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Fatigue Life Modelling Under Constant Amplitude Loading Using Adaptive Neuro - Fuzzy (ANFIS) Application

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Abstract: This work illustrates the using ANFIS model for predicting the fatigue life fatigue and crack growth rate (FCGR).The main purpose of this work is to consider a new crack propagation criterion based on prediction experimental tests on three point-bend (TPB) specimens, which allow predicting the fatigue life and FCG rate using ANFIS and FIS. The benefit of the proposed method is to estimate the initial FCGR with the number of cycle's relationship for case study. The aim of this work is to propose a novel architecture called a simply ANFIS, which can serve as a basis for constructing a set of fuzzy if-then a system with appropriate membership functions to generate the stipulated input-output pairs. The results achieved from the developed ANFIS system show a good matching with the experimental results when tested on ASTM A533 alloy.

Keywords: Fatigue crack growth rate, adaptive neuro-fuzzy inference system (ANFIS), fuzzy inference system (FIS), Constant amplitude loading.

INTRODUCTION

Crack growth in structures depends on the amplitude, stress ratio and frequency of the load. Due to the random nature of variable loading, it is difficult to model all these influential parameters correctly. It overloads are known to retard crack growth, while under loads accelerate crack growth relative to the background rate. These interactions which are highly dependent upon the loading sequence. The FCGR is greatly influenced by the order of loading most especially when it involves variable amplitude loading (VAL). The most basic form of VAL happens when high peak loads are intertwined in constant amplitude loading (CAL) history. There is a situation of reduction in the growth of fatigue crack due to reduced crack growth when compared to the average constant amplitude loading growth rate which often times is occasioned by the tensile nature of the loading sequence. Due to the absence of a thorough knowledge and mastering of the art of retardation micro-mechanisms, it has not really been easy in gauging life on the basis of the very complicated circumstances. This assertion is however premised on different approaches as well as on some theorised retardation equations (Ray A and Patanker P, 2001).

The fracture mechanics have been developed to support the economical fail-safe and damage tolerance concepts. These models may be categorised as global analysis and cycle-by-cycle analysis. The former's approach relies on forecasting the fatigue crack growth (FCG), having taken into consideration the mean of all the applied loading sequences, while the latter calculates the growth of crack for each loading sequence and the crack growth life is evaluated by the summation proposd by Elber as reported in (Kujawski 2001).The cycle-by-cycle analysis can be performed with or without involving the interaction effects, the effect of a load cycle on the crack growth in later cycles. A well-known interaction effect is caused by an overload on crack growth in the subsequent load cycle. The models that take interaction effects into account can be divided into three main categories namely yield zone models, crack closure models and strip yield models (Khan, 2004).

Many models had been developed to predict the fatigue life of components subjected to variable amplitude loading has been analysed by many different authors (Schijve J *et al*. 2004). The earliest of these are based on calculations of the yield zone size ahead of the crack tip and are still extensively used. The Wheeler and Willenborg models as reported in (Taheri *et al.* 2003), for example, both fall into this category, another category model based on the crack closure approach, which considers plastic deformation and crack face interaction in the wake of the crack, was subsequently proposed by Elber as reported in (Sander and Richard 2006), have been used to model crack growth rates under variable amplitude loads (Ray A, Patanker P, Part I,II, ,2001). The strip yield model is the most modern and it operates on the principles of the Dugdale model. The Dugdale model as reported in (Khan, 2004) was used to estimate the size of the plastic zone at the tip of the crack. Dugdale assumes that yielding occurs in a narrow strip ahead of the crack tip. The material response to plastic deformation is rigid-perfectly plastic, which leads to a constant stress (yield stress) in the plastic zone.

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Many methods involving algorithm- based programs such as artificial neural network (ANN), fuzzy-logic, adaptive neuro-fuzzy inference system (ANFIS) and optimization methods genetic algorithm (GA) can be used in the fatigue crack growth estimation. Although, GA and ANN have been performed in analyzing the numerous fatigue problem by more than a few researchers. Besides, ANFIS and fuzzy-logic only have been reported from the small number in research paper. Starting from (2002) Jarrah, Al-Assaf, and El Kadi, they were first group researchers to implement ANFIS into the field of fatigue. In their research, glass fiber/epoxy has been used and the ANFIS modeling results shown a good agreement with limited data set. Vassilopoulos and Bedi in (2008) have been successfully applied multidirectional composite laminates in modeling fatigue life using ANFIS. Moreover, mohanty, Verma in (2008, 2009 and 2010) have been applied a noval soft – computational method ANFIS in case mixed- mode overloads model – I to predict fatigue life, using 7020T7 and 2024T3 aluminum alloys. Crack propagation simulated with previously obtained experimental data may be more preferable in that under this situation, the fatigue tests are inexpensive, fast and non-complex in nature. It has proven to be a powerful and versatile soft-computing method which is quite efficient in modelling complicated linear and non-linear relationships based on of experimental data in a number of engineering fields (H. Al-Nashash *et al*.2001).

One of the most modern techniques designed to successfully check fatigue-induced problems is ANFIS and a new class of the computational intelligence system, useful to handle various complicated problems with a capacity to learn by examples.

Recently, ANFIS has been an intermediate between ANN and fuzzy inference system (FIS) models. While, each of the two models had their respective merits and demerits. However, ANFIS as an intermediate between the two possess the merits of both the two models with none of their demerits. It is an indispensable tool and has further advantage over the fuzzy system in that it does not require manual optimization in order to function. The system variables could be controlled automatically with the use of neural network mechanism. The focus of this study is on the prediction of fatigue life of ASTM A533 alloy under the different load histories by applying ANFIS model. The aim of this paper is to suggest a novel architecture called Adaptive Network- Based Fuzzy Inference System, or simply ANFIS, which can serve as a basis for constructing a set of fuzzy if-then rules with appropriate membership functions to generate the stipulated input-output pairs.

Fuzzy If-Then Rules and Fuzzy Inference Systems:

System modelling based on conventional mathematical tools (e.g., differential equations) is not well suited for dealing with ill-defined and unsure systems. By difference, a fuzzy inference system employing fuzzy if-then rules can model the qualitative facet of human knowledge and reasoning processes without employing precise quantitative analyses. This fuzzy modeling or fuzzy identification, first explored systematically by Takagi and Sugeno (R.Jang 1994), has found numerous practical applications in control prediction and inference (A.Kandel,1988,1992). However, there are some fundamental aspects of this approach which are in need of better understanding. More specifically:

1. No standard methods exist for transforming human knowledge or experience into the rule base and database of a fuzzy inference system.
2. There is a need for effective methods for tuning the membership functions (MF's) so as to minimize the output error measure or maximize performance index. The next section introduces the basics of fuzzy if-then rules and fuzzy inference systems.

Fuzzy 'IF-THEN' Rules:

Fuzzy conditional statements are expressions of the form if A then B, where A and B is labels of fuzzy sets proposed by Zadah as reported in (Jang 1994) characterized by appropriate membership functions. Due to their concise form, fuzzy if-then rules are frequently employed to capture the imprecise modes of reasoning that play an essential role in the human ability to take decisions in an environment of uncertainty and imprecision. Another form of fuzzy rules, proposed by Takagi and Sugeno (R.Jang 1994), have fuzzy sets concerned only in the premise part. By using Takagi and Sugeno's fuzzy if-then rule, in this case, the product is a first-order polynomial while crisp function represents the fuzzy rules of the output (J. Roger,1992). Both types of fuzzy these rules have been used extensively in both modelling and control. Through the use of linguistic labels and membership functions, a fuzzy if-then rule can simply capture the spirit of a "rule of thumb" used by humans. From another angle, due to the qualifiers on the premise parts, each fuzzy if-then rule can be viewed as a local description of the system under consideration. The fuzzy –rule system as shown in figure 1.

Hybrid Learning Rule:

The back propagation computer-based programming where the equation's factors are continuously reviewed by a gradient descent optimization method is one of the simplest regulations guiding the knowledge of adaptive network. The application of the gradient approach which is used to discover the factors which play prominent roles in adaptive network, which at times is not fast and thus prone to being constrained to a specific place is highly restrained (Mohanty *et al.,* 2010). There is a method presently being put forward which seeks to amalgamate the gradient approach with the least squares estimate (LSE) in recognising the prominent factors.

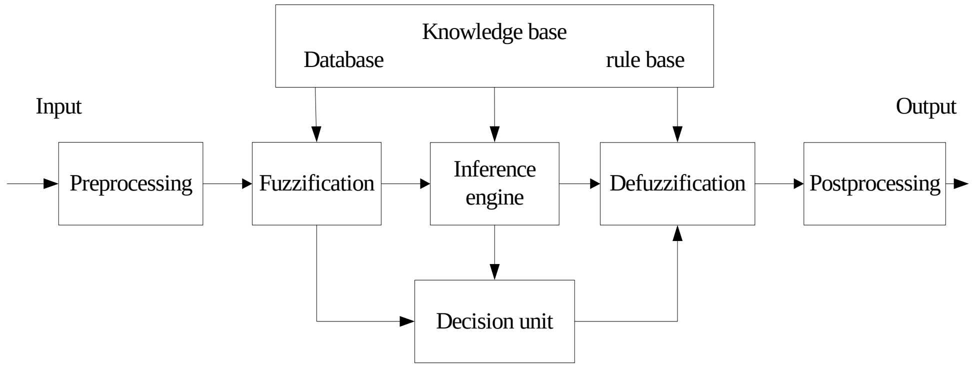


Fig. 1: Fuzzy –Rule – Based Systems

The method is referred to as Hybrid Learning Rule (HLR). As a simple illustration, imagine there is an output for the adaptive network being considered:

$$output = F\ (\vec{I}, S), \tag{1}$$

there are two sets involved, the first is designated as S, representing total parameter set while the second is designated as $\vec{I}$ i.e. the set of input variables. Provided there is a function designated as H whose composite function $H * F$ aligns linearly with some of the characters of S, the least square's method can be utilise to detect the presence of these characters provided also that the character set S can be resolved into two sets:

$$S = S_1 \oplus S_2 \tag{2}$$

For the remaining parameters, the linearity of their output depends only on the condition that the stated parameters remain unchanged in value. However, if the total parameter set is divided into three vis-à-vis S_1: comprising of the nonlinear parameters, S_2: comprising of linear parameters and $\oplus$: being the actual summation in the event that $H * F$ aligns linearly with elements of S_2, *b*ased on the application of H to equation (1), the equation becomes:

$$H(output) = H * F\ (\vec{I}, S), \tag{3}$$

It thus becomes linearized in the elements of S_2. Having known the values of the elements of set S_1 s and putting the training data into equation (3) lead to a matrix equation being obtained as shown:

$$AX = B \tag{4}$$

where X is an indefinite vector whose elements are parameters in S_2. Let $|S_2| = M$, then the dimensions of A, X and B are P x M, M x 1 and P x 1, respectively. Seeing as P (number of training data pairs) is usually greater than M (number of linear parameters), Equation 4 may not be able to be solved in the proper way because this may be an ambiguous and arduous task. However, the squared error $\|AX - B\|^2$ could be reduced by utilising the least square estimate (LSE). This problem may be a precursor to the principles of linear regression, adaptive filtering and signal processing. The pseudo-inverse of X represented by:

$$X^* = (A^T A)^{-1} A^T B, \tag{5}$$

Is used by a prominent formula for X^*, A^T refers to the transpose of A, and $(A^T A)^{-1} A^T$ represents the pseudo-inverse of A provided $A^T A$ is non-singular. Though, equation (5) appears quite simple in configuration, however, it's evaluation is ambiguous most especially when inverse matrix is involved and eventually becoming more cumbersome if $A^T A$ is singular.

The practise nowadays is to use in joint capacity the gradient method and the least square's estimate (LSE) to bring to the fore the prominent characters in an adaptive network. The respective steps in this new arrangement involve both forward and backward moves. Under the former, before the two matrices in equation (4) above could be obtained, input data and functional signals must be provided to ensure the determination of each nodal output and detect the characters in S_2 making use of the Least Squares; also, error measures for respective training pair are calculated. The least square approach is often used to isolate the different prominent characters in the output layer while the parameters in the hidden layer can be reviewed using the back-propagation learning rule. For each constant entity of set S_1 the characters in S_2 determined by this system are certainly programmed for an all-round most valuable point. The error from the output may be useful in preparing the most prominent characters by a generally acclaimed back propagation computer program. However, this special arrangement has been shown to be very appropriate in giving specific instructions to the ANFIS.

Structure of ANFIS:

The configuration of ANFIS structure used in the present work as shown in figure 2, which is a first-order Takagi–Sugeno type. The network analyses the system's output for given input data set through fuzzy if-then rules. The optimal model parameter is determined by hybrid-learning algorithms. The Adaptive-Network-Based Fuzzy Inference System is a fuzzy Sugeno model put in the framework of adaptive systems to assist learning and adaptation (Vassilopoulos *et al*., 2008). Framework makes the ANFIS modelling more systematic and less reliant on human expert knowledge. To present the ANFIS structure, three fuzzy if-then rules based on a first order Sugeno model are considered:

$$\text{Rule 1:} \, If \ (x \text{ is } A_1), (y \text{ is } B_1) \text{ and } (z \text{ is } C_1) \text{then } (f_1 = j_1 x + k_1 y + m_1 z + r_1) \tag{6}$$

$$\text{Rule 2:} \, If \ (x \text{ is } A_2), (y \text{ is } B_2) \text{ and } (z \text{ is } C_2) \text{ then } (f_2 = j_2 x + k_2 y + m_2 z + r_2) \tag{7}$$

$$\text{Rule 3:} \, If \ (x \text{ is } A_3), (y \text{ is } B_3) \text{ and } (z \text{ is } C_3) \text{ then } (f_3 = j_3 x + k_3 y + m_3 z + r_3) \tag{8}$$

the node functions in the same layer are of the same function family as described below:

Layer 1:

Every node *i* in this layer is a square node with a node function,

$$O_i^1 = \mu_{Ai}(x) \tag{9}$$

where x is the input to node i, and A_i is the linguistic label associated with this node function. In other words, O_i^1 is the membership function of A_i and it specifies the degree to which the given x satisfies the quantifier A_i. Usually has been choosing $\mu_{Ai}(x)$ to be bell-shaped with maximum equal to 1 and minimum equal to 0 given by:

$$\mu_A(x) = \frac{1}{1+\left|\frac{x-c_i}{a_i}\right|^{2b_i}} \tag{10}$$

where a_i, b_i, c_i are parameters to be learnt. These are the premise parameters.

Layer 2:

Every node in this layer is a circle node labeled *M* which multiplies the incoming signals and sends the product out.

$$O_i^2 = \mu_{Ai}(x) \times \mu_{Bi}(y) \times \mu_{Ci}(z) \tag{11}$$

Every node in this layer is fixed. Other T-norm operators that achieve generalized AND can be used as the node function in this layer.

Layer 3:

It contains fixed nodes, which calculate the ratio of the firing strengths of the rules, labeled *N*. The i^{th} node analyses the ratio of the i^{th} rule's firing strength (degree of fulfilment) to the sum of all rule firing strengths. Outputs of this layer will be described normalized firing strengths.

$$O_i^3 = w_i^3 = \frac{w_i}{w_1 + w_2 + w_3} \tag{12}$$

Layer 4:

The nodes in this layer are adaptive and perform the consequent of the rules.

$$O_i^4 = \overline{w}_i f_i = w_i \left(j_i x + k_i y + m_i z + r_i \right) \tag{13}$$

where $\overline{w_i}$ is the output of layer 3, and $\{ j_i, k_i, m_i, r_i \}$ is the parameter set. Parameter in this layer to be determined and are referred to as the consequent parameters.

Layer 5: The single node in this layer is computes the overall output as the summation of all incoming signals.

$$O_i^5 = \sum_i \overline{w}_i f_i = \frac{\sum w_i f_i}{\sum w_i} \tag{14}$$

Membership functions in the proposed ANFIS topology associated with each of the three inputs, respectively. So the input space is partitioned into five sub-domains, each of which is governed by fuzzy 'IF-THEN 'rules. The premise part of a rule (layer 1) defines a fuzzy sub-domain, while the following part (layer 4) specifies the output within this sub-domain. The performances of the model during training and testing were evaluated using a range of standard statistical performance assessment criteria such as, coefficient of determination (R^2) and root mean square error (RMSE) presented by Eqs. (15), (16).

$$\text{RMSE} = \sqrt{\left(\frac{\sum_{i=1}^{n} (x-y)^2}{(n-1)} \right)} \tag{15}$$

$$\text{R}^2 = \left(\frac{\sum_{i=1}^{n} (x-y)^2}{\sum_{i=1}^{n} (y)^2} \right) \tag{16}$$

where x is the target value , y is the output value and n is the number of data items .

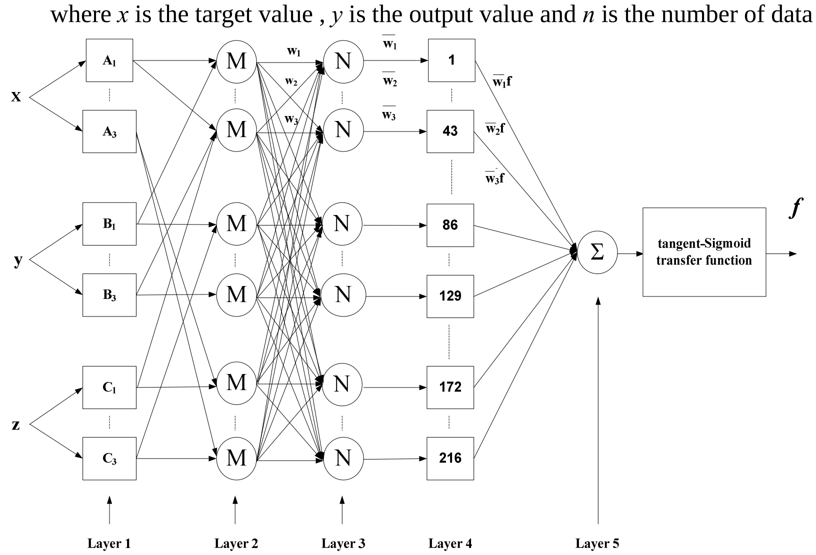


Fig. 2. Structure of the ANFIS model

Simulation and Experiments:

In this work, is to assess a new crack propagation criterion based on prediction experimental tests on three point-bend (TPB) specimens, which allow predicting the fatigue life and FCG rate. This research was carried out on ASTM A533 steel materials, following the ASTM E647 (2000) standard. In this analysis, the ASTM A533 steel material has been used. The fatigue crack growth tests were performed using the single edge notch bending (SENB), loaded by three point-bend specimens shows in figure. 3, was used for the simulation and experiments of ASTM A533 steel materials, with a thickness of 10 mm. All the cyclic tests were carried out using servo-hydraulic dynamic testing machine (Instron 8874) shows in figure 4; having a load capacity of 17 KN was used for the present investigation. Fatigue pre-cracking was introduced under constant amplitude load (CAL) to an *a*/*w* ratio of 0.1. Crack lengths were measured and recorded using a crack opining displacement (COD) technique based on the software from the machine as shown in figure 5, and were also using the Microscope to make sure and make a comparison between deferent measurement data. All fatigue tests were run at a frequency of 10 Hz with a sinusoidal wave form under the ambient laboratory conditions.

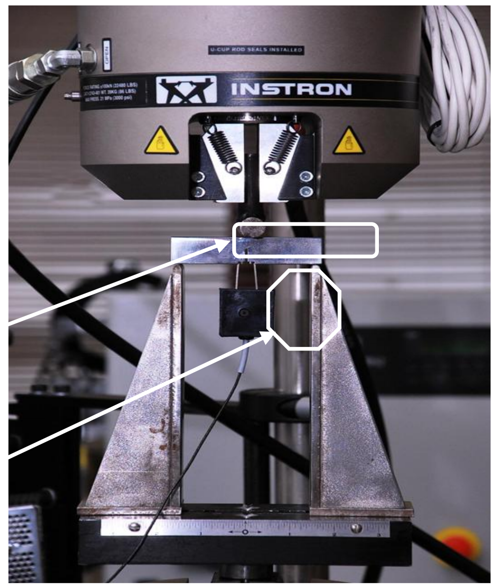


Fig. 3: The experiment setup of the INSTRON machine model 8874.

Fig. 4: The three-point bend specimen with COD under testing.

The present's prediction and experimental works is to achieve the main contribution of the work in order to propose a suitable fatigue crack growth model. Then to validate that based on experimental data collected from a set of experiments. Crack growth retardation depends on the magnitude of the stress ratio (*R*). The crack growth rate *(da/dN)* has been chosen as the output for the present ANFIS model (K. Donald and P.C. Paris, 1999; D. Kujawski, 2001). The most important principle of ANFIS is that the input/output data must be normalized (pre-processing) before applying the model to obtain optimum results. As far as normalization of input and output parameters are concerned. To make the input amenable to successful learning to minimize the overall sum-squared error (Mohanty *et al.,* 2010). ANFIS model is implemented by using the fatigue crack growth utility code has been developed under MATLAB environment software. In addition, the code contains other methods can be used in order to fatigue life prediction. It was developed to create a static customized, easy to use, more flexibility; fast learning and training for the data. (for example, ANN different architecture and different algorithms) and also GA. The interface for this code can see in figure 5.

Result And Discussion:

The crack driving forces: Three different types of input have been identified for the system vis-a-vis extent of stress intensity factor (ΔK), totally attainable stress intensity factor (K_{max}) and the ratio of load *(R).* The coverage of each input has been further categorised into six sub-domains and with each sub-domain being given 'gbell' type membership function. A node in input layer might be having connections to three nodes in layer 1, and thus given nomenclature according to the term nodes. However, there is no difference either in the total number of nodes present in the fuzzy division's layer or in the rule layer or in one of the fuzzy rules, this is so because besides each node standing in for one rule, it also determines the degree of fulfilment of the rule using membership category from layer 1.

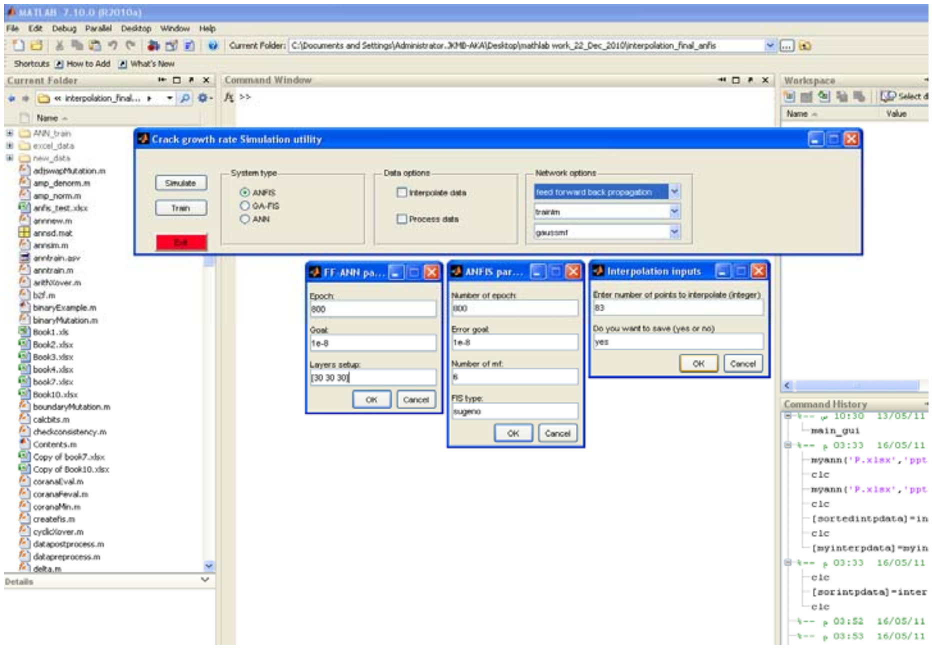


Fig. 5: The Matlab interface code

The background of each respective fuzzy rule bears a lot of things in common with the link between the two layers 1 and 2. The firing potentials are normalized by the Layer 3, it is referred to as the Normalization Layer. The first fuzzy division possess six membership function for each parameter while that of the three inputs parameters happens to be 18 (6×3) membership functions. The membership function were chosen to be 6 – 6 – 6 being the inputs for $Kmax$, $\Delta Kmax$ and R. The number of fuzzy rule 'IF-THEN'rules (6^3)$= 216$ were constituted in which parameters were connected by T- norm (fuzzy AND). The model was trained for 800 epochs as a maximum. Achieving the mandatory error tolerance limit or attaining the utmost epoch is the target of the training of the system, once these are achieved, the system will stop.

The step size for parameter adaptation had an initial value of 0.01. Figure 7 displays the flow chart of trained ANFIS model while Figur.8 shows the initial and final membership functions of the second and the attuned (after training) membership functions as regards the initial (before training) membership functions of the same inputs respectively. Checking on the initial and final membership functions shows that there are reasonable changes worthy of being effected in the final membership functions. The initial setup of the ANFIS network model is shown in table 1. Upon getting to the 600th training epoch, the mean square error (MSE) convergence curve of ANFIS exercise was determined as displayed in Figure. 9. From the plotted graph, the final convergence value is 0.8×10^{-6}. The training error got stabilised just after epoch 200.

Table 1: Characteristics of the ANFIS network

Type Of Membership Function	Generalized Bell
Number of input nodes (n)	3
Number of fuzzy partitions of each variable (p)	6
Total number of membership functions	18
Number of fuzzy rules (pn)	216
Total number of nodes	474
Total number of parameters	918
Max Number of epochs	800
Initial step size for parameter adaptation	0.01

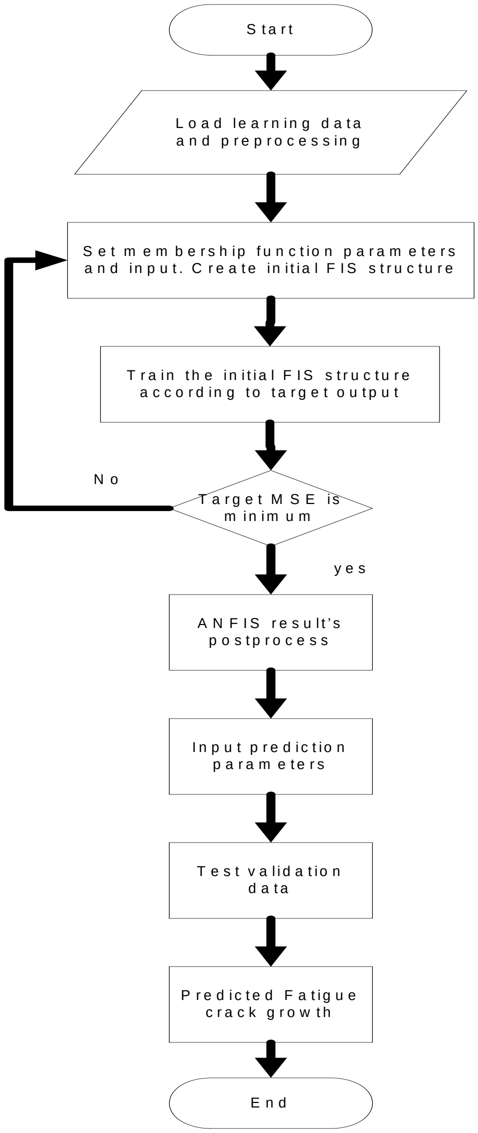
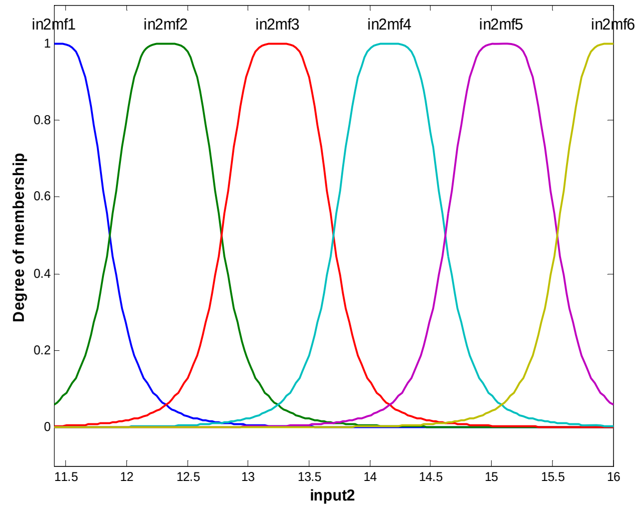
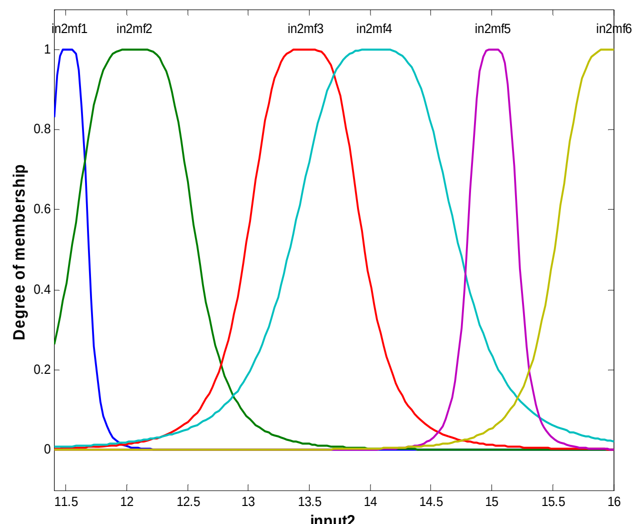


Fig. 7. Flow chart of trained ANFIS



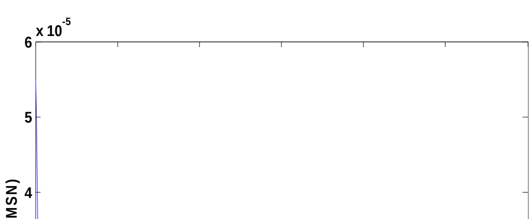
(a)



(b)

Fig. 8. (a) Initial and (b) final generalize membership function shape of Input 2

The performance of the training, testing and validation is shown in Figure.10. Figure. 11 also show the crack growth when compared to the experimental data. The FIS system results show a good matching with the experimental results with increased error at a higher number of cycles. ANFIS system shows a better matching with the experimental data in the entire data domain compared to FIS results. Figures. 12 and 13 show the relation between FCGR with life and with crack length respectively. The ANFIS and FIS models result are in good agreement with the experimental. It is important to note that ANFIS system closely emulates the FCGR experimental data in the entire data domain.



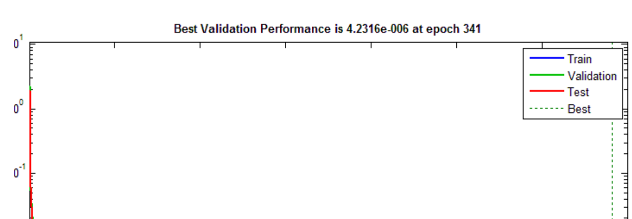


Fig. 9. ANFIS network error convergence curve.

Fig. 10. Performance curves of the training, testing and validation of the proposed system training.

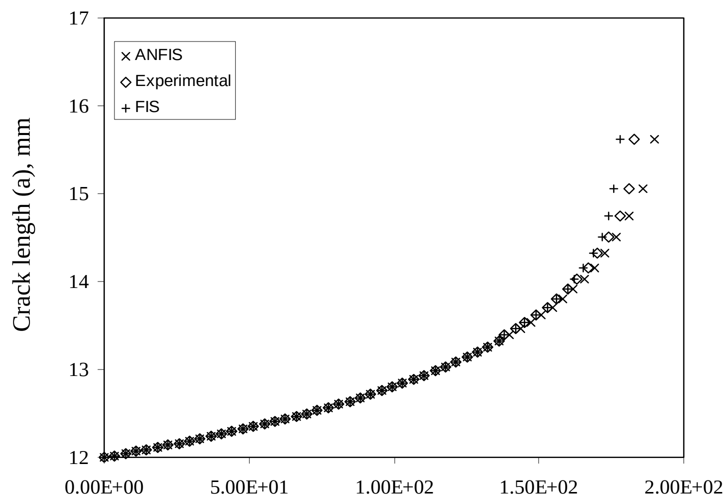


Fig. 11 Comparison of predicted (FIS - ANFIS) and experimental data

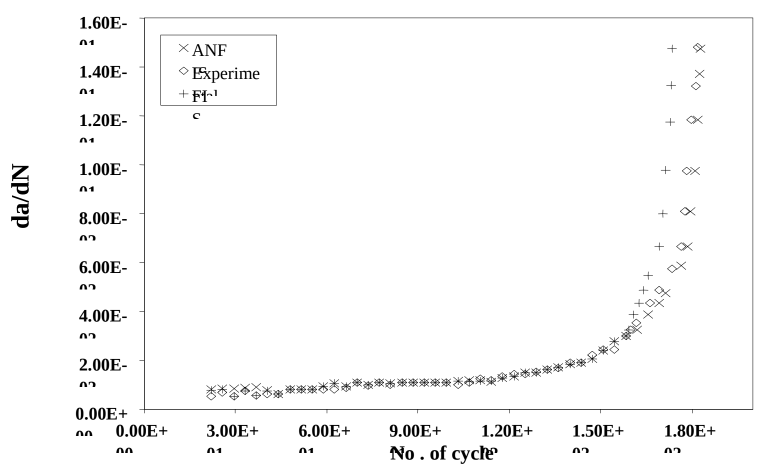


Fig. 12: Comparison of predicted (FIS - ANFIS), experimental delay cycle with crack growth rate.

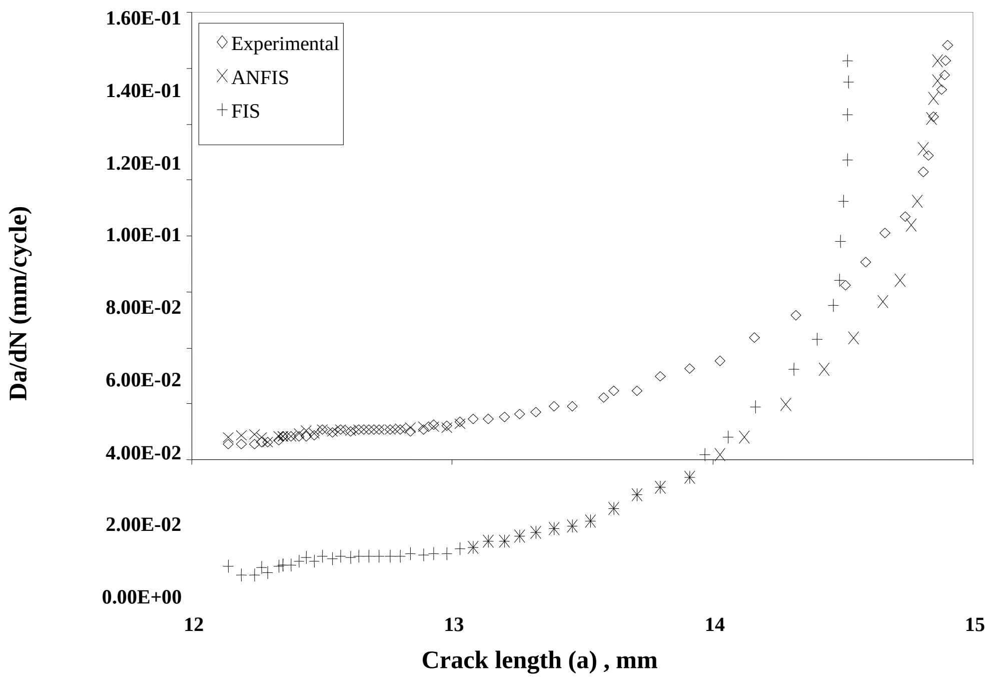


Fig .13: Comparison of predicted (FIS - ANFIS), experimental crack length with crack growth rate

Conclusion:

The adaptive neuro-fuzzy inference system (ANFIS), a new and innovative soft-computing application improvised method was utilized in forecasting fatigue life and fatigue crack growth rate while considering the circumstances under focus. The predictability the proposed model was reviewed alongside with that of standard experiments for ASTM A533 steel materials. This system gives a detailed and precise account and data of the rate of the crack growth, the extent of the stress intensity and the ratio of stress obtained experimentally with a great precision at an appropriate learning rate when compared with other systems used for evaluations. There are other preliminary operational approaches in which the freedom and exclusivity of dataset inputs not in doubt.

The learning rates and the overall ability of ANFIS become better through these preliminary mechanisms, hence ensuring more merits than the classical scaling approach. Generally, in the construction of ANFIS, the use of data that had been initially worked upon gives a better precision and results compared to data that were not pre-processed.

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