

## Solving Fuzzy Multi-Objective Linear Programming Problems With Linear Membership Functions

<sup>1</sup>C. Veeramani, <sup>2</sup>C. Duraisamy and <sup>3</sup>A. Nagoorgani

<sup>1</sup>Department of Mathematics & Computer Applications, PSG College of Technology,  
Coimbatore-641 004, India.

<sup>2</sup>Department of Mathematics, Kongu Engineering College, Erode - 641 046, India.

<sup>3</sup>PG& Research Department of Mathematics, Jamal Mohamed College (Autonomous),  
Tiruchirappalli-20, India.

---

**Abstract:** Fuzzy multi-objective linear programming (FMLOP) problem has its vast applications in the field of science and engineering. Many authors proposed different method to solve those problems. In this paper, FMOLP problem in which both technological coefficient and resources are fuzzy with linear membership function was studied and a novel approach was proposed to solve the above problem using the technique proposed by Gasimov R. N. and Yenilmez K. (2002).

**Key words:** linear programming problem, decisive set, multi-objective optimization.

---

### INTRODUCTION

Modeling and optimization under a fuzzy environment are called fuzzy modeling and fuzzy optimization. Fuzzy multi-objective linear programming is one of the most frequently applied in fuzzy decision making techniques. Although, it has been investigated and expanded for more than decades by many researchers and from the varies point of view, it is still useful to develop new approaches in order to better fit the real world problems within framework of fuzzy multi-objective linear programming.

However, when formulating the multi-objective programming problem which closely describes and represents the real decision situation, various factors of the real system should be reflected in the description of the objective functions and the constraints. Naturally, these objective functions and the constraints involve many parameters whose possible values may be assigned by the experts. In the traditional approaches, such parameters are fixed at some values in an experimental or subjective manner through the expert's understanding of the nature of the parameters. Unfortunately, real world situations are often not deterministic. There exist various types of uncertainties in social, industrial and economic systems, such as randomness of occurrence of events, imprecision and ambiguity of system data and linguistic vagueness, etc. which come from many ways, including errors of measurement, deficiency in history and statistical data, insufficient theory, incomplete knowledge expression and the subjectivity and preference of human judgment, etc. As pointed out by Zimmermann (1978), various kinds of uncertainties can be categorized as stochastic uncertainty and fuzziness.

Stochastic uncertainty relates to the uncertainty of occurrences of phenomena or events. Its characteristics, lie in that descriptions of information are crisp and well defined; however, they vary in their frequency of occurrence. The systems with this type of uncertainty are called stochastic systems, which can be solved by stochastic optimization techniques using probability theory.

In some other situations, the decision-maker does not think about the frequently used probability distribution which is always appropriate, especially when the information is vague. It may be related to human language and behavior, imprecise/ ambiguous system data. Such types of uncertainty are called fuzziness. It cannot be formulated and solved effectively by traditional mathematics-based optimization techniques and probability based stochastic optimization approaches. The idea of fuzzy set was first proposed by Zadeh (1965), as a mean of handling uncertainty that is due to imprecision rather than to randomness.

#### *Literature Review:*

In 1970 the concept of fuzzy decision and the decision model under fuzzy environments were proposed by Bellman and Zadeh. Zimmerman (1978) first considered MOLP problems with fuzzy goals. Tanaka and Asai (1984) introduced fuzzy linear programming problem in fuzzy environment. There are several methods in the literature for solving multi-objective linear programming models, by adopting fuzzy programming approaches.

---

**Crossponding Author:** C. Veeramani, Department of Mathematics & Computer Applications, PSG College of Technology,  
Coimbatore- 641 004, India.  
E-mail: [veerasworld@yahoo.com](mailto:veerasworld@yahoo.com)

In 1980, Sakawa and Yano introduced the concept of  $\alpha$  –multi-objective programming and (M-)  $\alpha$ – Pareto optimality based on the  $\alpha$  – level sets of the fuzzy numbers. Sakawa, *et al.* (1989) presented an interactive decision making method for multi-objective nonlinear programming problems with fuzzy parameters. They presented several interactive decision making methods not only in objective spaces but also in membership spaces to derive the satisfying solution for the decision maker efficiently from an (M-)  $\alpha$  –Pareto optimal solution set for multi-objective linear, linear fractional and nonlinear programming problems as a generalization of their previous results. Chanas,(1989) proposed a fuzzy programming in MOLP problem and it was solved by parametric approach. Lai-Hawng (1992) considered MOLP problem with all parameters, having a triangular possibility distribution. They used an auxiliary model and it was solved by MOLP methods. Sakawa *et al.* (1994) Presented an interactive fuzzy satisfying method for large-scale FMOLP problems with the block angular structure. Saad (1995) suggested a procedure for solving FMOLP problems and some basic stability notions have been characterized for FMOLP problems. Sakawa *et al.* (1996) focused on large-scale FMOLP problems with the block angular structure. Stanculescu et.al. (2003) have proposed a new methodology that considers fuzzy decision variables for solving FMOLP problems. The FMOLP problem has been transformed to its crisp equivalent, using possibility programming. The crisp MOLP problems, has been solved using the global criterion method and the distance functions method is proposed by M.G. Iskander (2008).

There have been a number of studies on applications of the FMOLP problems to regional planning issues, such as regional environmental management, water resource management, and agricultural development planning etc. In the area of regional environmental management, Sommer and pollatschek (1978) applied a fuzzy programming approach for solving an air pollution regulation problem. Esogbue (1986) applied FDP methods to water pollution control planning. Sakawa (1984) formulated an interactive fuzzy multi-objective nonlinear programming model for water quality management. In water resource management, Slowinski (1986, 1987) proposed an interactive FMOLP method and applied it to water supply planning problems. Kindler (1992) proposed a fuzzy linear programming formulation for water resource planning problems. In agricultural development planning, Czyzak (1989) applied a fuzzy linear programming method for solving multi-criteria agricultural planning problems under uncertainty. Pickens and Hof (1991) applied fuzzy goal programming to forestry management and planning under uncertainty. Recently, David Peidro *et al* (2009) proposed fuzzy optimization for supply chain planning under supply, demand and process uncertainties. In this paper, we have proposed a FMOLP problem in which technological coefficient and resources are fuzzy. Using Bellman and Zadeh's fuzzy decision-making process, the FMOLP problem is converted into an equivalent crisp non-linear programming problem. The non-linear programming problem is solved by fuzzy decisive set method.

The rest of this paper is organized as follows. Section 3, MOLP problem and its solutions are discussed. Fuzzy model of MOLP is given in section 4. In section 5, solution methodology and algorithm for FMOLP are analyzed. In section 6, the example illustrated the fact that the developed method can be successfully applied and conclusion is drawn in Section 7.

**Multi-objective Linear Programming (MOLP) Problem:**

Multi-objective Linear Programming (MOLP) Problems is an interest area of research, since most real-life problems have a set of conflict objectives. A mathematical model of the MOLP problem can be written as follows:

$$\begin{aligned}
 & \text{Max } Z_1(x) = C_1x \\
 & \text{Max } Z_2(x) = C_2x \\
 & \text{Max } Z_k(x) = C_kx \\
 & \text{Subject to } x \in X = \{x \in R^n / Ax = b, x \geq 0\}
 \end{aligned}
 \tag{1}$$

where  $x$  is an  $n$  – dimensional vector of decision variables.  $Z_1(x) \dots Z_k(x)$  are  $k$  – distinct linear objective function of the decision vector  $x$ .  $C_1, C_2 \dots C_k$  are  $n$  – dimensional cost factor vectors,  $A$  is an  $m \times n$  constraint matrix,  $b$  is an  $m$  – dimensional constant vector.

Definition 3. 1. (Complete Optimal Solution) The point  $x^* \in X$  is said to be a complete optimal solution of the MOLP problem (1), if  $Z_i(x^*) \geq Z_i(x), i = 1, 2, \dots k$  for all  $x \in X$ .

In general, when the objective functions conflict with one another, a complete optimal solution may not exist and hence, a new concept of optimality, called Pareto optimality, is considered.

Definition 3. 2. (Pareto Optimal Solution) The point  $x^* \in X$  is said to be a Pareto optimal solution if there does not exist  $x \in X$  such that if  $Z_i(x) \geq Z_i(x^*)$  for all  $i$  and  $Z_j(x) > Z_j(x^*)$  for at least one  $j$ .

**Fuzzy Multi-objective Linear Programming (FMOLP) Problem:**

The model (1), all coefficients of A, b and C are crisp numbers. However, in the real-world decision problems, a decision maker does not always know the exact values of the coefficients taking part in the problem, and that vagueness in the coefficients may not be a probabilistic type. In this situation, the decision maker can model inexactness by means of fuzzy parameter. In this section we consider a FMOLP problem with fuzzy technological coefficients and fuzzy resources. A mathematical model of the FMOLP problem can be written as follows:

$$\begin{aligned} \text{Max } Z_1(x) &= C_1x \\ \text{Max } Z_2(x) &= C_2x \\ \text{Max } Z_k(x) &= C_kx \end{aligned} \tag{2}$$

$$\text{Subject to } x \in X = \{x \in E^n / \tilde{A}x = \tilde{b}, x \geq 0\}$$

where  $x$  is an  $n$  – dimensional vector of decision variables.  $Z_1(x) \dots Z_k(x)$  are  $k$  – distinct linear objective function of the decision vector  $x$ ,  $C_1, C_2 \dots C_k$  are  $n$  – dimensional cost factor vectors,  $\tilde{A}$  is an  $m \times n$  constraint fuzzy matrix,  $\tilde{b}$  is an  $m$  – dimensional constant fuzzy vector (fuzzy resources).

The membership function of the fuzzy matrix  $\tilde{A}$  is

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x \leq a_{ij} \\ (a_{ij} + d_{ij} - x)/d_{ij}, & a_{ij} \leq x \leq a_{ij} + d_{ij} \\ 0, & x \geq a_{ij} + d_{ij} \end{cases}$$

where  $x \in R$  and  $d_{ij} > 0$  (tolerance level) for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

The membership function for the fuzzy resources  $\tilde{b}$  is

$$\mu_{\tilde{b}}(x) = \begin{cases} 1, & x \leq b_i \\ (b_i + p_i - x)/p_i, & b_i \leq x \leq b_i + p_i \\ 0, & x \geq b_i + p_i \end{cases}$$

where  $x \in R$  and  $p_i > 0$  (tolerance level), for  $i = 1, 2, \dots, m$ .

**Solution Methodology and Algorithm:**

In this section, we first fuzzify the objective function in order to defuzzificate the problem (2). It is done by calculating the lower and upper bounds of the optimal values. The bounds of the optimal values  $Z_q^l$  and  $Z_q^u$  are obtained by solving the standard linear programming problems.

$$Z_q^1 = \max \sum_{j=1}^n c_j x_j, \quad q = 1, 2, \dots, k$$

Subject to

$$\begin{aligned} \sum_{j=1}^n (a_{ij} + d_{ij})x_j &\leq b_i \quad i = 1, 2 \dots m \\ x_j &\geq 0 \end{aligned} \tag{3}$$

$$Z_q^2 = \max \sum_{j=1}^n c_j x_j, \quad q = 1, 2, \dots, k$$

Subject to

$$\begin{aligned} \sum_{j=1}^n a_{ij}x_j &\leq b_i + p_i \quad i = 1, 2 \dots m \\ x_j &\geq 0 \end{aligned} \tag{4}$$

$$Z_q^3 = \max \sum_{j=1}^n c_j x_j, \quad q = 1, 2, \dots, k$$

Subject to

$$\sum_{j=1}^n (a_{ij} + d_{ij})x_j \leq b_i + p_i, \quad i = 1, 2 \dots m \tag{5}$$

$$x_j \geq 0$$

$$Z_q^4 = \max \sum_{j=1}^n c_j x_j, \quad q = 1, 2, \dots, k$$

Subject to

$$\sum_{j=1}^n a_{ij} x_j \leq b_i \quad i = 1, 2 \dots m \tag{6}$$

$$x_j \geq 0$$

Let  $Z_q^l = \min (Z_q^1, Z_q^2, Z_q^4, Z_q^4)$  and  $Z_q^u = \max (Z_q^1, Z_q^2, Z_q^4, Z_q^4)$ . The objective function takes the values between  $Z_q^l$  and  $Z_q^u$  while the technological coefficients take values between  $a_{ij}$  and  $a_{ij} + d_{ij}$  and the right-hand side numbers takes the values  $b_i$  and  $b_i + p_i$ .

Then, the fuzzy set of  $j^{th}$  optimal value,  $G_j$  which subset for  $R^n$ , is defined by

$$\mu_{G_j}(x) = \begin{cases} 0, & \text{if } \sum_{j=1}^n c_j x_j \leq Z_q^l \\ (\sum_{j=1}^n c_j x_j - Z_q^l) / (Z_q^u - Z_q^l), & \text{if } Z_q^l \leq \sum_{j=1}^n c_j x_j \leq Z_q^u \\ 1, & \text{if } \sum_{j=1}^n c_j x_j \geq Z_q^u \end{cases} \tag{7}$$

The fuzzy set of the  $j^{th}$  constraint,  $C_j$  which subset for  $R^n$ , is defined by

$$\mu_{C_j}(x) = \begin{cases} 0, & b_i \leq \sum_{j=1}^n a_{ij} x_j \\ (b_i - \sum_{j=1}^n a_{ij} x_j) / (\sum_{j=1}^n d_{ij} x_j + p_i), & \sum_{j=1}^n a_{ij} x_j \leq b_i \leq \sum_{j=1}^n (a_{ij} + d_{ij})x_j + p_i \\ 1, & b_i \geq \sum_{j=1}^n (a_{ij} + d_{ij})x_j + p_i \end{cases} \tag{8}$$

By using the definition of the fuzzy decision proposed by Bellman and Zadeh, we have

$$\mu_D(x) = \min_j (\mu_{G_j}(x), \min (\mu_{C_j}(x))) \tag{9}$$

In this case the optimal fuzzy decision is a solution of the problem

$$\max_{x \geq 0} (\mu_D(x)) = \max_{x \geq 0} (\min_j (\mu_{G_j}(x), \min (\mu_{C_j}(x)))) \tag{10}$$

Consequently, the problem (2) is reduced to the following optimization problem

$$\begin{aligned} \max \quad & \lambda \\ \lambda (Z_q^u - Z_q^l) - \sum_{j=1}^n c_j x_j + Z_q^l & \leq 0 \\ \sum_{j=1}^n (a_{ij} + \lambda d_{ij})x_j + \lambda p_i - b_i & \leq 0 \\ x_j & \geq 0, \quad 0 \leq \lambda \leq 1 \end{aligned} \tag{11}$$

Notice that, the problem (11) containing the cross product terms  $\lambda x_i$  are not convex Therefore, the solution of the problem requires the special approach adopted for solving general non-convex optimization problems.

**The Algorithm of the Fuzzy Decisive Set Method:**

This method is based on the idea that, for a fixed value of  $\lambda$ , the problem (11) is converted in to linear programming problem. Obtaining the optimal solution  $\lambda^*$  is equivalent to determining the maximum value of  $\lambda$  so that the feasible set is nonempty. The algorithm of this method for the problem (11) is presented below.

Algorithm:

**Step 1:**

Set  $\lambda=1$  and test whether a feasible set satisfying the constraints of the problem (11) exists or not using phase one of the simplex method. If a feasible set exists, set  $\lambda = 1$ . Otherwise, set  $\lambda^L = 0$  and  $\lambda^R = 1$  and go to the next step.

**Step 2:**

For the value of  $\lambda = \frac{\lambda^L + \lambda^R}{2}$ , update the value of  $\lambda^L$  and  $\lambda^R$  using the bisection method as follows:

$$\lambda^L = \lambda, \text{ if feasible set is nonempty for } \lambda$$

$$\lambda^R = \lambda, \text{ if feasible set is empty for } \lambda.$$

Consequently, for each  $\lambda$ , test whether a feasible set of the problem (11) exists or not using phase one of the Simplex method and determine the maximum value  $\lambda^*$  satisfying the constraints of the problem (11)

**Numerical Example:**

Consider the following FMOLPP

$$\max Z_1(x) = 10x_1 + 11x_2 + 15x_3$$

$$\max Z_2(x) = 4x_1 + 5x_2 + 9x_3$$

Subject to

$$\tilde{1}x_1 + \tilde{1}x_2 + \tilde{1}x_3 \leq \tilde{15} \tag{12}$$

$$\tilde{7}x_1 + \tilde{5}x_2 + \tilde{3}x_3 \leq \tilde{80}$$

$$\tilde{3}x_1 + \tilde{4.4}x_2 + \tilde{10}x_3 \leq \tilde{100}$$

$$x_1, x_2, x_3 \geq 0$$

which takes fuzzy parameters as:  $\tilde{1} = L(1,1)$ ,  $\tilde{7} = L(7,4)$ ,  $\tilde{5} = L(5,3)$ ,  $\tilde{3} = L(3,1)$ ,  $\tilde{4.4} = L(4.4, 2)$ ,  $\tilde{10} = L(10,4)$ ,  $\tilde{15} = L(15,5)$ ,  $\tilde{80} = L(80,40)$ ,  $\tilde{100} = L(100,30)$ , as used by Shaocheng (1994).

$$\text{That is } (a_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 7 & 5 & 3 \\ 3 & 4.4 & 10 \end{pmatrix}, (d_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix} \Rightarrow (a_{ij} + d_{ij}) = \begin{pmatrix} 3 & 3 & 3 \\ 11 & 8 & 4 \\ 4 & 6.4 & 14 \end{pmatrix}$$

$$(b_i) = \begin{pmatrix} 15 \\ 80 \\ 100 \end{pmatrix}, (p_i) = \begin{pmatrix} 5 \\ 40 \\ 30 \end{pmatrix} \Rightarrow (b_i + p_i) = \begin{pmatrix} 20 \\ 120 \\ 130 \end{pmatrix}$$

For defuzzification of the problem (12), we first fuzzify the objective function. This is done by calculating the lower and upper bounds of the optimal values first. The bounds of the optimal values  $Z_q^L$  and  $Z_q^u$  are obtained by solving the standard linear programming problems

$$\max Z_1(x) = 10x_1 + 11x_2 + 15x_3$$

$$\max Z_2(x) = 4x_1 + 5x_2 + 9x_3$$

Subject to

$$x_1 + x_2 + x_3 \leq 15 \tag{13}$$

$$7x_1 + 5x_2 + 3x_3 \leq 80$$

$$3x_1 + 4.4x_2 + 10x_3 \leq 100$$

$$x_1, x_2, x_3 \geq 0$$

$$\max Z_1(x) = 10x_1 + 11x_2 + 15x_3$$

$$\max Z_2(x) = 4x_1 + 5x_2 + 9x_3$$

Subject to

$$\begin{aligned}
 x_1 + x_2 + x_3 &\leq 20 \\
 7x_1 + 5x_2 + 3x_3 &\leq 120 \\
 3x_1 + 4.4x_2 + 10x_3 &\leq 130 \\
 x_1, x_2, x_3 &\geq 0 \\
 \max Z_1(x) &= 10x_1 + 11x_2 + 15x_3 \\
 \max Z_2(x) &= 4x_1 + 5x_2 + 9x_3
 \end{aligned}
 \tag{14}$$

Subject to

$$\begin{aligned}
 2x_1 + 2x_2 + 2x_3 &\leq 15 \\
 11x_1 + 8x_2 + 4x_3 &\leq 80 \\
 4x_1 + 6.4x_2 + 14x_3 &\leq 100 \\
 x_1, x_2, x_3 &\geq 0 \\
 \max Z_1(x) &= 10x_1 + 11x_2 + 15x_3 \\
 \max Z_2(x) &= 4x_1 + 5x_2 + 9x_3
 \end{aligned}
 \tag{15}$$

Subject to

$$\begin{aligned}
 2x_1 + 2x_2 + 2x_3 &\leq 20 \\
 11x_1 + 8x_2 + 4x_3 &\leq 120 \\
 4x_1 + 6.4x_2 + 14x_3 &\leq 130 \\
 x_1, x_2, x_3 &\geq 0
 \end{aligned}
 \tag{16}$$

Optimal values of these problems are  $Z_1 = (189.29, 250, 110, 145)$  and  $Z_2 = (99.29, 130, 65, 85)$  respectively. Therefore,  $Z_1^l = 110$ ,  $Z_1^u = 250$ ,  $Z_2^l = 65$  and  $Z_2^u = 130$ . By using these optimal values, the problem (12) can be reduced by the following non-linear programming problem:

$$\begin{aligned}
 \max \lambda \\
 \frac{10x_1 + 11x_2 + 15x_3 - 110}{250 - 110} &\geq \lambda \\
 \frac{4x_1 + 5x_2 + 9x_3 - 65}{130 - 65} &\geq \lambda \\
 \frac{15 - x_1 - x_2 - x_3}{2x_1 + 2x_2 + 2x_3 + 5} &\geq \lambda \\
 \frac{80 - 7x_1 - 5x_2 - 3x_3}{4x_1 + 3x_2 + x_3 + 40} &\geq \lambda \\
 \frac{100 - 3x_1 - 4.4x_2 - 10x_3}{x_1 + 2x_2 + 4x_3 + 30} &\geq \lambda \\
 x_1, x_2, x_3 &\geq 0 \quad 0 \leq \lambda \leq 1
 \end{aligned}$$

that is,

$$\begin{aligned}
 \max \lambda \\
 10x_1 + 11x_2 + 15x_3 &\geq 110 + 140\lambda \\
 4x_1 + 5x_2 + 9x_3 &\geq 65 + 65\lambda \\
 (2\lambda + 1)x_1 + (2\lambda + 1)x_2 + (2\lambda + 1)x_3 &\leq 15 - 15\lambda
 \end{aligned}
 \tag{17}$$

$$\begin{aligned} (4\lambda + 7)x_1 + (3\lambda + 5)x_2 + (\lambda + 3)x_3 &\leq 80 - 40\lambda \\ (\lambda + 3)x_1 + (2\lambda + 4.4)x_2 + (4\lambda + 10)x_3 &\leq 100 - 30\lambda \\ x_1, x_2, x_3 &\geq 0 \quad 0 \leq \lambda \leq 1 \end{aligned}$$

Let us solve the problem (17) by using fuzzy decisive set method.

For  $\lambda = 1$ , the problem can be written as

$$\begin{aligned} 10x_1 + 11x_2 + 15x_3 &\geq 250 \\ 4x_1 + 5x_2 + 9x_3 &\geq 130 \\ 3x_1 + 3x_2 + 3x_3 &\leq 10 \\ 11x_1 + 8x_2 + 4x_3 &\leq 40 \\ 4x_1 + 6.4x_2 + 14x_3 &\leq 70 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Since the feasible set is empty, by taking  $\lambda^L = 0$  and  $\lambda^R = 1$ , the new value of  $\lambda = \frac{0+1}{2} = \frac{1}{2}$  is tried.

For  $\lambda = \frac{1}{2} = 0.5$ , the problem (17) can be written as

$$\begin{aligned} 10x_1 + 11x_2 + 15x_3 &\geq 180 \\ 4x_1 + 5x_2 + 9x_3 &\geq 97.5 \\ 2x_1 + 2x_2 + 2x_3 &\leq 12.5 \\ 9x_1 + 6.5x_2 + 3.5x_3 &\leq 60 \\ 3.5x_1 + 5.4x_2 + 12x_3 &\leq 85 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Since the feasible set is empty, by taking  $\lambda^L = 0$  and  $\lambda^R = 0.5$ , the new value of  $\lambda = \frac{0+1/2}{2} = \frac{1}{4}$  is tried.

For  $\lambda = 0.25$ , the problem (17) can be written as

$$\begin{aligned} 10x_1 + 11x_2 + 15x_3 &\geq 145 \\ 4x_1 + 5x_2 + 9x_3 &\geq 81.25 \\ 1.5x_1 + 1.5x_2 + 1.5x_3 &\leq 13.75 \\ 8x_1 + 5.75x_2 + 3.25x_3 &\leq 70 \\ 3.25x_1 + 4.9x_2 + 11x_3 &\leq 92.5 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Since the feasible set is empty, by taking  $\lambda^L = 0$  and  $\lambda^R = 1/4$ , the new value of  $\lambda = \frac{0+1/4}{2} = \frac{1}{8}$  is tried.

For  $\lambda = 0.125$ , the problem (17) can be written as

$$\begin{aligned} 10x_1 + 11x_2 + 15x_3 &\geq 127.5 \\ 4x_1 + 5x_2 + 9x_3 &\geq 73.125 \\ 1.25x_1 + 1.25x_2 + 1.25x_3 &\leq 14.375 \\ 7.5x_1 + 5.375x_2 + 3.125x_3 &\leq 75 \\ 3.125x_1 + 4.65x_2 + 10.5x_3 &\leq 96.25 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Since the feasible set is nonempty, by taking  $\lambda^L = 1/8$  and  $\lambda^R = 1/4$ , the new value of  $\lambda = \frac{1/8+1/4}{2} = \frac{3}{16}$  is tried.

For  $\lambda = 0.19$ , the problem (17) can be written as

$$10x_1 + 11x_2 + 15x_3 \geq 136.25$$

$$4x_1 + 5x_2 + 9x_3 \geq 77.19$$

$$1.38x_1 + 1.38x_2 + 1.38x_3 \leq 14.06$$

$$7.75x_1 + 5.5x_2 + 3.19x_3 \leq 72.5$$

$$3.19x_1 + 4.8x_2 + 10.75x_3 \leq 94.38$$

$$x_1, x_2, x_3 \geq 0$$

Since the feasible set is nonempty, by taking  $\lambda^L = 3/16$  and  $\lambda^R = 1/4$ , the new value of  $\lambda = \frac{3/16+1/4}{2} = \frac{7}{32}$  is tried.

Similarly, we continue the above process, the following values of  $\lambda$  are obtained:

$$\lambda = 7/32 = 0.2188$$

$$\lambda = 13/64 = 0.2031$$

$$\lambda = 27/128 = 0.2109$$

$$\lambda = 53/256 = 0.2070$$

$$\lambda = 107/512 = 0.2089$$

$$\lambda = 213/1024 = 0.2080$$

$$\lambda = 427/2048 = 0.2085$$

$$\lambda = 853/4096 = 0.2083$$

$$\lambda = 1705/8192 = 0.2081$$

$$\lambda = 3409/16384 = 0.2081$$

Consequently, we obtain the optimal value of  $\lambda$  at the fifteenth iteration by using the fuzzy decisive set method. The optimal solution is  $x_1 = 1.67$ ,  $x_2 = 0$ ,  $x_3 = 8.16$ ,  $Z_1 = 139.1$ ,  $Z_2 = 80.12$  and  $\lambda = 0.2081$ .

**Conclusion:**

In this paper, fuzzy multi-objective linear programming problem in which both the resources and the technological coefficients are fuzzy with linear membership function was studied. Further a FMLOP problem was converted into an equivalent crisp non-linear programming problem using the concept of max-min principle. The resultant non-linear programming problem was solved by fuzzy decisive set method. The discussed method was illustrated through an example. In future proposed method can be extended to solve problems like FMLOP with triangular or trapezoidal membership function and linear fuzzy fractional programming problems .

**REFERENCES**

A.Q., Esogbue, 1986, Optimal Clustering of Fuzzy Data via Fuzzy Dynamic Programming, Fuzzy Sets and Systems, 18, 283-298.  
 Bellman R.E., Zadeh L.A. 1970, Decision making in a fuzzy environment, Management Science 17, 141-164.  
 C., Stanculescu Ph., Fortemps, M. Installee and V. Wertz 2003, Multi-objective fuzzy linear programming problems with fuzzy decision variables, European Journal of Operational Research, 149, 654-675.  
 D., Chanas 1989, Fuzzy programming in multiobjective linear programming-a parametric approach, Fuzzy Set and system 29, 303-313.  
 G., Sommer and Pollatschek 1978, A fuzzy programming approach to an air pollution regulation problem. In: R. Trappl and G.J. Klir, Editors, Progress in Cybernetics and Systems Research Vol. III , 303-313.  
 H., Ishibuchi H. Tanaka 1990, Multi-objective programming in optimization of the interval objective function, European Journal of Operational Research 48, 219-225.

- H., Tanaka and K., Asai 1984, Fuzzy linear programming problems with fuzzy numbers, *Fuzzy Sets and Systems*, 13, 1-10.
- H.J., Zimmermann 1978, Fuzzy programming and linear programming with several objective functions, *Fuzzy sets and System* 1, 45- 55.
- J., Kindler 1992, Rationalizing water requirements with aid of fuzzy allocation model. *J. Water Res. Plng. and Mgmt.*, ASCE, 118, 308-323.
- J.B., Pickens and Hof J.G. 1991, Fuzzy goal programming in forestry: an application with special solution problems, *Fuzzy Sets and Systems*, 39, 239-246.
- L.A., Zadeh 1965, Fuzzy sets, *Information and Control*, 8, 69-78.
- M., Sakawa and H., Yano 1985, Interactive decision making for multi-objective linear fractional programming problems with fuzzy parameters. *Cybernetics Systems* 16, 377-394.
- M., Sakawa and H., Yano, 1989, Interactive fuzzy satisficing method for multi-objective nonlinear programming problems with fuzzy parameters, *Fuzzy Sets and Systems*, 30, 221-238.
- M., Sakawa and K., Sawada 1994, An interactive fuzzy satisficing method for large-scale multi-objective linear programming problems with block angular structure, *Fuzzy Sets and Systems*, 67, 5-17.
- M., Sakawa Inuiguchi, K., Sawada 1996, A fuzzy satisficing method for large-scale multi-objective linear programming problems with block angular structure, *Fuzzy Sets and Systems* 78, 279- 288.
- O.M., Saad 1995, Stability on multi-objective linear programming problems with fuzzy parameters, *Fuzzy Sets and Systems*, 74, 207-215.
- P., Czyzak 1989, Multicriteria agricultural problem solving under uncertainty, *Foundations of Control Engineering*, 14, 61-80.
- Peidro Mula Poler R. and José-Luis Verdegay 2009, Fuzzy optimization for supply chain planning under supply, demand and process uncertainties *Fuzzy Sets and Systems*, 160, 2640-2657.
- R., Slowinski 1986, A multi-criteria fuzzy linear programming method for water supply system development planning, *Fuzzy Sets and Systems*, 19, 217-237.
- R., Slowinski, 1987, An interactive method for multi-objective linear programming with fuzzy parameters and its application to water supply planning. In: Kacprzyk, J. and Orlovski, S.A., Editors.
- R.N., Gasimov and K., Yenilmez 2002, Solving fuzzy linear programming with linear membership functions. *Turk J. Math.* 26, 375-396.
- S., Tong 1994, Interval number and fuzzy number linear programming, *Fuzzy Sets and Systems* 66, 301-306.
- S.H., Nasser M., Sohrabi 2010, Solving Fuzzy Linear Programming by Using Revised Tsao's Method, *Australian Journal of Basic and Applied Sciences*, 4, 4592-4600.
- S.K., Das Goswami A. and S.S., Alam 1999, Multi-objective transportation problem with interval; cost, source and destination parameters., *European Journal of Operational Research* 117, 100-112.
- T., Shaocheng, 1994, Interval number and Fuzzy number linear programming, *Fuzzy Sets and Systems* 66, 301-306.
- V, Iskander 2008, A computational comparison between two evaluation criteria in fuzzy multi-objective linear programs using possibility programming, *Computers and Mathematics with Applications* 55, 2506-2511
- Y.J., Lai and C.L., Hawng 1992, *Fuzzy Mathematical Programming*, Lecture notes in Economics and Mathematical systems, Springer-Verlag.