

Modeling and Optimal Solution of *Open Capacitated Vehicle Routing Problem (Ocvrp)* in Garbage Transportation in Kecamatan Seberang Ulu I Kota Palembang

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Abstract: Garbage transportation system in Palembang (Irmeilyana et al., 2007) usually been done in many phases. Garbage collected from homes usually been collected to the nearest temporary garbage collection place (TPS). This garbage will be delivered to final garbage collection place (TPA). This system divides into working area (WK). The garbage truck, in this case, does not go back to the depot (TPA) after accomplishing its job. The driver usually takes the truck with him. This happens since it is more efficient to bring the truck home and the route will be simpler. In this problem, we deal with open capacitated vehicle routing problem (OCVRP). In this paper, all OCVRPs obtain its initial integer solution so we cannot apply branch and cut algorithm. But for symmetric capacitated vehicle routing problem (SCVRP) the optimal solution yields non integer solution so we have to apply branch and cut algorithm. In Kecamatan Seberang Ulu I, all of initial optimal solution yield integer solution, but this solution is not valid so we have to add new constraints (adding constraints) to obtain valid optimal solution.

Key words: OCVRP, garbage transportation, branch and cut algorithm, optimal solution.

INTRODUCTION

Garbage transportation system in Palembang Irmeilyana, (2007) usually been done in many phases. Garbage collected from homes usually been collected to the nearest temporary garbage collection place (TPS). This garbage will be delivered to final garbage collection place (TPA). This system divides into working area (WK).

In conventional CVRP problem as discussed in Irmeilyana, (2007) and Puspita, (2006), transport of commodities required to return to the depot after completing the work. But, for some vehicle routing problems (VRP) such as garbage transportation, the conditions mentioned above can not be done.

Transport vehicle usually does not return to the depot after performing their duties, but returned to another place, such as driver home. This condition occurs, since the route must pass by becoming more efficient if the vehicle had been taken earlier by a driver who transports garbage.

Another important incidents are usually occur when schedule visit demand is not once, that is, the traveling vehicle must be divided into several sections to regulate demand. Furthermore, if period time in the visit demand is limited, it will lead to limitation of delivery time. The problems which cause new problems to be solved because the trajectories are formed not closed, but the path is open, and the demand which is divided, so the problem becomes OCVRP-st (Open CVRP-split, time deadlines) Letchford *et al.*, (2006) OCVRP-st problem is important to be developed because it deals with the transport of commodities such as garbage.

Garbage transportation system in Palembang city done gradually. Garbage from households is transported by a man to collect on TPS which is provided by DKP nearby. Furthermore, the existing garbage in TPS are transported by DKP' officers by using garbage transport vehicle (dump truck, amroll) to one of two TPAs, namely TPA Karya Jaya and TPA Sukawinatan. Garbage transportation been done by the DKP officers are divided according to the Working Area (WK) for each driver.

Caccetta, (2000) emphasized that for CVRP problem consisting of 100 vertices, Branch and cut as deemed appropriate CVRP completion method as its solution method. So, based on last research on the transportation of garbage and also on the exact solution method of branch and cut are necessary to formulate an appropriate model for OCVRP-st and also its solution techniques by branch and cut method. If the branch and cut method is applied, then preprocessing and probing techniques used to detect the tight of a model which is formed

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should also be applied.

Based on the background described earlier, it needs to create models and solve them with application software on garbage transportation problems in Palembang city. It also needs the algorithm of Branch and Cut method for OCVRP on garbage transportation which focuses on Kecamatan Seberang Ulu I Palembang.

MATERIAL AND METHODS

Design and methodology of research can be arranged in the working steps as follows:

1. The study is begun by preparing materials from various sources, including books, journals, and information on the internet relating to CVRP and their variations such as the open path conditions, delivery of commodities which are divided, the limited time of delivery between each customer, branch and cut algorithm.
2. Garbage transportation data surveys in Palembang related to vehicle and TPS capacities, time collection, and collection time sharing.

A general description of the work stages was done, as follows:

- a. From transportation data, set assumptions to develop CVRP to OCVRP-st, i.e.:
 - identical vehicle capacity unknown
 - demand is known with certainty and must be divided
 - single depot
 - travel expenses of all customer locations in both directions (symmetric) and one-way (anti symmetric).
- b. Develop OCVRP-st models by defining the problem and the notation, by assuming the problem can be represented in a graph
 - to establish some basic reduction basic rules
 - trying to eliminate the arc which causes unfeasibility of OCVRP-st
 - trying to eliminate the arc current that is not correct solution.
- c. Developing the model, consisting of
 - two-index vehicle flow formulation
 - OCVRP-st formulation which became the basis of settlement with Branch and cut algorithm
- d. Forming valid inequalities that are used to support the branch and cut algorithm in solving OCVRP-st, such as:
 - Develop a modified adjusted comb inequalities for the problem OCVRP-st.
 - Using the sub tour elimination
- e. Analyzing the results obtained in the form of optimal routes and draw conclusions from the model.

RESULTS AND DISCUSSION

In this section, it was discussed about additional OCVRP-st models if a solution formed is invalid, i. e. resulting in a integer solution but the fact, there are TPS which did not visit yet or TPS are visited more than once. It means breaking CVRP condition that the customer should be visited only once. In this chapter, it was also described the model and the optimal route.

OCVRP model can be obtained by modifying the standard formulation of CVRP. Mathematically, OCVRP can be stated as follows Letchford *et al.*, (2006):

Minimize

$$Z = \sum_{i,j \in V_c} c_{ij}x_{ij} + \sum_{i \in V_c} c_{0i}y_{0i}$$

Subject to:

$$x(\bar{\delta}(i)) + y_{0i} + y_{i0} = 2 \quad (i = 1, 2, \dots, n) \tag{1}$$

$$x(\bar{\delta}(S)) + y^-(S) + y^+(S) \geq 2k(S) (S \subseteq V_c, |S| \geq 2) \quad (2)$$

$$y^-(V_c) = K, \text{ with } y^-(V_c) = \sum_{i \in V_c} y_{0i} \quad (3)$$

$$y^+(V_c) = K, \text{ with } y^+(V_c) = \sum_{i \in V_c} y_{i0} \quad (4)$$

$$x_{ij} \in \{0, 1\}, \text{ for } 1 \leq i < j \leq n \quad (5)$$

$$y_{0i}, y_{i0} \in \{0, 1\} \quad (i = 1, 2, \dots, n) \quad (6)$$

where Z is objective function, c_{ij} is distance from location i to j , x_{ij} is route from i to j , c_{0i} is distance from the depot to the customer location i , y_{0i} is travel route from the depot to customer i , y_{i0} is route from customer i to the depot,

$$x(\bar{\delta}(S)) = \sum_{i, j \in S} x_{ij}, \text{ for } \bar{\delta}(S) = \{\{i, j\} : i \in S, j \in \bar{S}\} \text{ and } \bar{S} = V_c \setminus S,$$

$$x(\bar{\delta}(i)) = \sum_{i \in S} x_{i0}, \text{ for } \bar{\delta}(i) = \{\{i, 0\} : i \in S, 0 \in V\},$$

K is number of vehicles, V_c is a set of customers, where $V_c = V \setminus \{0\}$, $k(S)$ = lower bound of the minimum number of vehicles required to visit the customer S , $k(S) = \frac{q(S)}{Q}$, $q(S)$ is customer demand and Q is vehicle capacity.

To prevent the solution is not valid, it is necessary to add a constraint which is called the balancing inequality constraint:

$$x(\bar{\delta}(S)) + y^+(S) \geq y^-(S) \quad (S \subseteq V_c, |S| \geq 2) \quad (7)$$

The absence of such constraints on CVRP standard formula, shows that OCVRP is more complex problem than CVRP. Marina *et al.*, (2007) describes the additional constraints used in the completion CVRP if the optimal solution is proved invalid. Suppose the given number of vehicles is K , the vehicle capacity is C , a distance matrix is symmetric and the average volume of each TPS is d_i , where $i = 1, 2, \dots, n$, then SCVRP model is formulated as follows:

$$\text{Minimumize: } Z = \sum_{0 \leq i \leq j \leq n} c_{ij} x_{ij} \quad (8)$$

Subject to:

$$\sum_{i \leq j \leq n} x_{0k} = 2K \quad (9)$$

The trip starts from the depot to TPS and immediately returned to the depot again. K is the number of garbage vehicles, in a case where the vehicle is equal to 1 ($K = 1$) per working area (WK).

$$\sum_{e \in \delta(i)} x_e = 2$$

Because of symmetric, a trip from i to j equal to the trip from j to i , while the trip i to j is calculated as 1, then the trip from i to j and the trip from j to i counted 2.

$$\sum_{0 \leq j \leq n} x_{ij} = 2 \text{ for all } 1 \leq i \leq n \quad (10)$$

Travel is not started from the depot

$$\sum_{j \in S} x_{oj} + \sum_{(i,j) \in S} x_{ij} = 2 \leq 2b(S) \text{ for all}$$

$$S \ni \forall \{0\}; |S| > 2 \tag{11}$$

$b(S)$ = lower bound (LB) of the number of vehicles required to visit customers S is obtained from $b(S) =$

$\frac{\sum_{i \in S} d(i)}{C}$ where $d(i)$ is the total volume of TPS, C is the capacity of vehicles (trucks), and S is identical set of TPS visited.

$x_{ij} \in \{0,1,2\}$ for all $e \in \delta(0)$ which is the value of travel routes or nonnegative binary constraints.

For WK cases if the equation on the optimal solution is not formed, the minimum route and there are

more than or the same as TPS that is not visited, then $b(S)$ was changed to $b(S) = \left\lceil \frac{\sum_{i \in S} d(i)}{C} \right\rceil$ where:

$\left\lceil \frac{\sum_{i \in S} d(i)}{C} \right\rceil$ is the smallest integer that is greater than or equal $\frac{d(i)}{C}$.

Thus Equation (11) is also broken into its multiple permutations $b(S)$ as follows:

$$\sum_{j \in S} x_{ij} = 1 \text{ for all } 1 \leq i \leq n \tag{12}$$

$$\sum_{i \in S} x_{ij} = 1 \text{ for all } 1 \leq j \leq n \tag{13}$$

$$\sum_{j \in S} x_{ij} = 2 \text{ for all } 1 \leq i \leq n \tag{14}$$

$$\sum_{i \in S} x_{i0} = 2 \tag{15}$$

Additional OCVRP-st Models:

For the OCVRP-st model in the form (1) until (6) often occurs in the form of solutions that form the route is invalid, then it was made its permutation $b(S)$ to OCVRP-st as follows:

$$y_{oi} + y_{io} = 1, (i = 1, \dots, n) \tag{16}$$

$$y_{oi} + \sum_{j \in S} x_{ij} = 1, (i = 1, \dots, n) \tag{17}$$

$$y_{oi} = 1, (i = 1, \dots, n) \tag{18}$$

$$y_{i0} = 1, (i = 1, \dots, n) \tag{19}$$

$$\sum_{i \in S} x_{ij} = 1 \text{ for all } 1 \leq j \leq n \tag{20}$$

Furthermore, based on model (1) - (6) and (16) - (20), data were found, then they are modeled and found their solution by LINDO software application tools to get the optimal solution; i. e. the route of vehicle in

every WK.

The results of research that have conducted data of garbage transportation in every district (kecamatan) in Palembang. Data processing was done by modeling the distance data into the OCVRP model (if the vehicle starts at driver home) and CVRP (if the vehicle departs from DKP office because the trajectory formed is closed).

Listed below is the results that have been obtained with the initial routes are acquired but not yet valid and further improved by the addition of balancing constraints and additional constraints for Kecamatan Seberang Ulu I. Optimum route is obtained and depicted began moving vehicle until the expiry of the transport vehicle in TPA Sukawinatan.

Puspita, (2006) developed SCVRP algorithms to be solved by branch and cut method. Furthermore, for OCVRP formulations (1) until (6), solutions with branch and cut method can also be applied similar to SCVRP that discussed in Puspita, (2006) and Sepputra *et al.*, (2007). $B(S)$ is the lower bound of the number of vehicles required to visit all the locations S in the optimal solution. Some vehicles that serve the customer set S , and some of the activities of these vehicles can be described as below:

- Vehicles leaving the depot, serving customers S and back again to the depot.
- Vehicles leaving the depot, serve customers S and serve a subset of customers S .
- Vehicles serving the customer subset S before and after serving customers S .

With a note $b(S)$ was transferred by the lower bound (LB) should be capacity restrictions. In practice, the determination is done by calculating LB each different restrictions then form $b(S)$ as the maximum value.

In Branch and Cut scheme, $b(S)$ was calculated as the maximum value of LB resulting from restrictions capacity or distance limitation. Below are intended algorithm for solving the OCVRP problem obtained from the above explanation.

Branch and Cut Algorithm in Solving OCVRP:

Step 1. Begin by considering LP (Linear Programming) relaxation. Where the number of vehicles is variable, K based on the restricted $b(S)$. Enter the distance between customers and depot, then customer demand and the capacity of the vehicle.

Step 2. Solve LP. If the objective function value is at least or equal to the upper bound (UB) then it stops. If not proceed to Step 3.

Step 3. Remove some restrictions of the LP, which has a basic slack variables.

Step 4. If LP solution meet sub tour elimination constraints and integer, then the solution is the UB from the real problem, then stop. If not proceed to Step 5.

Step 5. If tour S is not connected with the depot, the violation will be instantly added. When it's connected to the depot, if the tour S does not violate capacity and distance limitation then the search should be attempted to find a violation of the restrictions on the capacity of the subset S . Restricted of merging violations sets are also added to the LP. Add some violation restrictions to restriction LP set and proceed to Step 2, if the violation restriction was not found then the objective function value is lower bound; stop.

Kecamatan Seberang Ulu I Model:

In Kecamatan Seberang Ulu I, both two WKs have one garbage truck that is amroll (a truck with open container), so the route that is formed can not be analyzed because each vehicle transporting only one container.

a. WK 1

Use the Equation (1) - (6) thus becomes

Minimize

$$Z = 22,4 x_{01} + 17,4 x_{02} + 22,4 x_{10} + 8 x_{12} + 17,4 x_{20} + 8 x_{21}$$

Subject to :

$$x_{01} + x_{02} = 2$$

$$x_{10} + x_{12} + x_{20} + x_{21} = 2$$

$$x_{01} + x_{02} + x_{12} + x_{21} \leq 2.933$$

$$x_{01}, x_{02}, x_{10}, x_{20}, x_{12}, x_{21} \geq 0$$

LINDO' Result

LP OPTIMUM FOUND AT STEP 3
OBJECTIVE FUNCTION VALUE

1) 79.62980

VARIABLE	VALUE	REDUCED COST
X01	0.000000	1.000000
X02	2.000000	0.000000
X10	2.000000	0.000000
X12	0.000000	3.000000
X20	0.000000	1.000000
X21	0.000000	5.000000

NO. ITERATIONS= 3

b. WK 2

Use the Equation (1) - (6) thus becomes

Minimize

$$18 y_{01} + 16.4 y_{02} + 15.4 y_{03} + 17.7 y_{04} + 17.6 y_{05} + 3.6 x_{12} + 2.6 x_{13} + 3.9 x_{14} + 5 x_{15} + 3.6 x_{21} + x_{23} + 3.3 x_{24} + 3 x_{25} + 2.6 x_{31} + x_{32} + x_{34} + 2 x_{35} + 3.9 x_{41} + 3.3 x_{42} + x_{43} + 3 x_{45} + 5x_{51} + 3 x_{52} + 2 x_{53} + 3 x_{54}$$

Subject to :

$$x_{14} + x_{15} = 1$$

$$y_{02} + y_{20} = 1$$

$$y_{03} + y_{30} = 1$$

$$x_{35} = 1$$

$$x_{42} = 1$$

$$y_{05} + y_{50} = 1$$

$$y_{01} + y_{02} + y_{03} + y_{04} + y_{05} + y_{10} + y_{20} + y_{30} + y_{40} + y_{50} + x_{12} + x_{13} + x_{14} + x_{15} + x_{21} + x_{23} + x_{24} + x_{25} + x_{31} + x_{32} + x_{34} + x_{35} + x_{41} + x_{42} + x_{43} + x_{45} + x_{51} + x_{52} + x_{53} + x_{54} \geq 5.06$$

$$y_{10} + y_{20} + y_{30} + y_{40} + y_{50} - y_{01} - y_{02} - y_{03} - y_{04} - y_{05} + x_{12} + x_{13} + x_{14} + x_{15} + x_{21} + x_{23} + x_{24} + x_{25} + x_{31} + x_{32} + x_{34} + x_{35} + x_{41} + x_{42} + x_{43} + x_{45} + x_{51} + x_{52} + x_{53} + x_{54} \geq 0$$

$$y_{10} + y_{20} + y_{30} + y_{40} + y_{50} = 1$$

$$y_{01}, y_{02}, y_{03}, y_{04}, y_{05}, y_{10}, y_{20}, y_{30}, y_{40}, y_{50} \geq 0$$

$$x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{23}, x_{24}, x_{25}, x_{31}, x_{32}, x_{34}, x_{35}, x_{41}, x_{42}, x_{43}, x_{45}, x_{51}, x_{52}, x_{53}, x_{54} \geq 0$$

LINDO' Result

LP OPTIMUM FOUND AT STEP 8
OBJECTIVE FUNCTION VALUE

1) 41.00000

VARIABLE	VALUE	REDUCED COST
Y02	1.000000	0.000000
Y03	1.000000	0.000000
X14	1.000000	0.000000
X35	1.000000	0.000000
X42	1.000000	0.000000
Y50	1.000000	0.000000

NO. ITERATIONS= 8

Formed the final route is

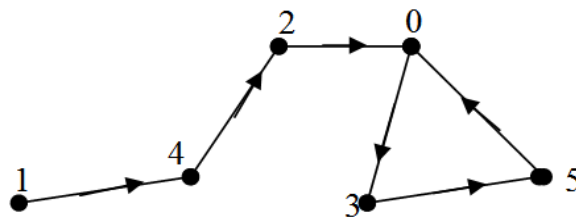
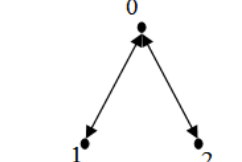
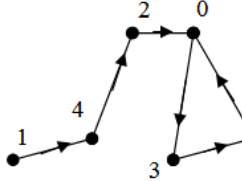
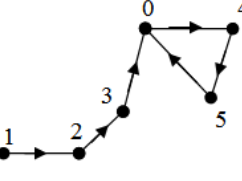
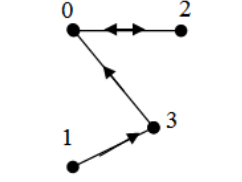
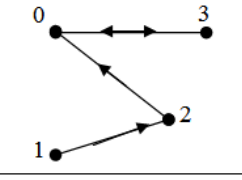
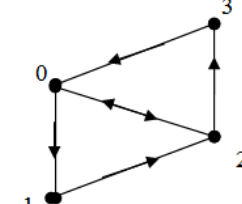
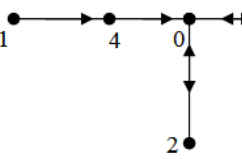
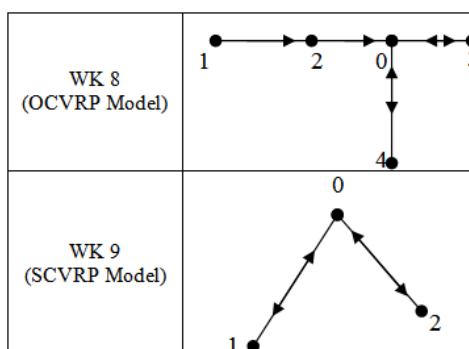


Fig. 1: End Route of Kecamatan SU I in the WK 2

All routes were completed as above, so that it can summarized in the table below which shows the end route of the vehicle after add the addition constraints.

Here, we list the routes of garbage transportation in Kecamatan Seberang Ulu I Kota Palembang.

Working Area (WK) And Model	Optimal Route
WK 1 (SCVRP Model)	
WK 2 (OCVRP Model)	
WK 3 (OCVRP Model)	
WK 4 (OCVRP Model)	
WK 5 (OCVRP Model)	
WK 6 (SCVRP Model)	
WK 7 (OCVRP Model)	



Description:

- WK 1 : TPA(0)- Jl. Ki Merogan (1)-TPA(0) - Jl. Gub. Bastari (2)-TPA(0)
- WK 2 : driver home Jl. Komperta (1) - dpn Lrg. Harapan Jaya I (4)- samping Poltabes(2)-TPA(0)- Jl. Bungaran (3) - Pasar 3 dan 4 Ulu (5) -TPA(0)
- WK 3 : driver home Jl. Mayor Santoso(1) - Depan Panca Usaha (2) - Depan RM. Omega Jaya (3) -TPA(0) - Pasar Kertapati(4) - Depan Tugu KB (5)-TPA(0)
- WK 4 : driver home Jl. Sukabangun II (1) - Belakang SMA Methodist (3)-TPA(0)- Pasar 7 Ulu (2)-TPA(0)
- WK 5 : driver home Jl. Kancil Putih (1) - Depan RS. BARI (2) -TPA(0) - Pasar Induk Jakabaring (3) -TPA(0)
- WK 6 : TPA(0)- Jl. A. Yani (1) - Jl. KH. Wahid Hasyim (2) - TPA(0) - Jl. KH. Wahid Hasyim (2)-Pasar Retail (3)- TPA(0)
- WK 7 : driver home 14 Ulu Jaya Indah (1)- TPS Kantor Camat SU I (4)-TPA(0)- PT. Ali (3)-TPA(0)-OPI (2) -TPA(0)
- WK 8 : driver home Jl. Sukawinatan (1) - Poltabes (3) -TPA(0) - Jl. Silaberanti (2) -TPA(0) - Pasar Sungki (4) -TPA(0)
- WK 9 : TPA(0) - Mutiara (1) -TPA(0) - OPI (2) - TPA(0)

Iv. Summary:

In accordance with the discussion to be worked, then it can be concluded from this study that the garbage transportation system in Kecamatan Seberang Ulu I Palembang have 25 TPS that are divided into 9 Wks (working areas), with 6 Wks are OCVRP, while the other three Wks are SCVRP . This is caused by the differences of where the driver put the vehicle. When the vehicle was in the depot (DKP) is formed SCVRP formulation and when positioned in the driver home, then it is formed OCVRP formulation.

Branch and cut algorithms can be applied for OCVRP as long as relaxation solutions obtained is non integer shape. In this discussion, all OCVRP problems generate integer initial solution, so the branch and cut method does not need to be applied in this OCVRP model. SCVRP model often produces a non integer optimal solution, so some SCVRP models need to be solved with branch and cut method, like on one working area in Kecamatan Seberang Ulu I Palembang.

OCVRP discussed in Palembang city produces an integer optimal solution early, but the initial solution is still not valid, so the need to add additional constraints (16) - (20) to obtain the valid optimal solution.

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