Australian Journal of Basic and Applied Sciences, 5(8): 1257-1268, 2011
ISSN 1991-8178

An Efficient Conjugate- Gradient Method With New Step-Size

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Abstract: In this paper, we derived a new formula for calculating the step-size α_k for conjugate gradient method. The new formula depend on the two formulas for Barzilai–Borwein(BB) method for convex quadratic function and the value of the variable θ which is generated by using uniform distribution. The convergence property of the new method and the numerical result of the new method comparative with the Barzilai -Borwein method are studied by using several test function.

Key words: conjugate gradient method**.** BB method, Step-size, uniform distributation.

INTRODUCTION

The steepest descent method, which can be traced back to Cauchy (Cauchy A. 1847), is the simplest gradient method for unconstrained optimization

$$\min f(x) \qquad x \in R^n \tag{1}$$

where f(x) is a continuous deferential function in R^n. The method has the following form:

$$x_{k+1} = x_k - \alpha_k g_k \tag{2}$$

where $g_k = g(x_k) = \nabla f(x_k)$ is the gradient vector of f(x) at the current iterate point x_k and $\alpha_k > 0$ is the step-size. Because the search direction in the method is the opposite of the gradient direction, it is the steepest descent direction locally, which gives the name of the method. Locally the steepest descent direction is the best direction in the sense that it reduces the objective function as much as possible.

The step-size α_k can be obtained by exact line search:

$$\alpha_k = \mathrm{argminf}\{(x_k - \alpha_k g_k\} \tag{3}$$

or by some line search conditions, such as Goldstein conditions or Wolfe conditions (Dai Y.H. and H. Zhang. 2001). It is easy to show that the steepest descent method is always convergent. That is, theoretically the method will not terminate unless a stationary point is found**.**

2. The Barzilai and Borwein's Method:

In 1988 Barzilai and Borwein (Barzilai, j and J.M, Borwein,. 1988) proposed a gradient descent method that uses a different strategy for choosing the step length. This is based on an interpretation of the quasi-Newton methods in a very simple manner.

The steplength along the negative gradient direction is computed from a two-point approximation to the secant equation from quasi-Newton methods.

The main idea of Barzilai-Borwein's approach (Yuan Ya-xiang,. 2006) is to use the information in the previous iteration to decide the step-size in the current iteration. The iteration is viewed as

$$x_{k+1} = x_k - D_k g_k \tag{4}$$

Where $D_k=\alpha_k I$. In order to force the matrix D_k to have certain quasi-Newton property , it is reasonable to require either

$$\min \left\| d_{k-1} - D_k y_{k-1} \right\|_2 \tag{5}$$

or

$$\min \left\| D_k^{-1} d_{k-1} - y_{k-1} \right\|_2 \tag{6}$$

where $d_{k-1} = x_k - x_{k-1}$ and $y_{k-1} = g_k - g_{k-1}$, because in a quasi-Newton we have that $x_{k+1} = x_k - B_k^{-1} g_k$ and the quasi –Newton matrix B_k satisfies the condition

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$$B_k d_{k-1} = y_{k-1} \tag{7}$$

Now, from $D_k = \alpha_k I$ and the relation (5)(6) two step-sizes can be obtained

$$\alpha_k = \frac{d_{k-1}^T y_{k-1}}{\|y_{k-1}\|_2^2} \tag{8}$$

$$\alpha_k = \frac{\|d_{k-1}\|_2^2}{d_{k-1}^T y_{k-1}} \tag{9}$$

Respectively.

For convex quadratic function in two variables, Barzilai and Borwein [1]shows that the gradient method (2) with α_k given by (8) converges R-super linearly and R-order is $\sqrt{2}$. It is proved by Raydan (Raydan, M. 1993) that the BB method is global convergent for any *n* if the objective function is a convex quadratic.

However, for $n > 2$ no superliner convergence results have been established for the BB method, though numerical results indicates quite often the BB method converges superlinearly (Yuan,Ya-xiang).

The BB method (Yuan Ya-xiang,. 2006) performs quite well for high dimensional problems. The BB method is not monotone, and it is not easy to generalized to general nonlinear functions unless certain non-monotone techniques being applied. Therefore, it is very desirable to find step-size formula which enables fast convergence and possesses the monotone property.

The result of Barzila–Borwein (Yuan,Ya-xiang) has triggered off many researches on the steepest descent method. For example, see(Dai, Y.H. 2003), (Dai, Y. H. Yuan, J. Y. and Y, Yuan,. 2002), (Dai, Y.H. and Y ,Yuan. 2003), (Friedlander, A. J. M, Martnez. B, Molina and M, Raydan,. 1999), (Nocedal, J. A, Sartenaer. and C, Zhu. 2002),(Fletcher, R., 1987),(Raydan, M. 1997),(Fletcher, R. 2005).

In this work, we suggest the method for the unconstrained minimization of a differentiable function $f(\chi): R^n \to R$ is defined by

$$x_{k+1} = x_k - \frac{1}{\alpha_k} g_{k,} \tag{10}$$

Where g_k is the gradient vector of f at x_k and the scalar α_k is given by

$$\alpha_k = \frac{\theta \|d_{k-1}\|^2 + (1-\theta) d_{k-1}^T y_{k-1}}{\theta\, y_{k-1}^T d_{k-1} + (1-\theta)\|y_{k-1}\|^2} \tag{11}$$

Where $d_{k-1} = x_k - x_{k-1}$, $y_{k-1} = g_k - g_{k-1}$.

When $$f(x) = \frac{1}{2} x^t G x - b^t x + c$$

is a quadratic function and G is a symmetric positive definite matrix and by using ELS then α_k in (11) becomes

$$\alpha_k = \frac{d_{k-1}^t G d_{k-1}}{d_{k-1}^t d_{k-1}} \tag{12}$$

In this case, α_k is the Rayleigh quotient of G at the vector d_{k-1} .

Since G is symmetric positive definite then

$$0 < \lambda_{\min} \le \alpha_k \le \lambda_{\max} \tag{13}$$

Where $\lambda_{\min}$ and $\lambda_{\max}$ are respectively the smallest and the largest eigenvalues of G, Hence ,there is no danger of dividing by zero in eq. (10).

3. *Derivation of the new step Size*

We know that QN-like condition is

$$H_k y_{k-1} = d_{k-1} \tag{14}$$

multiplying eq. (14) by the convex combination $(\theta\ d_{k-1} + (1-\theta) y_{k-1})$

we get

$$\alpha_k I(y_{k-1}\theta\, d_{k-1} + (1-\theta) y_{k-1}^T y_{k-1}) = d_{k-1}(\theta\, d_{k-1} + (1-\theta) y_{k-1}) \qquad (15a)$$

Simplify eq.(15a) to get

$$\alpha_k I(\theta\, y_{k-1}^T d_{k-1} + (1-\theta)\|y_{k-1}\|^2) = \theta\|d_{k-1}\|^2 + (1-\theta) d_{k-1}^T y_{k-1} \qquad (15b)$$

Solving eq. (15b) for α to have the equation for the new step-size as it defined in eq.(11).

$\theta \in (0,1)$ is a parameter generated by Uniform distribution, and $H_k = \alpha_k I$.

When $\theta = 1$ eq. (11) reduced to $\alpha_k = \dfrac{\|d_{k-1}\|^2}{y_{k-1}^T d_{k-1}}$

and

when $\theta = 0$ eq. (11) reduced to $\alpha_k = \dfrac{d_{k-1}^T y_{k-1}}{\|y_{k-1}\|_2^2}$,

otherwise and for any value of the parameter $\theta \in (0,1)$ which is generated by uniform distribution we will get the new step-size formula as it defined in eq.(11).

4. Convergence Property:

For any quadratic function:

$f(x) = \frac{1}{2} x^t G x - b^t x + c$

with an SPD(symmetric positive definite) Hessian matrix G, let x be the unique minimizer of f, $\{x_k\}$ the sequence generated by the new method from a given vector x_0, and

$$e_k = x - x_k \qquad \text{for all k} \qquad (16)$$

Then, using eq.(10) and the fact that $g_k = Ax_k - b$, where $b = Ax$, we have

$$Ge_k = \alpha_k d_k \qquad \text{for all k} \qquad (17)$$

Substituting $d_k = e_k - e_{k+1}$ in eq.(17) we obtain for any k

$$e_{k+1} = \frac{1}{\alpha_k}(\alpha_k I - G)e_k. \qquad (18)$$

Now for any initial error e_0, there exist constants $w_1^0, w_2^0, w_3^0, \ldots, w_n^0$ such that

$$e_0 = \sum_{i=1}^{n} w_i^0 v_i$$

Where $\{v_1, v_2, \ldots, v_n\}$ are orthogonal eigenvectors of G associated with eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. Using eq.(14) we obtain for any integer k,

$$e_{k+1} = \sum_{i=1}^{n} w_i^{k+1} v_i \qquad (19)$$

Where $$w_i^{k+1} = \left(\frac{\alpha_k - \lambda_i}{\alpha_k}\right) w_i^k = \prod_{j=0}^{k}\left(\frac{\alpha_j - \lambda_i}{\alpha_j}\right) w_i^0. \qquad (20)$$

The convergence properties of the sequence $\{e_k\}$ will depend on the behavior of each of the sequence $\{w_i^k\}, 1 \le i \le n.$ In general, these sequence will increase at some iterations. However, the following lemma shows that the sequence $\{w_1^k\}$ will decrease Q-linearly to zero.

Lemma 4.1:

The sequence $\{w_1^k\}$ converges to zero Q-linearly with converges factor $\hat{c} = 1 - (\frac{\lambda_{\min}}{\lambda_{\max}})$.

Proof:

For any positive integer k,

$$w_1^{k+1} = \left(\frac{\alpha_k - \lambda_{\min}}{\alpha_k}\right) w_1^k$$

Since α_k satisfy eq. (13), we have

$$\left|w_1^{k+1}\right| = \left(1 - \frac{\lambda_{\min}}{\lambda_{\max}}\right)\left|w_1^k\right| \le c\left|w_1^k\right|,$$

Where $\hat{c} = 1 - (\frac{\lambda_{\min}}{\lambda_{\max}})$

Lemma 4.2:

if the sequence $\{w_1^k\}, \{w_2^k\}, \ldots, \{w_i^k\}$ all converges to zero for a fixed integer l, $1 \le l \le n$. Then, $\lim\inf\left|w_{l+1}^k\right| = 0.$

Proof:

Suppose , by the way of contraction, that there exists a constant $\varepsilon > 0$ such that

$$(w_{l+1}^k)^2 \lambda_{l+1}^2 \ge \varepsilon \quad for\ \ all\ \ k \tag{21}$$

By eq.(12) and eq.(17) it follows that the Rayleigh quotient α_{k+1} can be written as

$$\alpha_{k+1} = \frac{e_k^l G^3 e_k}{e_k^l G^2 e_k}.$$

Then, by eq. (19) and the orthonormality of the eigenvectors$\{v_1, v_{2,\ldots,} v_n\}$, we obtain

$$\alpha_{k+1} = \frac{\sum_{i=1}^n (w_i^k)^2 \lambda_i^3}{\sum_{i=1}^n (w_i^k)^2 \lambda_i^2}. \tag{22}$$

Since the sequence $\{w_1^k\}, \{w_2^k\}, \ldots, \{w_i^k\}$ all convergence to zero, there exist $\hat{k}$ sufficiently large such that

$$\sum_{i=1}^{l} (w_l^k)^2 \lambda_i^2 \le \frac{1}{2} \in \quad \text{for all } k \ge \hat{k}. \tag{23}$$

By eq.(22) and eq.(23) ,for any k$\ge \hat{k}$

$$\frac{\left(\sum_{i=l+1}^n (w_i^k)^2 \lambda_i^2\right)\lambda_{i+1}}{\frac{1}{2} \in + \left(\sum_{i=l+1}^n (w_i^k)\lambda_i^2\right)} \le \alpha_{k+1} \le \lambda_{\max} \tag{24}$$

Since

$$\sum_{i=l+1}^{n} (w_i^k)^2 \lambda_i^2 \ge \in,$$

Then it follows from eq.(24) that

$$\frac{2}{3\lambda_{l+1}} \le \alpha_{k+1} \le \lambda_{\max} \quad \text{for all } k \ge \hat{k},$$

Which implies the bound

$$\left|1 - \frac{\lambda_{l+1}}{\alpha_k}\right| \le \max\left(\frac{1}{2}, 1 - \frac{\lambda_{l+1}}{\lambda_{\max}}\right) \quad \text{for all } k \ge \hat{k} + 1, \tag{25}$$

Finally, using eq.(21) and the first part of eq.(20), we obtain, for all k$\ge \hat{k} + 1$,

$$\left|w_{l+1}^{k+1}\right| = \left|1 - \frac{\lambda_{l+1}}{\alpha_k}\right|\left|w_{l+1}^k\right| \le \hat{c}\left|w_{l+1}^k\right|,$$

Where

$$\hat{c} = \max\left(\frac{1}{2}, 1-\frac{\lambda_{\min}}{\lambda_{\max}}\right) < 1 \tag{26}$$

Because this conclusion contradicts the hypothesis (21) we find that the lemma is true.

Theorem (4.3) establishes the convergence of the new method when applied to quadratic function with a symmetric positive definite Hessian matrix.

Theorem 4.3:

Let f(x) be a strictly quadratic convex function .Let $\{x_k\}$be the sequence generated by the new gradient method and x^* unique minimize of f. Then, either $x_j = x^*$ for some finite j , or the sequence $\{x_k\}$ converges to x^*.

Proof:

We need only consider the case when there is no finite j such that $x_j = x^*$. Hence, it suffices to prove that the sequence $\{e_k\}$converges to zero.

From eq.(16) and the and the orthonormality of the eigenvectors we have

$$\|e_k\|_2^2 = \sum_{i=1}^{n} (w_i^k)^2$$

Therefore, the sequence of errors$\{e_k\}$converges to zero if and only if each one of the sequences $\{w_i^k\}$, for i= 1,…,n, converges to zero. Lemma (4.1) shows that $\{w_1^k\}$ converges to zero. We prove that $\{w_p^k\}$ converges to zero for $2 \le p \le n$ by induction on p. Therefore we let p be any integer from this interval, and we assume that $\{w_1^k\},\ldots,\{w_{p-1}^k\}$ all tend to zero. Then for any given $\in > 0$ there exists $\hat{k}$ sufficiently large such that $\sum_{i=1}^{p-1}(w_i^k)^2\lambda_i^2 < \frac{1}{2}\varepsilon$ for all

$$k \ge \hat{k} \tag{27}$$

From eq.(22) and eq.(27), we obtain

$$\frac{\left(\sum_{i=p}^{n}(w_i^k)^2\lambda_i^2\right)\lambda_p}{\frac{1}{2}\varepsilon + \left(\sum_{i=p}^{n}(w_i^k)^2\lambda_i^2\right)} \le \alpha_{k+1} \le \lambda_{\max} \tag{28}$$

For all integers $k \ge \hat{k}$. Moreover, by lemma 4.2, there exists $k_p \ge \hat{k}$ such that

$$(w_p^{k_p})^2\lambda_p^2 < \varepsilon$$

Now, let us say that $k_0 > k_p$ is any integer for which $(w_p^{k_0-1})^2\lambda_p^2 < \varepsilon$ and $(w_p^{k_0})^2\lambda_p^2 \ge \varepsilon$. Clearly,

$$\sum_{i=1}^{n}(w_i^k)^2\lambda_i^2 \ge (w_k^p)^2\lambda_p^2 \ge \varepsilon \quad \text{for} \quad k_0 \le k \le j-1 \tag{29}$$

Where j is the first than k_0 for which $(w_p^j)^2\lambda_p^2 < \varepsilon$. Then , by (28) and (29), we have

$$\frac{2}{3}\lambda_p \le \alpha_{k+1} \le \lambda_{\max} \quad \text{for} \quad k_0 \le k \le j-1 \tag{30}$$

Thus using eq.(30) and the first part of eq. (20), we obtain

$$\left|w_p^{k+2}\right| \le \hat{c}\left|w_p^{k+1}\right| \qquad \text{for} \quad k_0 \le k \le j-1$$

Where $\hat{c}$ is a constant as in eq.(26), which satisfies $\hat{c} < 1$. Finally, using the bound

$$\left|w_p^{k_0+1}\right| \le \left(\frac{\lambda_{\min}-\lambda_{\max}}{\lambda_{\min}}\right)^2 \left|w_p^{k_0-1}\right|$$

Which is implies by expression (13) and the first part of eq.(20), we conclude that

$$(w_p^k)^2 \le \left(\frac{\lambda_{\max}-\lambda_{\min}}{\lambda_{\min}}\right)^4 (w_p^{k_0-1})^2 \le \left(\frac{\lambda_{\max}-\lambda_{\min}}{\lambda_{\min}}\right)^4 \frac{\varepsilon}{\lambda_p^2}$$

For all $k_0+1 \le k \le j+1$. Further, eq.(10) provides the inequality $(w_p^{k_0})^2 \le [(\lambda_{\max}-\lambda_{\min})]^2 (w_p^{k_0-1})^2$. It follows from the conditions on k_0 and j that $(w_p^k)^2$ is bounded above by a constant multiple of ε for all $k \ge k_0-1$. Hence, since $\varepsilon > 0$ can be chosen arbitrarily small, we deduce $\lim_{k\to\infty}\left|w_p^k\right| = 0$ as required, which complete the proof.

5. Outlines of the New Method:

Step (1): Given $x_1 \in R^n$. Set k=1, $d_k = -g_k$.

Step(2): Set $x_{k+1}=x_k+\alpha_k d_k$. where α_k is the step size is computed by Using eq.(11).

Step (3): Check for convergence i.e. if $\|g_k\| < \in$, where $\in$ is small Positive tolerance, Stop; otherwise continue.

Step (4): Compute new search direction defined by $d_k=-g_k+\beta_k d_{k-1}$ where β_k is Computed by the following formula $\beta_k = \dfrac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}}$

Step (5): If k=n or if the restart criterion $\|g_k^T g_{k+1}\| > 0.2\|g_k\|^2$, [Powell, M.J.D., 1977] Set k=k+1, go to step(2).

6. Statistics Test:

6.1 Uniform Distribution:

The uniform distribution is simple probability distribution. It can be either discrete or continuous**.** In the discrete case, they can be characterized by saying that all possible values are equally probable. In the continuous case, one says that all intervals of the same length are equally probable. The continuous uniform distribution is called the rectangular distribution because of the shape of its probability density function. It is parameterized by smallest and largest values that the uniformly-distributed random variable can take, a and b. The general form of the formula of probability density function of the uniform random distribution is defined as {f(x)=(1/(b-a), if a<x<b), otherwise 0}.

The case where a=0 and b=1is called the standard uniform, so that the random variable can takes values only between zero and one (Steel, G.D. and J. H, Torrie. 1981), (Uniform distributation–definition).

6.2 Chi-Square Distribution:

The distribution of χ^2 (Greek chi: read chi square) because of its relation to s^2 and the very important student 's t distribution.

Chi-square is defined as a sum of squares of independent standard normally distributed variables with zero means and unit variance[Uniform distributation–definition].

6.3 Goodness-of-Fit Test:

It is one of chi -square applications which is used to determined whether or not data "fit" a particular distribution. The χ^2 distribution, when associated with discrete data ,is usually in conjunction with a test of goodness of fit. The test criterion is

$$\chi^2 = \sum \frac{(observed - \exp eced)^2}{\exp ected}$$

The sum is taken over all cells in the classification system. Observed refers to the numbers observed in the cells; expected refers to the average numbers of expected values when the hypothesis is true, that is the theoretical values [Susan Dean and Barbara Illowsky].

we use this application of chi -square to determine whether the results of the new step-sizes are uniformly distributed or not

Discussion:

The results of the new method are reported in Tables (1-5) in term of number of function evaluation, the number of iterations, the value of step size, and the minimum point. The comparison of the new method with BB-method involve five well known test functions .

All the result are obtained by using double precision using program written in FORTRAN(2000). The comparative Performance of the algorithm are evaluated by considering the number of function evaluation, the number of iterations, the value of step size, and the minimum point. The stopping criterion is taken to be :

$$\|g_{k+1}\|_2 < 1x10^{-6}$$

The numerical results of the new method showed that it is as efficient as BB-method and there are improving in the results for some values of the parameter θ**.**

Each tables (1-5) consist of (52) values for the parameter θ, the first two values of θ, that means when θ=1,and θ=0 give results of the two formulas for BB-Method, and the other values for $\theta \in (0,1)$ which is generated by uniform distribution give the results of the present work.

The results in table (1) due to the Powell function with n=1000. The best results at θ= 0.960687

The results in table (2) due to the Wolf function with n=1000. The best results at θ= 0.913459 , θ= 0.913081

The results in table (3) due to the Wood function with n=1000. The best results at θ= 0.913081 , θ= 0.960687

The results in table (4) due to the Shallow function with n=1000. The best results at θ=0.416577 , θ=0.415696

The results in table (5) due to the Reciep function with n=1000. The best results at θ= 0.017795

we use goodness - of- fit test which is one of chi –square application to determine whether the results of the new step-sizes are uniformly distributed or not.

We choose Powell function to check if the 50 values of the new step-sizes for this function distributed uniformly or not. The results of using SPSS package to determine whether these step-sizes results are uniformly distributed or not are declared in table (6).

To explain this let

H_0: new step- sizes results of Powell function distributed uniformly(Null hypothesis)

$H_{1:}$ new step-sizes results of Powell function are not distributed uniformly (Alternate hypothesis)

α =0.05 (Level of significance)

The Asymp.sig. is (1>0.05), it means we accept null hypothesis and the new step-sizes results of Powell function distributed uniformly.

Conclusion:

New formula for calculating step-sizes for conjugate gradient method is proposed in this work with its numerical results which showed that it is an efficient as BB-method and there are improving in the results for some values of the parameter θ**.** In future research we should seek more approaches for estimating the step-size as exactly as possible and find some available technique to guarantee both the global convergence and quick convergence rate of conjugate gradient methods**.**

Table 1: Comparative between BB-Method and the New Method , (Powell Function(N=1000))

	θ	**NOF**	**NOI**	**STEP- SIZE**	**MIN**
1-	1.000000	274	68	Exceed one	0.39231712924404441612E-08
2-	0.000000	300	80	0.4536351555898647E-01	0.35997299471692455874E-08
3-	0.758880	177	44	0.3455805425357337E-01	0 .34347910227531331295E-09
4-	0.889442	184	53	0.8978502288662730E-02	0.74136783292327153176E-09
5-	0.299216	202	57	0.2791886865786126E-01	0 .10411210418838407979E-09
6-	0.515259	199	54	0.1988711821554071E-01	0.39433718745134366257E-08
7-	0.682900	196	48	0.1164780143835216E-01	0.45089411131467953575E-08
8-	0.345030	205	53	0.1565458663102686E-01	0.27077499972639696174E-09
9-	0.944100	166	44	0.4070408939696699E-01	0.12686061002205954519E-10
10-	0.416577	178	48	0.3995085215197194E-01	0.22789077728352667405E-08
11-	0.583346	200	58	0.7456758519681845E-02	0.10752637591578031291E-09
12-	0.282370	220	61	0.4468254944623997E-01	0.72287021254473007317E-09
13-	0.254747	265	68	0.4446498683938308E-01	0.67315638817203871770E-09
14-	0.455153	196	54	0.5039711638240738E-02	0.15775042499898694071E-08

15-	0.913459	179	46	0.3409866530908531E-01	0.6584468145510882 6887E-09
16-	0.857628	172	48	0.3158084988679190E-01	0.2925792277877164 9403E-08
17-	0.983443	174	42	0.4949014296624476E-02	0.3226347340717656 8016E-09
18-	0.833983	192	46	0.6865276068924238E-02	0.3421598013146568 4554E-08
19-	0.238529	262	66	0.3405106131838419E-01	0.6274644051871883 8818E-09
20-	0.721548	193	52	0.9479292460801966E-02	0.3592313645445955 2740E-11
21-	0.855916	143	37	0.3978650049608504E-01	0.8287309348539468 9007E-08
22-	0.268054	251	65	0.1008027050965276E-01	0.3934683150873410 1594E-08
23-	0.913081	227	52	0.4276672751692611E-01	0.4873486712162081 4967E-09
24-	0.452672	163	47	0.5015200339897601E-02	0.6489415685016659 2341E-08
25-	0.197661	253	63	0.4505697722637841E-01	0.2821010090675533 3316E-08
26-	0.110385	231	62	0.3952832260565500E-01	0.2934978002993753 6988E-10
27-	0.2701170	213	57	0.4992316405639558E-02	0.6265299932693643 3170E-10
28-	0.287721	187	50	0.5698223191934776E-02	0.8978243781726677 3786E-09
29-	0.844633	162	40	0.4913291180153890E-02	0.1043416934068634 5379E-09
30-	0.687444	163	42	0.1077994810515671E-01	0.9051824862233114 9049E-09
31-	0.415696	229	54	0.1573024310900187E-01	0.2561350002991233 1093E-09
32-	0.284473	205	55	0.8893924596391939E-02	0.1642149819380953 4204E-10
33-	0.710829	175	45	0.1492728855160544E-02	0.1148430205961434 6282E-08
34-	0.832882	209	51	0.4263651434031269E-01	0.5248495082382864 3857E-09
35-	0.692989	201	51	0.6114566488266081E-02	0.2599354861132225 6547E-08
36-	0.854430	172	44	0.5174376440474348E-02	0.2166371736254033 3235E-08
37-	0.703774	236	56	0.4972581945087762E-02	0.3222217048990810 2413E-08
38-	0.960687	131	35	0.5694430931747250E-02	0.4793908925011340 6922E-13
39-	0.842926	188	48	0.2815206316047036E-01	0.2081770602675768 1089E-08
40-	0.279672	205	59	0.1905060690397669E-01	0.9149690813300515 5353E-09
41-	0.522343	203	57	0.1068602424125075E-01	0.1265902911189286 0388E-08
42-	0.440175	173	47	0.9144402601324951E-02	0.2199279732864296 2111E-08
43-	0.017795	246	68	0.3073602260315203E-01	0.1253722671861245 9123E-08
44-	0.383943	223	57	0.4417710289007266E-01	0.7775292725244998 8821E-08
45-	0.265080	267	68	0.2052766406911024E-01	0.9718199233648102 7505E-10
46-	0.897284	219	53	0.3719474521090655E-01	0.6006713044196326 8790E-09
47-	0.719055	234	55	0.7713482508437058E-02	0.3529811526471301 0936E-08
48-	0.509061	213	56	0.5030182645809706E-01	0.1731728607452708 4570E-08
49-	0.870169	225	57	0.5161229914976979E-02	0.7689855126416093 5585E-09
50-	0.94302	279	77	0.4272181723408262E-01	0.8254948330739668 5439E-10
51-	0.830864	191	48	0.8572696781151388E-02	0.2519005290686085 3694E-13
52-	0.446836	219	59	0.3563233997293407E-01	0.1816695617752568 0831E-09

Table 2: Comparative between BB-Method and the New Method ,(Wolf Function(N=1000))

	θ	NOF	NOI	STEP- SIZE	MIN.
1-	1.000000	202	71	0.2579794987944429E-01	0.16218396334555630007E-13
2-	0.000000	236	79	0.1451639202830650E-01	0.33123470256249211106E-13
3-	0.758880	240	80	0.1418645085707849E-01	0.23623427503245224891E-13
4-	0.889442	227	76	0.7567892780029888E-02	0.22069148420338474548E-13
5-	0.299216	233	78	0.1292946926480419E-01	0.33007843104048153114E-13
6-	0.515259	237	79	0.1420895302045916E-01	0.35379309195830479974E-13
7-	0.682900	237	79	0.1397956879555691E-01	0.36734201412313373415E-13
8-	0.345030	233	78	0.1256081974464295E-01	0.27532493969154152871E-13
9-	0.944100	201	71	0.2812539471558881E-02	0.30350149039301318241E-13
10-	0.416577	230	77	0.1113855781725194E-01	0.29797095451950373974E-13
11-	0.583346	237	79	0.1413408563724071E-01	0.35735672084586725197E-13
12-	0.282370	233	78	0.1301135449999879E-01	0.34886878715529068144E-13
13-	0.254747	233	78	0.1308921577707646E-01	0.37788774144349424592E-13
14-	0.455153	237	79	0.1426156025173853E-01	0.35172561060055339729E-13
15-	0.913459	201	69	0.9206610512477452E-02	0.19960343438004038939E-13
16-	0.857628	220	74	0.1206702217649164E-01	0.32536348685583608459E-13
17-	0.983443	218	76	0.2733319917680909E-01	0.20769984084847634643E-13
18-	0.833983	240	80	0.1391816824865560E-01	0.26231904030847042744E-13
19-	0.238529	236	79	0.1404450679341232E-01	0.24328836932417999177E-13
20-	0.721548	237	79	0.1389763748557148E-01	0.37432973791339663150E-13
21-	0.855916	220	74	0.1216608257453133E-01	0.30231734398674288761E-13
22-	0.268054	233	78	0.1305823652327145E-01	0.36419599610278839120E-13
23-	0.913081	201	69	0.8891224346873556E-02	0.23292507681822900500E-13
24-	0.452672	237	79	0.1426351139425866E-01	0.35165537661624632518E-13
25-	0.197661	236	79	0.1418238899778187E-01	0.26559143728209055810E-13
26-	0.110385	236	79	0.1416154352746675E-01	0.30272590157577989445E-13
27-	.2701170	233	78	0.1305253093001727E-01	0.36202481421569004371E-13
28-	0.287721	233	78	0.1298877811782541E-01	0.34298652781743164089E-13
29-	0.844633	240	80	0.1386138256841913E-01	0.26899339318935785528E-13
30-	0.687444	215	73	0.1216516238285387E-01	0.26076409218084318132E-13
31-	0.415696	230	77	0.1114725247456196E-01	0.29975393396043520119E-13
32-	0.284473	233	78	0.1300284164207498E-01	0.34656676943810024121E-13
33-	0.710829	237	79	0.1392196444802924E-01	0.37211686472130745702E-13

34-	0.832882	240	80	0.1392364971864321E-01	0.26169366890636262908E-13
35-	0.692989	237	79	0.1395966232165131E-01	0.36892089112710291449E-13
36-	0.854430	220	74	0.1224476014721929E-01	0.28334630923765065975E-13
37-	.703774	237	79	0.1393727463466978E-01	0.37078573800181049331E-13
38-	0.960687	206	76	0.2540843665681255E-01	0.29024176393517536767E-13
39-	0.842926	240	80	0.1387097258387463E-01	0.26784166330720662104E-13
40-	0.279672	233	78	0.1302160598684308E-01	0.35180345484405190835E-13
41-	0.522343	237	79	0.1420200810763416E-01	0.35409216934501206887E-13
42-	0.440175	237	79	0.1427312501563874E-01	0.35131650706749563630E-13
43-	0.017795	236	79	0.1446225846467974E-01	0.32779195348512121451E-13
44-	0.383943	230	77	0.1144432239872067E-01	0.36470935665247988824E-13
45-	0.265080	233	78	0.1306596083743730E-01	0.36730322292196373769E-13
46-	0.897284	214	73	0.1167165367922850E-01	0.28851143763141314118E-13
47-	0.719055	237	79	0.1390341148069628E-01	0.37379316314981491753E-13
48-	0.509061	237	79	0.1421488906786336E-01	0.35354241513327652445E-13
49-	0.870169	215	73	0.1232011393905307E-01	0.28264770652012783578E-13
50-	0.94302	236	79	0.1418876100341013E-01	0.30806667767888253004E-13
51-	0.830864	240	80	0.1393353774224282E-01	0.26057587153062779665E-13
52-	0.446836	237	79	0.1426804686986680E-01	0.35149407384995417341E-13

Table 3: Comparative between BB-Method and the New Method , (Wood Function(N=1000))

	θ	NOF	NOI	STEP- SIZE	MIN.
1-	1.000000	176	58	0.9985500740193882E-03	0.63254811874268345035E-13
2-	0.000000	412	136	0.4953703261515834E-03	0.12912251466128314124E-14
3-	0.758880	231	77	0.1175682249846358E-01	0.56773951594448537323E-14
4-	0.889442	178	60	0.2006952647697644E-03	0.25272276900609584509E-12
5-	0.299216	412	136	0.4934374573882487E-03	0.12933140270722845188E-14
6-	0.515259	412	136	0.4906704048443839E-03	0.12972422626866213567E-14
7-	0.682900	249	82	0.1131995492426532E-02	0.65606581329120018725E-14
8-	0.345030	412	136	0.4930037032012679E-03	0.12938940298173442587E-14
9-	0.944100	168	57	0.1120501457568170E-02	0.88252056932864230779E-13
10-	0.416577	412	136	0.4921743840566013E-03	0.12949764871744369398E-14
11-	0.583346	412	136	0.4892416530311105E-03	0.12996637987913457849E-14
12-	0.282370	412	136	0.4935875113700974E-03	0.12931399022064914449E-14
13-	0.254747	412	136	0.4938123382720596E-03	0.12928634744161141824E-14
14-	0.455153	412	136	0.4916466909242261E-03	0.12957028115904897397E-14
15-	0.913459	185	63	0.3182766561657411E+00	0.19174953896997365244E-20
16-	0.857628	208	71	0.3614466723022637E-03	0.10768305872235699644E-13
17-	0.983443	164	56	0.2880618443848735E+00	0.22849598759812824878E-16
18-	0.833983	343	115	0.7265099461637153E-01	0.20506331261550724485E-20
19-	0.238529	412	136	0.4939507851085536E-03	0.12927348138608421809E-14
20-	0.721548	409	136	0.3395130411049880E-03	0.15112719632085265091E-14
21-	0.855916	233	79	0.1644329544905034E-01	0.28354110952916681637E-14
22-	0.268054	412	136	0.4937179614526441E-03	0.12930040484598721129E-14
23-	0.913081	163	55	0.1104099918477753E-02	0.40569177427843748835E-13
24-	0.452672	412	136	0.4916763856714245E-03	0.12956569165407271904E-14
25-	0.197661	412	136	0.4942546500833442E-03	0.12923971524541346183E-14
26-	0.110385	412	136	0.4947956405539059E-03	0.12917920726630686471E-14
27-	0.2701170	412	136	0.4937003780286433E-03	0.12930318678973021909E-14
28-	0.287721	412	136	0.4935459884706845E-03	0.12932214928869355504E-14
29-	0.844633	322	108	0.3468967355565966E+00	0.13142505742845650375E-15
30-	0.687444	259	87	0.3377222710222345E+00	0.27809240619887526236E-18
31-	0.415696	412	136	0.4921820510842511E-03	0.12949430452012568951E-14
32-	0.284473	412	136	0.4935695651272817E-03	0.12931773692001501857E-14
33-	0.710829	409	136	0.4017752343188995E-03	0.40321381434673746258E-14
34-	0.832882	364	122	0.1294136673823848E+00	0.19654407705552100127E-21
35-	0.692989	348	115	0.3149556562890194E+00	0.42518521969048686199E-20
36-	0.854430	222	75	0.1626012006908881E-01	0.54186467520752958474E-16
37-	0.703774	404	134	0.9687429918984165E-03	0.10902409549518213926E-13
38-	0.960687	162	55	0.1324414667811908E+00	0.18087602122686067856E-18
39-	0.842926	320	108	0.1004569018858116E-02	0.36164034822015608842E-17
40-	0.279672	412	136	0.4936226676439903E-03	0.12931414015256797590E-14
41-	0.522343	412	136	0.4905368980564051E-03	0.12974279816206090529E-14
42-	0.440175	412	136	0.4918637286944967E-03	0.12954237500882044807E-14
43-	0.017795	412	136	0.4952923893023006E-03	0.12912947545223201259E-14
44-	0.383943	412	136	0.4925789459590718E-03	0.12944368606510705316E-14
45-	0.265080	412	136	0.4937390084392951E-03	0.12929864460892938301E-14
46-	0.897284	219	75	0.2692835742805674E-01	0.12649897551562787848E-14
47-	0.719055	361	120	0.3512165910000160E+00	0.64493949863847123567E-20
48-	0.509061	412	136	0.4907702044202971E-03	0.12970287910728164332E-14

49-	0.870169	449	153	0.6692101429443915E-03	0.16553757828255075459E-12
50-	0.94302	412	136	0.4948932305808865E-03	0.12916860788381187287E-14
51-	0.830864	194	66	0.1920525459258463E-03	0.15384525352472158327E-12
52-	0.446836	412	136	0.4917621756209857E-03	0.12955440884362779050E-14

Table 4: Comparative between BB-Method and the New Method , (Shallow Function(N=1000))

	θ	NOF	NOI	STEP-SIZE	MIN.
1-	1.000000	29	9	0.3294557137708508E-06	0.24854378824557413530E-15
2-	0.000000	28	9	0.7443056586481205E-01	0.79043647384177600409E-18
3-	0.758880	29	9	0.2102331780577550E+00	0.15337414424812366346E-19
4-	0.889442	30	9	0.2102509686644478E+00	0.38681489851617430483E-20
5-	0.299216	45	15	0.1969606941919866E+00	0.43162194320690960287E-14
6-	0.515259	29	9	0.2102228465120987E+00	0.20140599544516306796E-19
7-	0.682900	29	9	0.2102298878082164E+00	0.18506051121875676964E-19
8-	0.345030	34	11	0.2937528125599109E-07	0.23106161159255156482E-22
9-	0.944100			fail	
10-	0.416577	25	8	0.1393879658942727E+00	0.13932216803629696146E-13
11-	0.583346	29	9	0.2102256282068168E+00	0.19999245771418073803E-19
12-	0.282370	27	9	0.1926314591203737E+00	0.20115106630512487631E-13
13-	0.254747	28	9	0.2403568186936820E-01	0.29871458290188522731E-20
14-	0.455153	32	10	0.1380141668954186E+00	0.23758192072170897914E-18
15-	0.913459	30	9	0.2102584812420884E+00	0.16314090335717621975E-21
16-	0.857628	26	8	0.1380875067676995E+00	0.11919589950489590049E-15
17-	0.983443	30	9	0.2910506176366385E-07	0.18730178636250574519E-18
18-	0.833983	28	9	0.2104566066609098E+00	0.50463612686124015466E-17
19-	0.238529	28	9	0.2813209665436819E-01	0.51374560104653297127E-20
20-	0.721548	29	9	0.2102315677467542E+00	0.17201841272458305200E-19
21-	0.855916	27	8	0.6220630554468042E-07	0.53169375870509172770E-14
22-	0.268054	27	9	0.1889115031906023E+00	0.42627153430239504707E-13
23-	0.913081	30	9	0.2102583707764057E+00	0.19190461469130509988E-21
24-	0.452672	32	10	0.1380167289027101E+00	0.17714984655240899704E-18
25-	0.197661	28	9	0.3780247111440686E-01	0.17100937377655217690E-19
26-	0.110385	28	9	0.5578082228874031E-01	0.12646245198416665465E-18
27-	0.2701170	27	9	0.1894396499658120E+00	0.38541226266280761431E-13
28-	0.287721	27	9	0.1940555595292543E+00	0.14643205052730642083E-13
29-	0.844633	28	9	0.2102989288146117E+00	0.80219459277514293624E-19
30-	0.687444	29	9	0.2098994611563974E+00	0.18277134706581309144E-19
31-	0.415696	25	8	0.1393840504719745E+00	0.14843240102726694718E-13
32-	0.284473	27	9	0.1931889170469061E+00	0.17805174411451013158E-13
33-	0.710829	29	9	0.2102311020987577E+00	0.17619575261691742885E-19
34-	0.832882	28	9	0.2104720402866398E+00	0.70305562710828858126E-17
35-	0.692989	29	9	0.2102303261984571E+00	0.18215870813420592951E-19
36-	0.854430	30	9	0.2101395591398366E+00	0.94085098234897703153E-23
37-	0.703774	29	9	0.2102307953359979E+00	0.17869500337117068878E-19
38-	0.960687	30	9	0.1530171278563778E-07	0.56990904364335882463E-19
39-	0.842926	28	9	0.2103252816291656E+00	0.18373054690967667845E-18
40-	0.279672	27	9	0.1919204653021477E+00	0.23408438397160802917E-13
41-	0.522343	29	9	0.2102231297490816E+00	0.20147564280351962491E-19
42-	0.440175	32	10	0.1380293785525302E+00	*0.32116534362994079429E-19*
43-	0.017795	28	9	0.7168167156023181E-01	0.61109991985461453735E-18
44-	0.383943	28	9	0.3626620468009686E-05	0.41714045577727883649E-14
45-	0.265080	28	9	0.2134288227478346E-01	0.20661122652041914659E-20
46-	0.897284	30	9	0.2102535210599164E+00	0.24982739349114183633E-20
47-	0.719055	29	9	0.2102314592601169E+00	0.17302531922134974334E-19
48-	0.509061	29	9	0.2102225999508419E+00	0.20132567313475539878E-19
49-	0.870169	29	9	0.2098449708319232E+00	0.56188177577028091069E-19
50-	0.94302	28	9	0.5875319201700478E-01	0.17185645819407161832E-18
51-	0.830864	28	9	0.2104999365640582E+00	0.12530953320592691727E-16
52-	0.446836	32	10	0.1380226887001152E+00	0.84160831965087243828E-19

Table 5 : Comparative between BB-Method and the New Method , (Reciep Function(N=1000))

	θ	NOF	NOI	STEP-SIZE	MIN.
1-	1.000000	59	17	0.5632333136952330E+00	0.12849892154908485670E-13
2-	0.000000	28	8	0.5000806648494674E+00	0.12537943585605010276E-13
3-	0.758880	46	13	0.5060006201945272E+00	0.22198457143619122976E-12
4-	0.889442	48	14	0.5463895332755995E+00	0.12946260755610710598E-11
5-	0.299216	39	11	0.2508947978062387E+00	0.56092366162555903687E-13
6-	0.515259	96	29	0.7612044616651237E+00	0.23171408294486234104E-12
7-	0.682900	46	13	0.5050277627484939E+00	0.62951712645309204524E-13
8-	0.345030	42	12	0.5054034165953742E+00	0.21296390113009474086E-14
9-	0.944100	47	14	0.5803347913768991E+00	0.28571775813764836767E-11
10-	0.416577	52	15	0.6216228749670106E+00	0.83292861088944811894E-13
11-	0.583346	43	12	0.5165237075142612E+00	0.99859756382192338778E-13
12-	0.282370	39	11	0.1711208248406427E+00	0.23752912362245204655E-13
13-	0.254747	39	11	0.4730423152643098E-01	0.51696961828318452532E-14
14-	0.455153	36	10	0.5420456314222428E+00	0.11154836248873686802E-13
15-	0.913459	47	14	0.5590414077918524E+00	0.18349816871027974899E-11
16-	0.857628	46	13	0.5084010833242879E+00	0.10878658582776057963E-11
17-	0.983443	59	17	0.5531179622420036E+00	0.80480346342597588580E-14
18-	0.833983	46	13	0.5076778451344978E+00	0.74560007039049283626E-12
19-	0.238529	61	18	0.9642777277371117E-01	0.18083823006498874231E-11
20-	0.721548	46	13	0.5054452664816025E+00	0.12020626619542720353E-12
21-	0.855916	46	13	0.5083449693405362E+00	0.10585474030353762967E-11
22-	0.268054	39	11	0.1057280715911943E+00	0.10987400677941396564E-13
23-	0.913081	47	14	0.5588175045806754E+00	0.18249649464707882365E-11
24-	0.452672	36	10	0.5257334467598656E+00	0.43231897158324012931E-14
25-	0.197661	39	11	0.5128617966044916E+00	0.23548199705254915254E-13
26-	0.110385	30	9	0.5003745445529639E+00	0.56321124511478388336E-12
27-	0.2701170	39	11	0.1149968777510642E+00	0.12308795231787504405E-13
28-	0.287721	39	11	0.1961627966275671E+00	0.31375300400212590100E-13
29-	0.844633	46	13	0.5079900936477351E+00	0.88401977976838702301E-12
30-	0.687444	48	14	0.5372271723227282E+00	0.93836468616922687473E-12
31-	0.415696	52	15	0.6787653249509041E+00	0.24807577510734695374E-12
32-	0.284473	39	11	0.1809264997127364E+00	0.26514760971082694216E-13
33-	0.710829	45	13	0.5488265997887078E+00	0.89649814665958458628E-12
34-	0.832882	46	13	0.5076467666152390E+00	0.73257834060309329067E-12
35-	0.692989	45	13	0.5072137677183731E+00	0.10871295953555236933E-12
36-	0.854430	46	13	0.5082967575910914E+00	0.10337369510595537937E-11
37-	0.703774	45	13	0.5115145358228258E+00	0.19528089891353175498E-12
38-	0.960687	47	14	0.5948779991677005E+00	0.36314437860527743009E-11
39-	0.842926	46	13	0.5079385999177202E+00	0.86023108382320548469E-12
40-	0.279672	39	11	0.1586119453624799E+00	0.20602227595194223019E-13
41-	0.522343	84	25	0.5844914967146411E+00	0.22482445873804008545E-11
42-	0.440175	36	10	0.5663740764807631E+00	0.21812840275397845977E-17
43-	0.017795	28	8	0.5000007509220310E+00	0.62260734922774240577E-15
44-	0.383943	41	12	0.5179063120706158E+00	0.28071478371182585558E-13
45-	0.265080	39	11	0.9246320961491485E-01	0.93139147091951479535E-14
46-	0.897284	48	14	0.5501865148816284E+00	0.14511706142519732987E-11
47-	0.719055	48	14	0.1586227368553413E+00	0.17855785501398358540E-11
48-	0.509061	82	25	0.6011777561281747E+00	0.11842176006036510106E-11
49-	0.870169	48	14	0.5382532963400497E+00	0.97667098664557239137E-12
50-	0.94302	30	9	0.5002415228752162E+00	0.29749513019730509376E-12
51-	0.830864	46	13	0.5075903747432291E+00	0.70929321283627147303E-12
52-	0.446836	49	14	0.6841780222668842E+00	0.93133071732970750441E-12

Table 6: Test Statistics

VAR00001	
1.840	Chi-square
47	Df
1.000	Asymp. Sig**.**

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