}{s} \sinh \beta \quad y = \frac{a}{s} \sinh \beta \]  

(9)

Where:
\[ a = (H^2 - r_c^2)^{1/2}, \quad s = \cosh \beta + \cos \alpha \]  

(10)

The metric coefficients are given by the following expressions:
\[ h_r = h_{\omega} = \frac{a}{s} \]  

(11)

And the co-ordinates of coronating conductor and ground plane are given by:
\[ \beta_c = \cosh^{-1}\left(\frac{H}{r_c}\right), \quad \beta_s = 0 \]  

(12)

d\(A\), is one of the cylindrical surface, where \(\beta = \cos \tan t\), then:
\[ dA = \hat{a}_{\beta} a (\cosh \beta + \cos \alpha)^{-1} d\alpha \]  

(13)

**Fig. 3:** bipolar co-ordination in conductor to plane configuration

Where \(\hat{a}_{\beta}\) is a unit vector in the \(\beta\) co-ordinate direction.

Because the electric field is assumed to have the same direction as that of the space-charge-free field (Deutsch assumption), the integration of space charges for updating them was performed along \(\hat{a}_{\beta}\) direction.

The space charge densities located at nodes \((i, 1)\), around the periphery of the conductor is assumed initially as:
\[ \rho_{i,1} = \rho_c \cos\left(\frac{\pi - \theta_i}{2}\right), i = 1, 2, \ldots, M \]  

(14)
\[ \rho_{e,1} = 0.1 \rho_{e,1} \]  \hspace{1cm} (15)

Where M is the total field lines (was equal to 30 in this work) and \( \rho_e \) is the value of \( \rho_i \) at \( \theta = 0 \);

The value of \( \rho_e \) was estimated using an approximate expression reported before for the charge density at the ground plane:

\[ \rho_e = \frac{E_D}{E_{cr}} \frac{V_0(V - V_0)}{D^2V(5 - 4V/V)} \]  \hspace{1cm} (16)

Where \( E_g \) the space charge free electric field at the ground plane is, \( E_{cr} \) is determined by analysis of model in each step in ANSYS and \( V_0 \) is the onset voltage Aboelsaad et al., (1989).

\[ V_0 \equiv \eta R(2 \times E_{cr}) \ln \left( \frac{2H}{R} \right) \]  \hspace{1cm} (17)

Using (1-5) the (18-19) can be obtained as:

\[ \frac{\partial \rho_{l+2\Delta l}}{\partial l} = \frac{\rho_{l+2\Delta l}}{E} \left[ \frac{\rho_{l+2\Delta l}}{d k} - \frac{\rho_{l+2\Delta l}}{\varepsilon_0} \right] \]  \hspace{1cm} (18)

\[ \frac{\partial \rho_{l+4\Delta l}}{\partial l} = \frac{\rho_{l+4\Delta l}}{E} \left[ \frac{\rho_{l+4\Delta l}}{d k} - \frac{\rho_{l+4\Delta l}}{\varepsilon_0} \right] \]  \hspace{1cm} (19)

Starting at conductor's surface, (18-19) is integrated along each Line to estimate the nodal space-charge densities. For the first step \( \rho \) is determined by (14), and initial value of \( E \), is \( E_{cr} \).

This estimate of the space charge density is utilized to determine the distributed space charges at the FE mesh nodes.

By (18-19) using Rung-Kutta method the value of \( \rho(l+\Delta l) \) was estimated.
- The two previous steps to calculate \( \rho(l+2\Delta l) \) and \( \rho(l+3\Delta l) \), was performed to estimate the space charge densities at all nodes in each integration nodes.
- The two previous steps was performed for all integration routes (M=30) to identify the space charges at all nodes around the conductor.

In Rung-Kutta integration method, the Simpson method is used. In each step of integration for calculation \( \rho(l+\Delta l) \), the four parameters K were used to determine it. For calculation \( \rho(l+\Delta l) \) along electric field by transferring bipolar co-ordinates to Cartesian co-ordinates with (9-13) the (20) is used:

\[ \rho_{l+1} = \rho_{l} + \frac{\Delta l}{6} [K1 + 2K2 + 2K3 + K4] \]  \hspace{1cm} (20)

Where K1 to K4 is determined from (21-24) (Afjayi S.Eb.'Numerical, 2000):

\[ K1 = h(\rho_{l}, l) \]  \hspace{1cm} (21)

\[ K2 = h(\rho_{l} + \frac{K1}{2}, l + \frac{\Delta l}{2}) \]  \hspace{1cm} (22)

\[ K3 = h(\rho_{l} + \frac{K2}{2}, l + \frac{\Delta l}{2}) \]  \hspace{1cm} (23)

\[ K4 = h(\rho_{l} + K3, l + \frac{\Delta l}{2}) \]  \hspace{1cm} (24)

After calculation \( \rho(l+\Delta l) \), the process was repeated to evaluate \( \rho(l+2\Delta l) \).
Estimation Electric field and Potentials:
In this step the estimated space charges in previous step were applied to model in ANSYS software to calculate the potentials and electric fields in all nodes. These values of potentials and electric fields were used in next step of updating space charges.

Space Charge densities correction:
The last two estimated of potentials at each node, $\Phi^{(i)}, \Phi^{(i+1)}$ are compared. Nodal potentials error $E_v$ Relative to the average value of nodal potential $\Phi_{av}$ is defined as:

$$E_v = \frac{|\Phi^{(i)} - \Phi^{(i+1)}|}{\Phi_{av}}$$  \hspace{1cm} (25)

Where

$$\Phi_{av} = \frac{|\Phi^{(i)} + \Phi^{(i+1)}|}{2}$$

If the maximum nodal potential error exceeds a prespecified error $\delta_1$, a correction of $\rho_i[N]$ which is the space charge density at the last node of each field line, was made. The correction follows by (27),

$$\rho_{i,new} = \rho_{i,old} \left[1 + f \max\{E_v\} \right]$$  \hspace{1cm} (26)

Where $f$ is an acceleration factor, take equal to 0.5.

Converge condition:
The steps 3-5 repeated until the maximum error in calculation of potential become less than a prespecified error $\delta_2$.

Calculation of corona current:
For each applied voltage above the onset value, corona current is equal to the sum of current flowing, i.e.

$$I = \sum_{i=1}^{M} J_{i} A_{i,1}$$ \hspace{1cm} (28)

As:

$$J = k \rho E$$ \hspace{1cm} (29)

Then:

$$I = \sum_{i=1}^{M} \left( k_+ \rho_{i+1} + k_- \rho_{i-1} \right) E_{i,1} A_{i,1} \hspace{1cm} (30)$$

2-HVAC Transmission Lines:
For analysis of corona due to ac applied voltage, each alternation cycle must to be divided into NT discrete time steps. In each time step the model seems to a HVDC lines. Because of the periodic reversal of the electric field, in practical AC transmission lines, the space charges created by corona are constrained to the near vicinity of the conductor. In each time step by integration Runge-Kutta method the space charges created by corona was calculated and applied to model. The liberated negative space charges move away from the stressed wire. At the same time; the positive space charges return back to the wire and meet the outgoing negative ones. Some of the positive charges are neutralized through a recombination process. To evaluate the loss of ions due to recombination over a time interval $\Delta t$, $\rho_+$ is the positive ions density where a positive line charge $Q_+$ exists.

Fig. 4: NT discrete time steps of alternation cycle of applied voltage

$$\rho_s = \frac{Q_s}{e\Delta V_s}$$  \hspace{1cm} (31)

After the recombination process, the magnitude of the line charges, $Q_s$, become:

$$Q_s = \rho_s(1 - \gamma \rho_s - \Delta t)e\Delta V_s$$ \hspace{1cm} (32)

Where the recombination coefficient $\gamma = 1.5 \times 10^{-12} \text{ m}^3 / \text{s}$

For air at N.P.T conditions.

After calculation of electric fields in all nodes around the conductor, the energy dissipation through a complete cycle is calculated as:

$$W = \sum \int Q_s E \, dr$$ \hspace{1cm} (33)

Where the integration range $A$ is over all line charges and $C$ is the line charge trajectory over cycle.

The corona power loss $PL$ is simply given as:

$$PL = \int W$$ \hspace{1cm} (34)

For solving above equations, two softwares MATLAB and ANSYS are implemented. Figure 5 shows the relation between two softwares.

In figure 6, the flaw-chart of proposed method is presented.

RESULT AND DISCUSSION

Proposed method was applied for two conductors AC and DC. A sample of grid for one investigated configuration is shown in Fig. 2.

The number of element is 645 with 1250 unknown nodal potentials.

V-I characteristics of this configuration for present method and previous method as well experimental result are shown in Fig. 6.

It is clearly that obtained results with proposed method is closer to the experimentally values and those obtained by previous method.

The specifications of studied line is height $H=161.29$ cm

$R=0.4635$ cm.

In addition to in this paper in the one case study the corona loss calculation in HVAC lines implemented with this methods and another case study was performed with the RMS value of applied voltage and simulation was performed similar to in HVDC unipolar lines.
Fig. 5: Relation between MATLAB and ANSYS for calculation of corona current

Fig. 6: The flow-chart of the proposed algorithm for corona calculation

Fig. 7: Present and previous calculated V-I characteristics of a conductor to plane configuration in comparison with that obtained experimentally
The Table I shows the comparison between corona losses calculated with this method in HVDC line and HVAC lines. The simulation was performed for lines with height, \( H = 9.15 \text{m} \) and radius, \( R = 0.0102 \text{ m} \) under voltage equal to 350 kV in DC and 350 kV (rms) in ac lines.

<table>
<thead>
<tr>
<th>Line Type</th>
<th>HVDC</th>
<th>HVAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corona Loss (kW/km)</td>
<td>0.669</td>
<td>0.801</td>
</tr>
</tbody>
</table>

This shows that the corona power loss in HVDC transmission lines is less than in HVAC lines.

In second case study calculated corona loss with ac voltage and dc equivalent voltage equal to its rms value was compared.

The rms value voltage applied in the case is 130 kV. The difference between two values of corona power loss in figure 8 is due to this fact that the nature of corona current is multi frequency and this don’t allow to calculate corona power loss with multiple \( V(\text{rms}) \) and \( I(\text{rms}) \).

In addition to the effect of recombination phenomena in the state of equivalent dc voltage instead of ac voltage is not considered and finally leads to incurrent results.

![Fig. 8: Corona loss calculated with ac voltage and dc equivalent voltage equal to its rms value](image)

Figure 9 shows the great effect of changing the conductor radius on corona current. A large conductor size results in a lower onset field, by (5), but a higher applied voltage is required to set the onset field.

![Fig. 9: Effect of conductor radius on corona current](image)

At large radius, corona current has disappeared.

The transmission line height above the ground plane also affects the corona current. This is shown in Fig.10.

It has been found that when the distance of two electrodes was increased, the onset field decreases and this effect decreases the probability of ionization and leads to lower corona current.

Another case that is worthy to investigate is the effect of surface factor and roughness of the surface on corona current.
Fig. 10: Effect of conductor height on corona current

The large surface factor means that the surface is better and the corona current is lower.

Fig. 11: Effect of quality of conductor surface on corona current

Figure 12 shows depict the effect of changing the spacing between the pole conductors.

Fig. 12: Effect of distance between two poles of conductors on corona current

The surface factor is depended of quality of conductor surface and when it washed this factor improved and reduced.
It has been found that the larger the spacing between the conductors the less is the interaction between two poles and consequently the less is the corona loss.

**Conclusion:**

In this paper, a method has been suggested that has differences with other references:

- A new method for updating space charges
- Reduce the number of loops in algorithm of calculation corona current.
- These changes lead to these results:
  - Not only the more accurate results are obtained but also the number of iterations is reduced comparing with previous method.
  - The agreement between the present calculated characteristics the previous methods and measured values are obtained.
- In this paper, also the effect of various parameters on corona current were investigated that the results of these investigation listed as follow:
  - With increase the height of conductor the corona current decreases.
  - With decrease the radius of conductor the corona current decreases.
  - With decrease the surface factor, i.e. increasing of the roughness of conductor surface the corona current increases.
  - With increase the space between of two poles of bipolar HVDC conductor the corona loss decreases.
  - The corona power loss in HVDC transmission lines is less than in HVAC lines.
- In AC power corona loss, calculation shouldn't do with its equivalent dc value.

**REFERENCES**

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