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Manipulator Control Using Adaptive Structure and Payload Estimation

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Abstract*:* In this study, we investigate a simple structure of adaptive control and finally provide a closed loop adaptive control for a 2-link pay-loaded manipulator. Then, Using the designed adaptive controller and through an expressed algorithm, we estimate the maximum payload of the 2-link manipulator. We also investigate the performance of proposed controller and algorithm using several simulations. In order to validate the results, we arrange some comparisons with similar works.

Key words: Adaptive Control, Robot, Payload, Manipulator.

INTRODUCTION

Adaptive control is a design approach tailored for high performance applications in control systems with uncertainty in the parameters. That is, uncertainty in the dynamic system is assumed to be characterized by a set of unknown *constant* parameters. However, the design of adaptive controllers requires the precise knowledge of the structure of the system being controlled.

Certainly one may consider other variants such as adaptive control for systems with time-varying parameters, or *robust adaptive* control for systems
with structural and parameter uncertainty.

A key step in the study of this controller, and by the way, also in that of the succeeding controllers in the literature, was the use of the linear parameterization property of the robot model (Spong *et al.,* 1989). This first adaptive controller needed *a priori* knowledge of bounds on the dynamic parameters as well as the measurement of the vector of joint accelerations $\ddot{\boldsymbol{q}}$. After this first adaptive controller a series of adaptive controllers that did not need knowledge of the bounds on the parameters nor the measurement of joint accelerations were developed (Lozano *et al.,* 1992; Canudas *et al.,* 1992; Hsu *et al.,* 1993; Yu *et al.,* 1994)

Nowadays, we also count on several textbooks that are devoted in part to the study of adaptive controllers for robot manipulators (Lewis *et al*., 1993; Arimoto , 1996).

A detailed description of the basic concepts of adaptive control systems may be found in (Marino et al., 1995; Khalil, 1996; Landau *et al.,* 1998).

In the following sections, we concentrate specifically on adaptive control of robot manipulators with *constant* parameters and for which we assume that we have no structural uncertainties. We have use the designed adaptive control for a 2-link pay-loaded manipulator and, the maximum payload has been investigated using a proposed algorithm.

Parameterization of the Dynamic Model:

The dynamic model of robot manipulators, as we know, is given by Lagrange's equations, which we repeat here in their compact form:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \tag{1}$$

In this equation, we have not emphasized the fact that the elements of the inertia matrix $M(\boldsymbol{q})$, the centrifugal and Coriolis forces matrix $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and the vector of gravitational torques $\boldsymbol{G}(\boldsymbol{q})$, depend not only on the geometry of the corresponding robot but also on the numerical values of diverse parameters such as masses, inertias and distances to centers of mass.

The scenario in which these parameters and the geometry of the robot are exactly known is called in the context of adaptive control, *the ideal case*. A more realistic scenario is usually that in which the numerical values of some parameters of the robot are unknown. Such is the case, for instance, when the object manipulated by the end-effecter of the robot (which may be considered as part of the last link) is of uncertain mass and/or inertia. The adaptive controllers are useful precisely in this more realistic case.

To emphasize the dependence of the dynamic model on the dynamic parameters, from now on we write the dynamic model (1) explicitly as a function of the vector of unknown dynamic parameters, θ, that is,

$$M(q,\theta)\ddot{q} + C(q,\dot{q},\theta) + G(q,\theta) = \tau \tag{2}$$

The vector of parameters θ may be of any dimension, that is, it does not depend in any specific way on the number of degrees of freedom or on whether the robot has revolute or prismatic joints *etc.* Notwithstanding, an upperbound on the dimension is determined by the number of degrees of freedom. Therefore, we simply say

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that $\theta \in \mathrm{IR}^m$ where m is some known constant. It is also important to stress that the dynamic parameters, denoted here by θ, *do not* necessarily correspond to the individual physical parameters of the robot, as is illustrated in the following example.

Example 1:

Consider the example of an ideal pendulum with its mass m concentrated at the tip, at a distance l from its axis of rotation. Its dynamic model is given by

$$ml^2\ddot{q} + mgl\sin(q) = \tau \quad (3)$$

hence, compared to (2) we identify $M(q, \theta)=ml^2$, $g(q, \theta)=mgl\sin(q)$. Hence, assuming that both the mass m and the length from the joint axis to the center of mass l, are unknown, we identify the vector of dynamic parameters as

$$\theta = \begin{bmatrix} ml^2 \\ mgl \end{bmatrix} \quad (4)$$

which is, strictly speaking, a nonlinear vectorial function of the physical parameters m and l, since θ depends on products of them.

Note that here, the number of dynamic parameters coincides with the number of physical parameters, however, this is in general not the case as is clear from the examples below.

Linearity in the Dynamic Parameters:

Example 1 also shows that the dynamic model (2) is linear in the parameters θ. To see this more clearly, notice that we may write

$$ml^2\ddot{q} + mgl\sin(q) = \begin{bmatrix} \ddot{q} & \sin(q) \end{bmatrix} \begin{bmatrix} ml^2 \\ mgl \end{bmatrix} := \Phi(q,\ddot{q})\theta \quad (5)$$

That is, the dynamic model (2) with zero input ($\tau = 0$), can be rewritten as the product of a vector function θ which contains nonlinear terms of the state (the generalized coordinates and its derivatives) and the vector of dynamic parameters, θ.

This property is commonly known as "linearity in the parameters" or "linear parameterization". It is a property possessed by many nonlinear systems and, in particular, by a fairly large class of robot manipulators. It is also our standing hypothesis for the subsequent sections hence, we enunciate it formally below.

Property 1:

Linearity in the dynamic parameters: For the matrices $M(q,\theta)$, $C(q,\dot{q},\theta)$ and the vector $G(q,\theta)$ from the dynamic model (2), we have the following:

For all $\boldsymbol{q} \in \mathrm{IR}^n$ it holds that

$$M(q,\theta)\ddot{q} + C(q,\dot{q},\theta) + G(q,\theta) = \Phi(q,\dot{q},\ddot{q})\theta + \kappa(q,\dot{q},\ddot{q}) \quad (6)$$

where $\kappa(q,\dot{q},\ddot{q})$ is a vector of $n\times 1$, $\Phi(q,\dot{q},\ddot{q})$ is a matrix of $n\times m$ and the vector $\theta \in \mathrm{IR}^m$ depends only on the dynamic parameters of the manipulator and its load.

It is worth remarking that one may always find a vector $\theta \in \mathrm{IR}^m$ for which $\theta \equiv \mathbf{0} \in \mathrm{IR}^n$.

The following example shows that, as expected, the model of the pay-loaded 2-link manipulator satisfies the linear parameterization property. This is used extensively later to design adaptive control schemes for this prototype in succeeding examples.

Example 2:

Consider the manipulator shown in Fig. **1**. Different dynamic equations of the manipulator can be formulated as below:

$$\tau_1 = a_1\ddot{q}_1 + a_2\ddot{q}_1 + 2a_3\ddot{q}_1\cos(q_2) + a_2\ddot{q}_2 + a_3\ddot{q}_2\cos(q_2) - 2a_3\dot{q}_1\dot{q}_2\sin(q_2) - a_3\dot{q}_2^2\sin(q_2) + ga_4\cos(q_1) + ga_5\cos(q_1+q_2) \quad (7)$$

$$\tau_2 = a_2\ddot{q}_1 + a_3\ddot{q}_1\cos(q_2) + a_2\ddot{q}_2 + a_3\sin(q_2)\dot{q}_1^2 + ga_5\cos(q_1+q_2) \quad (8)$$

where

$$\begin{aligned} a_1 &= I_1 + m_1L_{c1}^2 + (m_2+m_p)L_1^2 \\ a_2 &= I_2 + m_2L_{c2}^2 + m_pL_2^2 \\ a_3 &= L_1(m_2L_{c2} + m_pL_2) \\ a_4 &= m_1L_{c1} + (m_2+m_p)L_1 \\ a_5 &= m_2L_{c2} + m_pL_2 \end{aligned} \quad (9)$$

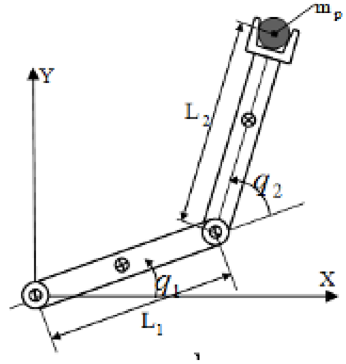


Fig. 1: Pay-loaded 2-link Manipulator.

Different dynamic matrixes of this manipulator are arranged as follows:
the inertia matrix $M(\boldsymbol{q})$:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix}$$

$$M_{11} = a_1 + a_2 + 2a_3 \cos(q_2) \qquad (10)$$

$$M_{12} = a_2 + a_3 \cos(q_2)$$

$$M_{22} = a_2$$

the centrifugal and Coriolis forces matrix $C(\boldsymbol{q},\boldsymbol{q}^{\cdot})$:

$$C = \begin{bmatrix} -a_3 \sin(q_2)(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) \\ a_3 \sin(q_2)\dot{q}_1^2 \end{bmatrix} \qquad (11)$$

and the vector of gravitational torques $\boldsymbol{G}(\boldsymbol{q})$:

$$G = \begin{bmatrix} g(a_4 \cos(q_1) + a_5 \cos(q_1 + q_2)) \\ g a_5 \cos(q_1 + q_2) \end{bmatrix} \qquad (12)$$

The dynamic parameters in the model are the masses a_1, a_2, a_3, a_4 and a_5. Define the vector θ of dynamic parameters as $\theta = [a_1\ a_2\ a_3\ a_4\ a_5]^T$. The set of dynamic equations (7 and (8) may be rewritten in linear terms of θ, that is, in the form

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} & \varphi_{24} & \varphi_{25} \end{bmatrix} \theta \qquad (13)$$

where

$$\begin{aligned} \varphi_{11} &= \ddot{q}_1 \\ \varphi_{12} &= \ddot{q}_1 + \ddot{q}_2 \\ \varphi_{13} &= 2\ddot{q}_1 \cos(q_2) + \ddot{q}_2 \cos(q_2) \\ &\quad - 2\dot{q}_1\dot{q}_2 \sin(q_2) - \dot{q}_2^2 \sin(q_2) \\ \varphi_{14} &= g \cos(q_1) \\ \varphi_{15} &= g \cos(q_1 + q_2) \end{aligned} \qquad (14)$$

and

$$\begin{aligned} \varphi_{21} &= 0 \\ \varphi_{22} &= \ddot{q}_1 + \ddot{q}_2 \\ \varphi_{23} &= \ddot{q}_1 \cos(q_2) + \sin(q_2)\dot{q}_1^2 \\ \varphi_{24} &= 0 \\ \varphi_{15} &= g \cos(q_1 + q_2) \end{aligned} \qquad (15)$$

In the above example, we chose the parameters such that $\theta \equiv 0$;

The Nominal Model:

We remark that for any given robot the vector of dynamic parameters θ is not unique since it depends on how the parameters of interest are chosen. In the context of adaptive control, the parameters of interest are those whose numerical values are unknown. Usually, these are the mass, the inertia and the physical location of the center of mass of the last link of the robot. For instance, as mentioned and illustrated through examples above, the vector $\kappa(q,\dot{\mathbf{q}},\ddot{\mathbf{q}})$ and the matrix $\Phi(q,\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ are obtained from knowledge of the dynamic model of the robot under study, as well as from the vector $\theta \in \mathrm{IR}^m$ formed by the selection of the *m* dynamic parameters of interest.

Naturally, it is always possible to choose a vector of dynamic parameters θ for which (6) holds with $\kappa(q,\dot{\mathbf{q}},\ddot{\mathbf{q}}) = \mathbf{0} \in \mathrm{IR}^n$.

Example 3:

Consider the payloaded 2-link manipulator in Fig. **1**. If the dynamic parameter in the model is m_p, then the set of dynamic equations (7 and (8) may be rewriten in linear terms of θ and κ, that is, in the form

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}\varphi_{11}\\ \varphi_{12}\end{bmatrix}\theta + \begin{bmatrix}\kappa_{11}\\ \kappa_{12}\end{bmatrix} \quad , \quad \theta = m_p \tag{16}$$

and

$$\begin{aligned}\varphi_{11} &= (L_2^2 + L_1^2 + 2L_1L_2\cos(q_2))\ddot{q}_1 + (L_2^2 + L_1L_2\cos(q_2))\ddot{q}_2 - L_1L_2\sin(q_2)(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) + gL_1\cos(q_1) + gL_2\cos(q_1+q_2)\\ \varphi_{21} &= (L_2^2 + L_1L_2\cos(q_2))\ddot{q}_1 + L_2^2\ddot{q}^2 + L_1L_2\sin(q_2)\dot{q}_1^2 + gL_2\cos(q_1+q_2)\end{aligned} \tag{17}$$

We may identify the *nominal model* or nominal part of the model as:

$$\kappa(q,\dot{q},\ddot{q}) = \begin{bmatrix}\kappa_{11}\\ \kappa_{12}\end{bmatrix} = M_0(q)\ddot{q} + C_0(q,\dot{q}) + G_0(q)$$

$$\begin{aligned}M\,0_{11} &= I_1 + m_1L_{c1}^2 + m_2L_1^2 + 2L_1m_2L_{c2}\cos(q_2) + I_2 + m_2L_{c2}^2\\ M\,0_{12} &= I_2 + m_2L_{c2}^2 + L_1m_2L_{c2}\cos(q_2)\\ M\,0_{21} &= I_2 + m_2L_{c2}^2 + L_1m_2L_{c2}\cos(q_2)\\ M\,0_{22} &= I_2 + m_2L_{c2}^2\\ C\,0_{11} &= -L_1L_{c2}m_2(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2)\sin(q_2)\\ C\,0_{21} &= L_1L_{c2}m_2\dot{q}_1^2\sin(q_2)\\ G\,0_{11} &= g(m_1L_{c1} + m_2L_1)\cos(q_1) + gm_2L_{c2}\cos(q_1+q_2)\\ G\,0_{21} &= gm_2L_{c2}\cos(q_1+q_2)\end{aligned} \tag{18}$$

That is, the matrices $M0(\boldsymbol{q},\boldsymbol{q}^{\cdot},\boldsymbol{q}^{\cdot\cdot})$, $C0(\boldsymbol{q},\boldsymbol{q}^{\cdot})$ and the vector $G0(\boldsymbol{q})$ represent respectively, parts of the matrices $M(\boldsymbol{q},\boldsymbol{q}^{\cdot},\boldsymbol{q}^{\cdot\cdot})$, $C(\boldsymbol{q},\boldsymbol{q}^{\cdot})$ and of the vector $G(\boldsymbol{q})$ that do not depend on the vector of *unknown* dynamic parameters θ.

According with the parameterization (6) given a vector $\hat{\theta} \in \mathrm{IR}^m$, the expression $\Phi(q,\dot{q},\ddot{q})\hat{\theta}$ corresponds to:

$$\Phi(q,\dot{q},\ddot{q})\hat{\theta} = M(q,\hat{\theta})\ddot{q} + C(q,\dot{q},\hat{\theta}) + G(q,\hat{\theta}) - M_0(q)\ddot{q} - C_0(q,\dot{q}) - G_0(q) \tag{19}$$

A particular case of parameterization (14.9) is when $\dot{q} = \ddot{q} = 0 \in \mathrm{IR}^n$. In this scenario, we have the following parameterization of the vector of gravitational torques:

$$G(q,\theta) = \Phi(q,0,0)\theta + G_0(q) \tag{20}$$

The Adaptive Robot Control Problem:

We have presented and discussed so far the fundamental property of linear parameterization of robot manipulators. The adaptive controller that we study in the following section relies on the assumption that this property holds. In addition, it is assumed that uncertainty in the model of the manipulator consists only of the lack of knowledge of the numerical values of the elements of θ. Hence, the structural form of the model of the manipulator is assumed to be exactly known.

Formally, the control problem that we address in this text may be stated in the following terms. Consider the dynamic equation of *n*-DOF robots taking into account the linear parameterization (2) that is,

$$M(q,\theta)\ddot{q} + C(q,\dot{q},\theta) + G(q,\theta) = \tau \tag{21}$$

or equivalently,

$$\Phi(q,\dot{q},\ddot{q})\theta + M\,0(q)\ddot{q} + C\,0(q,\dot{q}) + G\,0(q) = \tau \tag{22}$$

Assume that the matrices $\Phi(q,\dot{q},\ddot{q}) \in IR^{n\times m}$, $M0(q)$, $C0(q,\dot{q}) \in IR^{n\times 1}$ *and* the vector $G0(q) \in IR^{n}$ *are known but that the constant vector of dynamic* parameters (which includes, for instance, inertias and masses) $\theta \in IR^{m}$ *is unknown.*

Given a set of vectorial bounded functions, $q_d, \dot{q}_d, \ddot{q}_d$ *referred* to as desired joint positions, velocities and accelerations, we seek to design controllers that achieve the position or motion control objectives.

Another important observation about the control problem formulated above is the following. We have said explicitly that the vector of dynamic parameters $\theta \in IR^{m}$ is assumed unknown but constant. This means precisely that the components of this vector do not vary as functions of time. Consequently, in the case where the parametric uncertainty comes from the mass or the inertia corresponding to the manipulated load by the robot, this must always be the same object, and therefore, it may not be latched or changed.

Obviously, this is a serious restriction from a practical viewpoint but it is necessary for the stability analysis of any adaptive controller if one is interested in guaranteeing achievement of the motion or position control objectives.

Parameterization of the Adaptive Controller:

The control laws to solve the position and motion control problems for robot manipulators may be written in the functional form

$$\tau = \tau_1(q,\dot{q},q_d,\dot{q}_d,\ddot{q}_d,M(q),C(q,\dot{q}),G(q)) \tag{23}$$

In general, these control laws are formed by the sum of two terms; the first, which does not depend explicitly on the dynamic model of the robot to be controlled, and a second one, which does. Therefore, giving a little 'more' structure to (23), we may write that most of the control laws have the form

$$\tau = \tau_1(q,\dot{q},q_d,\dot{q}_d,\ddot{q}_d) + M(q)\ddot{q} + C(q,\dot{q}) + G(q) \tag{24}$$

The term $\tau_1(q,\dot{q},q_d,\dot{q}_d,\ddot{q}_d)$ which does not depend on the dynamic model, usually corresponds to linear control terms of PD type,

$$\tau_1(q,\dot{q},q_d,\dot{q}_d,\ddot{q}_d) = K_p\left[q_d - q\right] + K_v\left[\dot{q}_d - \dot{q}\right] \tag{25}$$

where *Kp* and *Kv* are gain matrices of position and velocity (or derivative gain) respectively.
Certainly, the structure of some position control laws do not depend on the dynamic model of the robot to be controlled; *e.g.* such is the case for PD and PID control laws. Other control laws require only part of the dynamic model of the robot; *e.g.* PD control with gravity compensation.
In general, an adaptive controller is formed of two main parts: control law or controller; and adaptive (update) law.

At this point, it is worth remarking that we have not spoken of any particular adaptive controller to solve a given control problem. Indeed, there may exist many control and adaptive laws that allow one to solve a specific control problem. However, in general the control law is an algebraic equation that calculates the control action and which may be written in the generic form

$$\tau = \tau_1(q,\dot{q},q_d,\dot{q}_d,\ddot{q}_d) + M(q,\hat{\theta})\ddot{q} + C(q,\dot{q},\hat{\theta}) + G(q,\hat{\theta}) \tag{26}$$

Typically, the control law (26) is chosen so that when substituting the vector of adaptive parameters $\hat{\theta}$ by the vector of dynamic parameters $\theta \in IR^{m}$ (which yields a non-adaptive controller), the resulting closed-loop system meets the control objective.

The adaptive law allows one to determine $\hat{\theta}(t)$ and in general, may be written as a differential equation of $\hat{\theta}$. An adaptive law commonly used in continuous adaptive systems is the so-called integral law or gradient type:

$$\hat{\theta}(t) = \Gamma\int_0^t \psi(s,q,\ddot{q},\ddot{q},q_d,\dot{q}_d,\ddot{q}_d)ds + \hat{\theta}(0) \tag{27}$$

where $\Gamma = \Gamma^T \in IR^{m\times m}$ and $\hat{\theta}(0) \in IR^{m}$ are design parameters while ψ is a vectorial function to be determined, of dimension *m*.

The symmetric matrix Γ is usually diagonal and positive definite and is called 'adaptive gain'. The "magnitude" of the adaptive gain Γ is related proportionally to the "rapidity of adaptation" of the control system and the parametric uncertainty of the dynamic model. In practice one simply applies 'experience' to a trial-and-error approach until satisfactory behavior of the control system is obtained and usually, the adaptive gain is initially chosen to be "small".

On the other hand, $\hat{\theta}(0)$ is an arbitrary vector even though in practice, we choose it as the best approximation available to the unknown vector of dynamic parameters, θ.
Fig. **2** shows a block-diagram of the adaptive control of a robot.

An equivalent representation of the adaptive law is obtained by differentiating (27) with respect to time, that is,

$$\dot{\hat{\theta}}(t) = \Gamma \psi(s, q, \ddot{q}, \ddot{q}, q_d, \dot{q}_d, \ddot{q}_d) \quad (28)$$

It is desirable, from a practical viewpoint, that the control law as well as the adaptive law, do not depend explicitly on the joint acceleration $\ddot{\boldsymbol{q}}$.

PD Control with Adaptive Desired Gravity Compensation:

We start by recalling the PD control law with desired gravity compensation (Kelly, 1993; *et al.,* 1993):

$$\tau = K_p\tilde{q} + K_v\dot{\tilde{q}} + G(q_d) \quad (29)$$

We also recall that $K_p, K_v \in IR^{n\times n}$ are symmetric positive definite matrices chosen by the designer. As is customary, the position error is denoted by

$$\tilde{q} = q_d - q \quad (30)$$

while $\boldsymbol{q}_d \in IR^n$ stands for the desired joint position.

The practical convenience of this control law with respect to that of PD control with gravity compensation is evident. Indeed, the vector $\boldsymbol{G}(\boldsymbol{q}_d)$ used in the control law depends on $\boldsymbol{q}_d$ and not on $\boldsymbol{q}$.
Therefore, it may be evaluated "off-line" once ***qd*** is defined. In other words, it is unnecessary to compute $\boldsymbol{G}(\boldsymbol{q})$ in real time.

For further development it is also worth recalling *Property 1*, and also by virtue of the previous fact, notice that the following expression is valid

$$G(q,\theta) = \Phi(q,0,0)\theta + G_0(q) \quad (31)$$

Since $\hat{\theta}$ is estimated continuously, we can rewrite (31) as follows:

$$G(q_d,\hat{\theta}) = \Phi_g(q_d)\hat{\theta} + G_0(q_d)$$
$$\Phi_g(q_d) \triangleq \Phi(q_d,0,0) \quad (32)$$

Therefore, the PD control law with desired gravity compensation, (29), may also be written as

$$\tau = K_p\tilde{q} + K_v\dot{\tilde{q}} + \Phi_g(q_d)\theta + G0(q_d) \quad (33)$$

The solution that we consider in this paper to the position control problem formulated above consists in the so-called adaptive version of PD control with desired gravity compensation, that is, PD control with *adaptive desired gravity compensation.*

The structure of the motion adaptive control schemes for robot manipulators that are studied in this text, are defined by means of a control law like (26) and an adaptive law like (27). In the particular case of position control, these control laws take the form

$$\tau = K_p\tilde{q} + K_v\dot{\tilde{q}} + \Phi_g(q_d)\hat{\theta} + G0(q_d) \quad (34)$$

$$\dot{\hat{\theta}}(t) = \Gamma\Phi_g(q_d)^T \left[\frac{\varepsilon_0}{1+\|\tilde{q}\|}\tilde{q} - \dot{q} \right] \quad (35)$$

Notice that the control law, does not depend on the dynamic parameters θ but on the so-called adaptive parameters $\hat{\theta}$ that in their turn, are obtained from the adaptive law, which of course, does not depend either on θ.

Among the design parameters of the adaptive controller formed by Equations (34), only the matrix *Kp* and the real positive constant ε_0 must be chosen carefully.
Before proceeding to derive the closed-loop equation, we define the parameter errors vector as:

$$\tilde{\theta} = \hat{\theta} - \theta \quad (36)$$

The parametric errors vector $\tilde{\theta}$ is unknown since this is obtained as a function of the vector of dynamic parameters θ, which is assumed to be unknown. Nevertheless, the parametric error $\tilde{\theta}$ is introduced here only for analytical purposes and evidently, the controller does not use it.

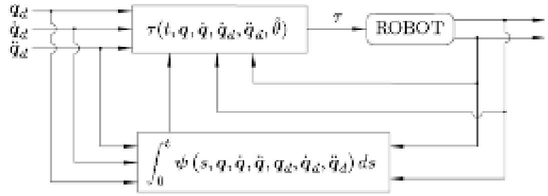


Fig. 2: Block diagram generic adaptive control of robots.

From the definition of the parametric errors vector, it may be verified that

$$\begin{aligned}\Phi_g(q_d)\hat{\theta} &= \Phi_g(q_d)\tilde{\theta} + \Phi_g(q_d)\theta \\ &= \Phi_g(q_d)\tilde{\theta} + G(q_d,\theta) - G0(q_d)\end{aligned} \tag{37}$$

Using the expression above, the control law (34) may be written as

$$\tau = K_p\tilde{q} + K_v\dot{\tilde{q}} + \Phi_g(q_d)\tilde{\theta} + G(q_d,\theta) \tag{38}$$

On the other hand, since the vector of dynamic parameters θ has been assumed to be constant, its time derivative is zero, $\dot{\theta} = 0$. Therefore, taking the derivative with respect to time of the parametric errors vector

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} - \dot{\theta} = \dot{\hat{\theta}} - 0 = \dot{\hat{\theta}} \tag{39}$$

In its turn, the time derivative of the vector of adaptive parameters $\hat{\theta}$ is obtained by derivation with respect to time.

The adaptive law (35), using these arguments changes to

$$\dot{\tilde{\theta}}(t) = \Gamma\Phi_g(q_d)^T\left[\frac{\varepsilon_0}{1+\|\tilde{q}\|}\tilde{q} - \dot{q}\right] \tag{40}$$

From all the above conclude that the closed-loop equation is formed by

$$\begin{aligned}\frac{d}{dt}\tilde{q} &= \dot{q}_d - \dot{q} \\ \frac{d}{dt}\dot{q} &= (M(q,\hat{\theta}))^{-1}\begin{bmatrix} K_p\tilde{q} + K_v\dot{\tilde{q}} + \Phi_g(q_d)\tilde{\theta} \\ -C(q,\dot{q},\hat{\theta}) + G(q_d,\hat{\theta}) - G(q,\hat{\theta})\end{bmatrix} \\ \frac{d}{dt}\tilde{\theta} &= \Gamma\Phi_g(q_d)^T\left[\frac{\varepsilon_0\tilde{q}}{1+\|\tilde{q}\|} - \dot{q}\right]\end{aligned} \tag{41}$$

Notice that this is a set of autonomous differential equations with state $\begin{bmatrix}\tilde{q} & \dot{q} & \tilde{\theta}\end{bmatrix}$ and the origin of the state space, *i.e.* $\begin{bmatrix}\tilde{q} & \dot{q} & \tilde{\theta}\end{bmatrix} = 0$. The stability analysis of PD control with adaptive compensation is presented in (Spong et al., 1990; Slotine et al., 1987; Lor´ıa, et al., 2003).

Example 4:

Consider the manipulator discussed in example 2 and 3. The closed loop equations can be formulated as (41), considering (10), (11), (12), (17), (18) and the following matrices and parameters:

$$\begin{aligned}&\theta = m_p \\ &\Phi_g(q_d) = \begin{bmatrix}\varphi_{11}(q_d) \\ \varphi_{21}(q_d)\end{bmatrix} \\ &\varphi_{11}(q_d) = gL_1\cos(q_{1d}) + gL_2\cos(q_{1d} + q_{2d}) \\ &\varphi_{21}(q_d) = gL_2\cos(q_{1d} + q_{2d}) \\ &K_p = \begin{bmatrix}K_{p11} & 0 \\ 0 & K_{p22}\end{bmatrix},\ K_v = \begin{bmatrix}K_{v11} & 0 \\ 0 & K_{v22}\end{bmatrix} \\ &\Gamma_p = \begin{bmatrix}\Gamma_{11} & 0 \\ 0 & \Gamma_{22}\end{bmatrix}\end{aligned} \tag{42}$$

Maximum Payload Estimation:

In the previous sections, we followed a procedure to find vector of torques in an adaptive manner. Maximum payload can be estimated using the algorithm described in Fig. **3**, (Korayem et al., 2008). This algorithm uses the closed loop adaptive system of (41) and calculates maximum payload for the manipulator described in example 2.

Simulation and Comparison:

In order to examine the performance of adaptive controller, several simulations have been arranged using MATLAB.

For the manipulator described in example 2 and 3 following physical parameters have been used.

Parameter	Value	Unit
Length of Links	$L_1 = 0.4, L_2 = 0.25$	m
Mass of Links	$m_1 = 29.58, m_2 = 15$	Kg
Moment of Inertia	$I_1 = 0.416739, I_2 = 0.205625$	$Kg.m^2$
Motor Capacity	$U_1^{\pm} = \pm 25, U_2^{\pm} = \pm 9$	N.m

In the following figures, results of different simulations have been arranged. Maximum Mp for each pass has been expressed in the figures. In all simulations, the gravitational torques have not been considered (g=0); Figure 4 shows a comparison; we compare proposed adaptive control with the approaches used by Korayen et al. (2008). They achieved the maximum payload of 5.9 kg. Maximum payload for the adaptive control is 5.4 kg. However, the signals have softer behavior. Variations in angles and angular velocities are much less, for adaptive controller and Torques are not always in extremes when adaptive control used. The results obviously show adaptive control superiority.

Figures 5, 6 and 7 express performance of manipulator when it is supposed to reach different points of working space.

In Figure 5, the signals express successful performance of adaptive controller when the manipulator is supposed to travel from first quarter to second one. In 1.2 second, the end effecter reaches the desired position while the torques are not always in extremes. The maximum payload for this path is 5.5 kg.

In Figure 6, travel from first quarter to fourth one takes 2 second, the end effecter reaches the desired position while the torques are not always in extremes. The maximum payload for this path is 5 kg.

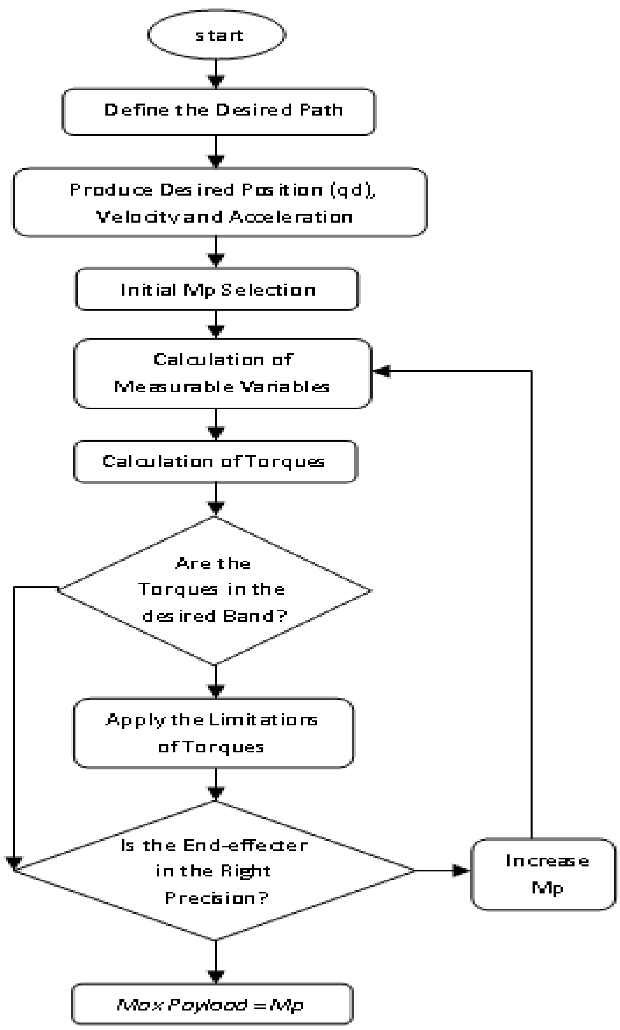


Fig. 3: Flowchart for Maximum Payload Estimation.

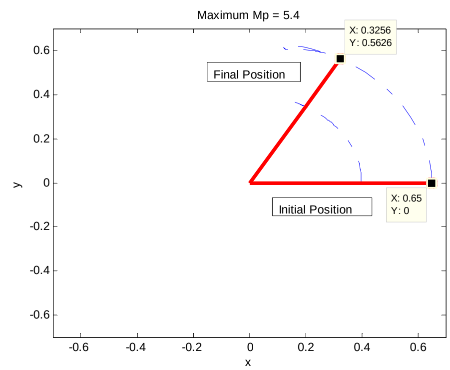
In Figure 7, in almost 1.2 second, the end effecter reaches the desired position while the torques are not always in extremes. However, the maximum payload in this case is less than that of Figure 5. The maximum payload for this path is 2.5 kg.

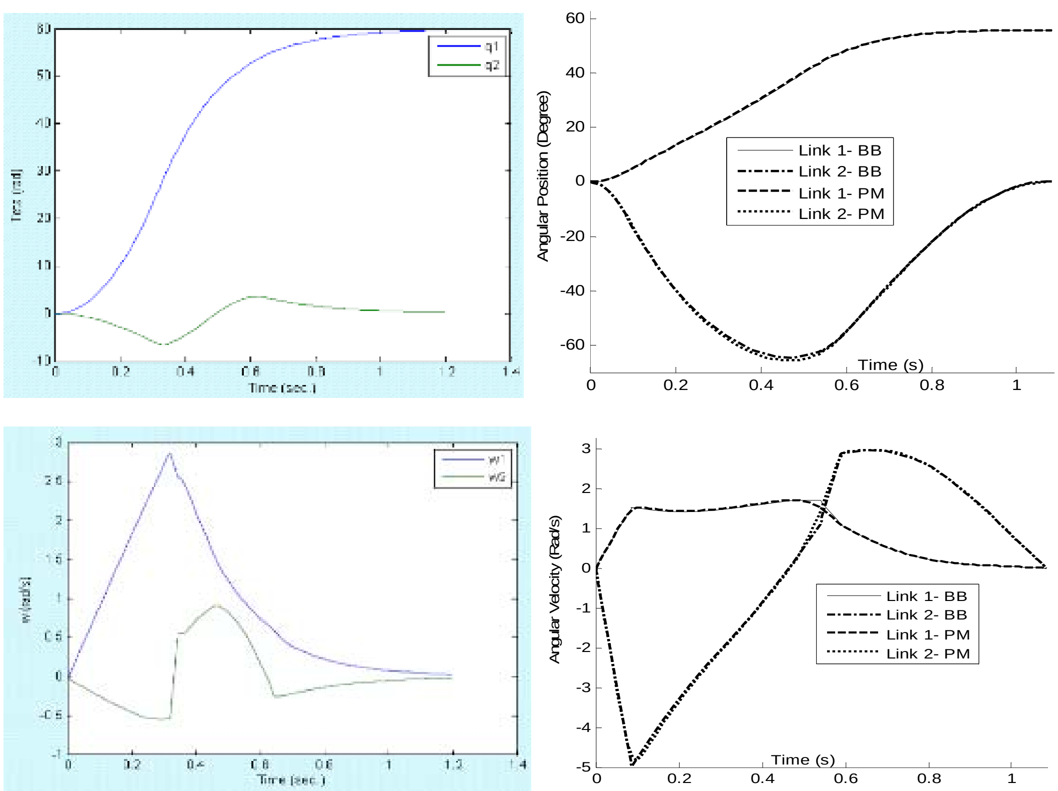
Conclusions:

In this paper, an adaptive controller has been designed for a 2-link pay-loaded manipulator. Comparison shows less ability to carry payload but softer behavior of control signals. The maximum payload was 5.5 kg when the control is adaptive.

The adaptive controller assumed that the parameters of the robot are time invariant, therefore, further investigation maybe arranged to extend and redesign the adaptive control for robots having time varying parameters.

Since the procedure has been arranged parametrically while considering the global dynamics of robots, it can be used for different types of robots.





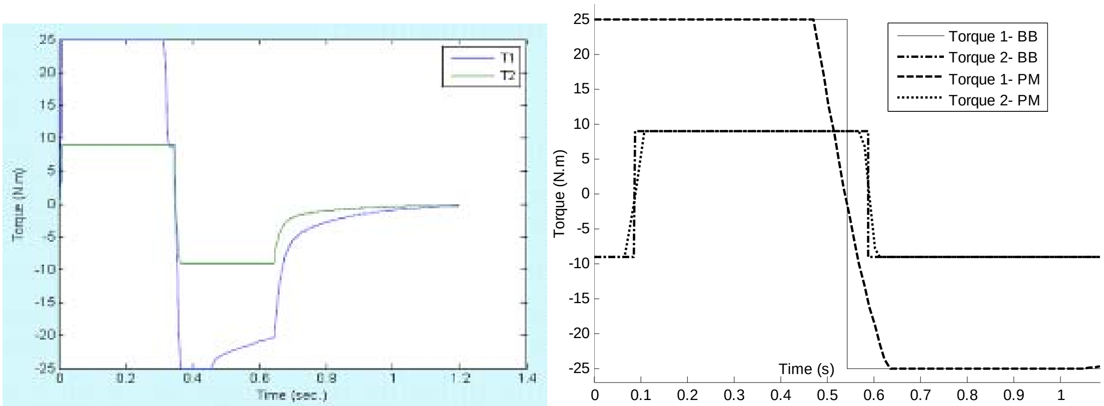
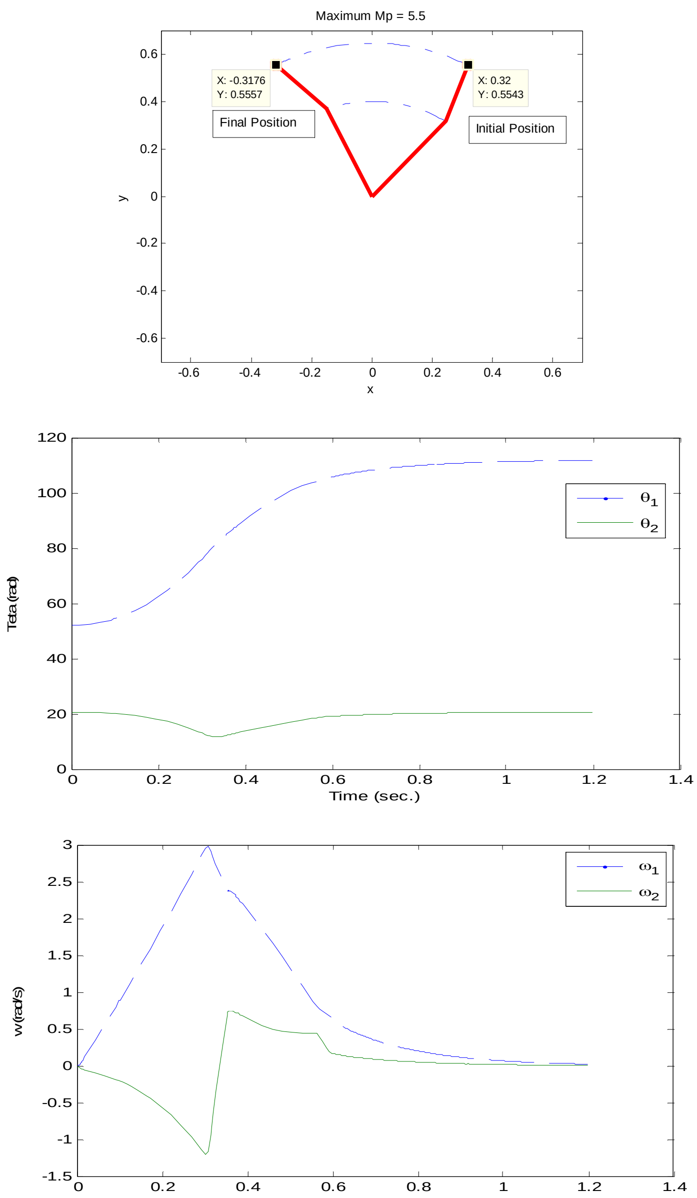


Fig 4: Performance Comparisons of proposed Adaptive Control (left) and Optimal Controls used by Korayem et al. (2008)



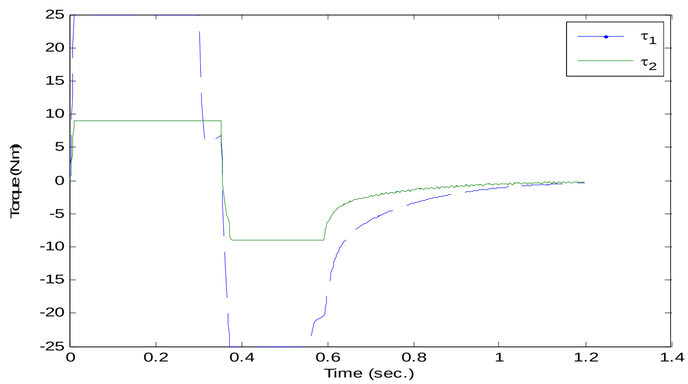
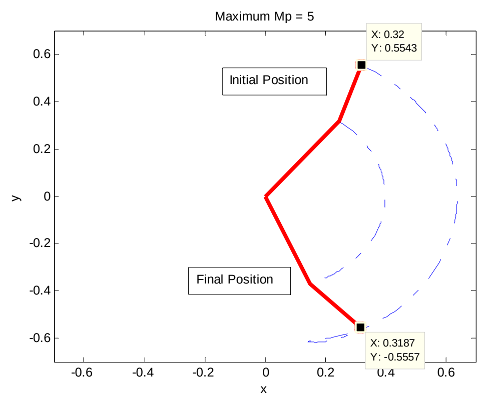
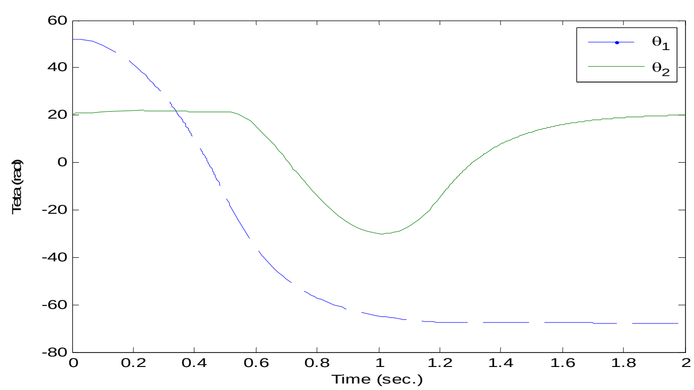
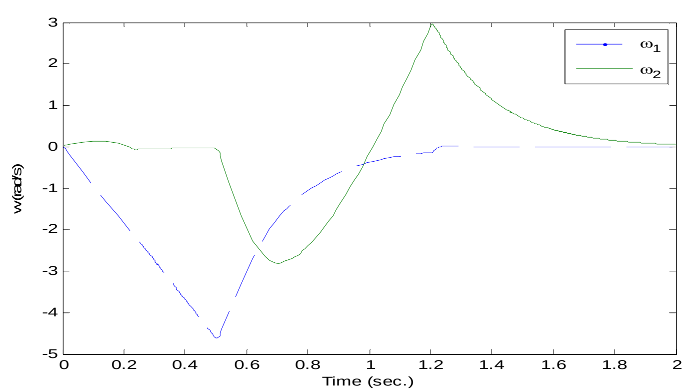


Fig. 5: Manipulator simulation results while passing from first quarter to second quarter.







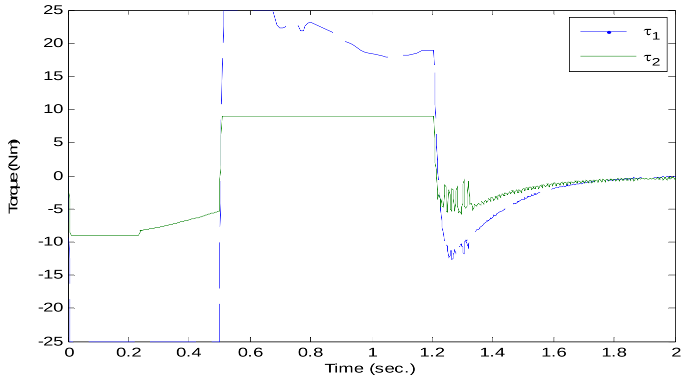
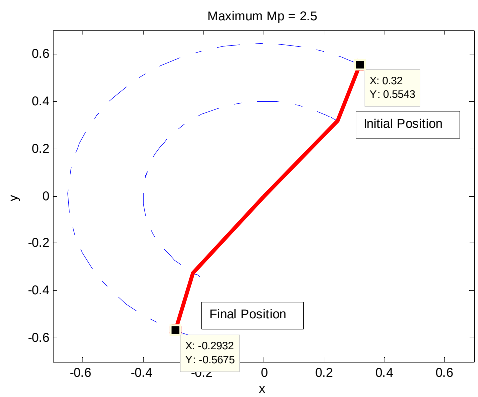
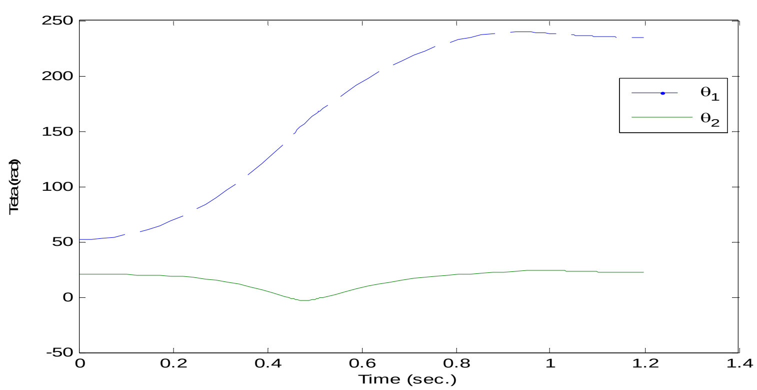
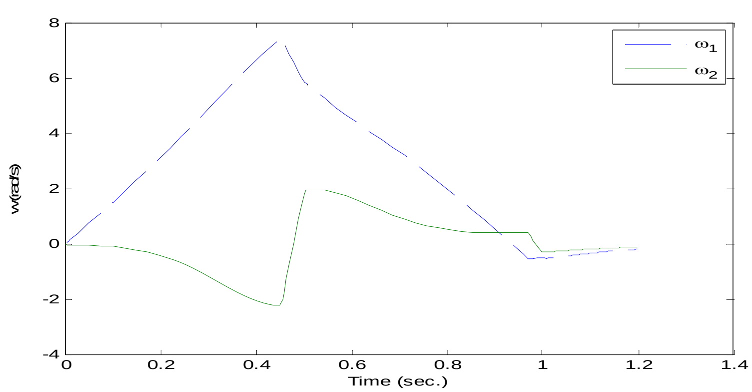


Fig. 6: Manipulator simulation results while passing from first quarter to fourth quarter.







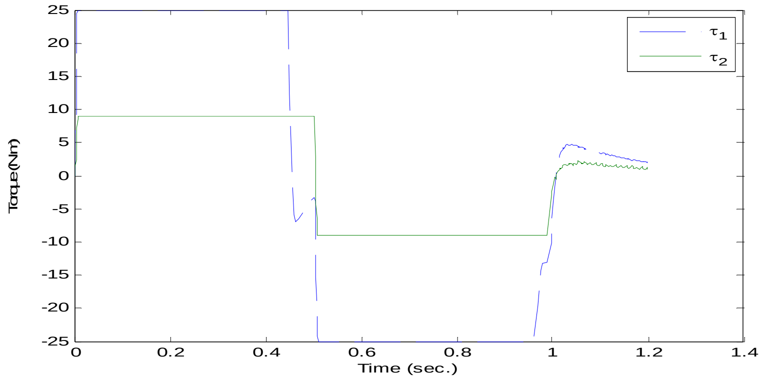


Fig. 7: Manipulator simulation results while passing from first quarter to third quarter.

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