

## Evaluation and Calculation of Overhead Line Impedance in Distribution Networks

R. Ebrahimi, A. Babae and M. Hoseynpoor

Bushehr Branch, Islamic Azad University, Bushehr, Iran.

---

**Abstract:** The determination of the impedance for overhead lines is a critical step before the analysis of distribution feeder such as load flow and short circuit calculation can begin. Because distribution systems consist of single-phase, two-phase, and untransposed three-phase lines serving unbalanced loads, it is necessary to retain the identity of the self and mutual impedance terms of conductors and take into account the ground return path for the unbalanced currents. In this paper, Carson's equations are developed by Kirchhoff's voltage and current laws. This developing that use a suitable impedance model of distribution systems is novel and easy to understand. Due to the aforementioned equations, a computer program by Matlab software has been developed to calculate the impedance of overhead distribution lines. Comparison of the results shows the validity of the proposed program and ATP software. Then, some effective parameters such as power frequency, skin effect, type of conductor and spacing distance between the phase conductors on impedance calculation are obtained and studied.

**Key words:** Impedance of distribution lines, Carson's equations, Distribution lines standard, effective parameters on impedance calculation.

---

### INTRODUCTION

The resistor, the reactance and the capacitance of capacitor are three significant lines parameters, where formatted in parallel or series impedances, to modeling them in networks. As usual, these parameters withdraw, regarding the low capacitance that the capacitors in overhead lines may have. So in analysis and modeling of these networks, only the series impedances of lines are evaluated.

The series impedances of distribution networks lines, are consists of the conductor resistance, self or mutual reactance where the reactance section is produced by the magnetic fields around the conductors. The resistor value depends on the conductor quality which expressed by the company.

Distinguishing the transposition between lines and balanced loads, the self and mutual sections of reactance can be combined In transportation networks, and only one phased-reactance are the calculated (*Stevenson, 1994*). If the distribution network used single-phase, double-phase, three-phase lines with unbalanced loads and without transposition, the phase currents are dramatically unbalanced and the phased-reactance is unused for debating proposes. For this reason, determination of self-impedance, inducted-impedance, besides distinguish to the ground consequences in a case of ground return line and the unbalanced currents line, is the significant task (*W. H. Kersting, 2002*). In 1926, Carson's paper presented equations to analyze the impedances of lines with unbalanced loads, where in these equations, the ground designated as the return path of lines current, by concerning self and mutual impedances (*W. Dommel, 1986*). Using the neutral line beside other phases is one of the distribution network properties, which may distinguished it in comparison with other voltage levels where changes the impedances of lines. Therefore, in order to compute impedances more accurate, this problem should be mentioned.

By considering special properties of distribution network, it can be seen that variety items may effect on calculation of series impedances in distribution lines. In recent years, in order to study about the system constructions, such as load allocation and short-circuiting computations, notice to above problem is more highlighted (*Rade M. Ciric, 2004; J. Teng, 2005*).

In this paper, first, the distribution networks impedances of lines are calculated by using the circuit voltage and current laws and with the aid of the desired model of the distribution networks impedances of lines, the Carson equations is proved in the way of a novel and simple method. Then, regarding to the presented equations and by using the MTLAB software, a program is expressed to calculate the accurate network impedances where, the correctness of this program, is studied by ATP software. Finally, concerning the standards of Iran, such as line type, the frequency and common formations in distribution networks, the effective items on impedances of lines value and the results are evaluated.

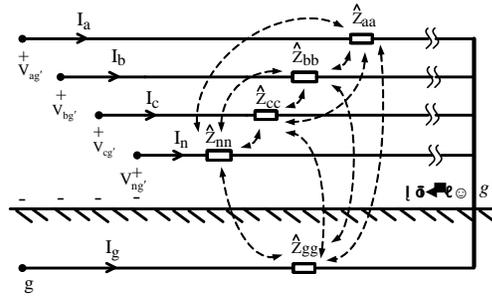
#### ***The Distribution Impedances Of Lines Of Lines Analyzing And Calculating:***

Today, for determination of distributed impedances of lines, using Carson equations are widely applicable. In this section, by designate an impedance model of four-wire (with neutral conductor), the Carson equation is proved

---

**Corresponding Author:** R. Ebrahimi, Bushehr Branch, Islamic Azad University, Bushehr, Iran.  
E-mail: ebrahimi.reza61@gmail.com

easily in the beginning. Figure-1 demonstrates a four-wire with self-impedances, inducted-impedances and the ground.



**Fig. 1:** Impedance model of four-wire.

A voltage supply in one side of the lines, and short-circuiting them in the other side is assumed, so following equation due to the voltage law is presented:

$$\begin{bmatrix} V_{ag'} \\ V_{bg'} \\ V_{cg'} \\ V_{ng'} \\ V_{gg'} \end{bmatrix} = \begin{bmatrix} \widehat{Z}_{aa} & \widehat{Z}_{ab} & \widehat{Z}_{ac} & \widehat{Z}_{an} & \widehat{Z}_{ag} \\ \widehat{Z}_{ba} & \widehat{Z}_{bb} & \widehat{Z}_{bc} & \widehat{Z}_{bn} & \widehat{Z}_{bg} \\ \widehat{Z}_{ca} & \widehat{Z}_{cb} & \widehat{Z}_{cc} & \widehat{Z}_{cn} & \widehat{Z}_{cg} \\ \widehat{Z}_{na} & \widehat{Z}_{nb} & \widehat{Z}_{nc} & \widehat{Z}_{nn} & \widehat{Z}_{ng} \\ \widehat{Z}_{ga} & \widehat{Z}_{gb} & \widehat{Z}_{gc} & \widehat{Z}_{gn} & \widehat{Z}_{gg} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \\ I_g \end{bmatrix} \quad (1)$$

Extending equation (1) for phase a and the ground may cause

$$V_{ag} = I_a \widehat{Z}_{aa} + I_b \widehat{Z}_{ab} + I_c \widehat{Z}_{ac} + I_n \widehat{Z}_{an} + I_g \widehat{Z}_{ag} - (I_g \widehat{Z}_{gg} + I_a \widehat{Z}_{ga} + I_b \widehat{Z}_{gb} + I_c \widehat{Z}_{gc} + I_n \widehat{Z}_{gn}) \quad (2)$$

by arranging the equation (2), another one is shown as:

$$V_{ag} = I_a (\widehat{Z}_{aa} - \widehat{Z}_{ga}) + I_b (\widehat{Z}_{ab} - \widehat{Z}_{gb}) + I_c (\widehat{Z}_{ac} - \widehat{Z}_{gc}) + I_n (\widehat{Z}_{an} - \widehat{Z}_{gn}) + I_g (\widehat{Z}_{ag} - \widehat{Z}_{gg}) \quad (3)$$

Also, considering the current law in  $g'$  point:

$$\begin{aligned} I_a + I_b + I_c + I_n + I_g &= 0 \\ I_g &= -I_a - I_b - I_c - I_n \end{aligned} \quad (4)$$

And then, substitution equation (4) into equation (3), displays as:

$$\begin{aligned} V_{ag} &= (\widehat{Z}_{aa} + \widehat{Z}_{gg} - \widehat{Z}_{ag} - \widehat{Z}_{ga}) \cdot I_a + (\widehat{Z}_{ab} + \widehat{Z}_{gg} - \widehat{Z}_{ag} - \widehat{Z}_{gb}) \cdot I_b + (\widehat{Z}_{ac} + \widehat{Z}_{gg} - \widehat{Z}_{ag} - \widehat{Z}_{gc}) \cdot I_c + \\ &(\widehat{Z}_{an} + \widehat{Z}_{gg} - \widehat{Z}_{ag} - \widehat{Z}_{gn}) \cdot I_n \end{aligned} \quad (5)$$

Arranging above equation and regarding to the  $\bar{Z}$  impedance definition which is the combination of initial impedances, the equation (6) can be written as:

$$V_{ag} = \bar{Z}_{aa} \cdot I_a + \bar{Z}_{ab} \cdot I_b + \bar{Z}_{ac} \cdot I_c + \bar{Z}_{an} \cdot I_n \quad (6)$$

In overall, Concentrating both (5) and (6) equations, for self and mutual impedances:

$$\begin{aligned} \bar{Z}_{xx} &= \widehat{Z}_{xx} + \widehat{Z}_{gg} - 2\widehat{Z}_{xg} \\ \bar{Z}_{xy} &= \widehat{Z}_{xy} + \widehat{Z}_{gg} - \widehat{Z}_{xg} - \widehat{Z}_{gy} \end{aligned} \quad (7)$$

$$x = y = a, b, c, n; \quad \forall x \neq y$$

At the right side of the (7) equation,  $\widehat{Z}_{gg} - \widehat{Z}_{xg} - \widehat{Z}_{gy}$  and  $\widehat{Z}_{gg} - 2\widehat{Z}_{xg}$  illustrates the ground efficiency on initial impedances of lines, named by correction part. Regarding to reference (Rade M. Ciric, 2004), the adjustment part of each phase (for instance phase-a) illustrates as following equation:

$$\begin{aligned} \widehat{Z}_{gg} &= \pi^2 \times 10^{-4} \cdot f - j0.0386.8\pi \times 10^{-4} \cdot f + j4\pi \times 10^{-4} \cdot f \cdot Ln \frac{2}{5.6198 \times 10^{-3}} \\ \widehat{Z}_{ag} &= j2\pi f \times 10^{-4} Ln \frac{h_a}{\sqrt{\rho/f}} \end{aligned} \quad (8)$$

Above equation contains:

$f$ : power system frequency

$h_a$ : The height of wire a, from ground's surface in terms of meter

$\rho$ : The ground particular resistance in terms of ohm-meter

In this manner, computation of phase-a-self -impedance and the phase-a-mutual-impedance with other phases, without distinguishing the ground efficiency (for the initial mode) in the distribution network overhead lines can be debated as [Rade M. Ciric, 2004]:

$$\bar{Z}_{aa} = r_a + j4\pi \times 10^{-4} f \cdot \text{Ln} \left( \frac{2h_a}{GMR_a} \right) \Omega/km \tag{9}$$

$$\bar{Z}_{ab} = j4\pi \cdot 10^{-4} f \cdot \text{Ln} \left( \frac{\sqrt{d_{ab}^2 + (h_a + h_b)^2}}{\sqrt{d_{ab}^2 + (h_a - h_b)^2}} \right) \Omega/km$$

**The network Lines Where:**

$r_a$ : The resistance of phase-a wire in a unit of length ( $\Omega / km$ )

$h_b, h_a$ : The height of wires a and b in terms of meter

$d_{ab}$ : The horizontal distance between a and b wire in terms of meter

Nevertheless, the adjusted mutual-impedance and self-impedance (considering the ground efficiency) for phase-a can be extracted, by substituting equation (8) and (9) into (7) one and simplified as equation 10.

$$\bar{Z}_{ii} = r_i + 9.8696 \times 10^{-4} \cdot f + j1.2566 \times 10^{-3} \cdot f \cdot \left( \ln \frac{1}{GMR_i} + 6.4905 + \frac{1}{2} \ln \frac{\rho}{f} \right), (\Omega / km) \tag{10}$$

$$\bar{Z}_{ij} = 9.8696 \times 10^{-4} \cdot f + j1.2566 \times 10^{-3} \cdot f \cdot \left( \ln \frac{1}{D_{ij}} + 6.4905 + \frac{1}{2} \ln \frac{\rho}{f} \right), (\Omega / km)$$

Where,  $GMR_i$  and  $D_{ij}$  are the geometrical radius average of conductor and the distance between both conductors in terms of meter, respectively. The correctness of this equation, were studied in reference (W. H. Kersting, 2002).By using equation (10) for four-wired line shown in figure-1, the adjusted impedance matrix can be demonstrated as bellow:

$$[\bar{Z}_{line}] = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac} & \bar{Z}_{an} \\ \bar{Z}_{ba} & \bar{Z}_{bb} & \bar{Z}_{bc} & \bar{Z}_{bn} \\ \bar{Z}_{ca} & \bar{Z}_{cb} & \bar{Z}_{cc} & \bar{Z}_{cn} \\ \bar{Z}_{na} & \bar{Z}_{nb} & \bar{Z}_{nc} & \bar{Z}_{nn} \end{bmatrix} \tag{11}$$

Generally, a 3\*3 phased matrix is used, in order to review the distribution network parts, such as calculation of the load or determination of the short-circuiting current. The Kron reduction method is used to reduce the matrix size extracted from equation (11). Then, the phased-lines matrix can be written as:

$$[Z_{abc}] = [\bar{Z}_{ij}] - [\bar{Z}_{in}] \cdot [\bar{Z}_{nn}]^{-1} \cdot [\bar{Z}_{nj}] \tag{12}$$

Where,  $[Z_{abc}]$  is the reduced phased matrix and  $[\bar{Z}_{xx}]$  matrices are used as sub-matrices of equation (11) matrices.

In addition, if the lines formation were in the shape of single-phase, double-phase and three-phase without the neutral conductor, then non-attendances of each phases in the mentioned matrix in equation (11), can change its column and row to zero.

**RESULTS AND DISCUSSION**

In this paper, considering presented equations and with the MATLAB software, a program is expressed to calculate the accurate network impedances.

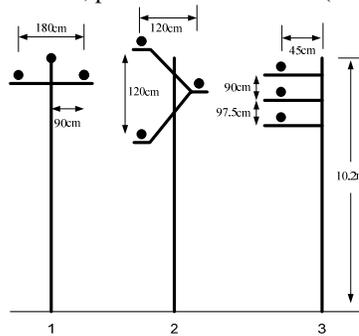
Conductors and conventional formations corresponding the power ministry standards of Iran are used in medium voltages lines, due to computation of the impedance and analyzing all the effective items on it (Overhead line standard).

Table-1 is displayed the technical properties of conventional conductors using in medium voltages level distribution networks.

**Table 1:** Technical properties of studied conductors

| Conductor name | DC resistivity | Outside diameter (mm) | Rea (mm <sup>2</sup> ) | MR (mm) |
|----------------|----------------|-----------------------|------------------------|---------|
| Fox            | 0.7            | 8.37                  | 2.77                   | .37     |
|                | 822            |                       |                        |         |
| Mink           | 0.4            | 10.98                 | 3.65                   | 0.98    |
|                | 545            |                       |                        |         |
| Hyna           | 0.2            | 14.57                 | 26.43                  | 4.57    |
|                | 712            |                       |                        |         |

In addition, figure 2 defines variety types of formations using in this voltage level (20 kilovolt). The correctness of mentioned program is proved, by ATP software, presented in reference (W. H. Kersting, 2002).



**Fig. 2:** Variety types of conductors formations, 1- 2 meter cross arm shape (horizontal), 2- Jenaghi shape, 3- Flaggy shape

The result information of ATP software for a kilometer of three-phase line with Fox conductor within horizontal linear formation in contrast with a 336.4 conductor within four-wire IEEE formation mentioned in reference (W. H. Kersting, 2002), is gathered and shown in Table-2. The maximum acceptable tolerance of program results is about 0.19 percentages.

**Table 2:** Study about the correctness of impedance calculation for the ATP program and reference (W. H. Kersting, 2002).

|                        | The fox conductor with horizontal form |           |        | The 336.4 conductor with IEEE 4 wired form |                       |
|------------------------|--|-----------|--------|--|-----------------------|
|                        | The MATLAB program                     | software  | ATP    | The MATLAB program                         | Reference [ Kersting] |
| Three-phase impedances | 0.8315+j0.7874                         | +j0.7889  | 0.8304 | 0.4577+j1.07                               | 0.4576+j1.            |
|                        | 0.8315+j0.7874                         | +j0.7889  | 0.8304 | 0.4667+j1.04                               | 0.4666+j1.            |
|                        | 0.8315+j0.7874                         | +j0.7889  | 0.8304 | 0.4616+j1.06                               | 0.4615+j1.            |
|                        | 0.0493+j0.4531                         | +j0.4531  | 0.0482 | 0.1561+j0.50                               | 0.1560+j0.            |
|                        | 0.0493+j0.4086                         | +j0.4096  | 0.0482 | 0.1536+j0.38                               | 0.1535+j0.            |
|                        | 0.0493+j0.4531                         | +j0.45131 | 0.0482 | 0.1581+j0.42                               | 0.1580+j0.            |

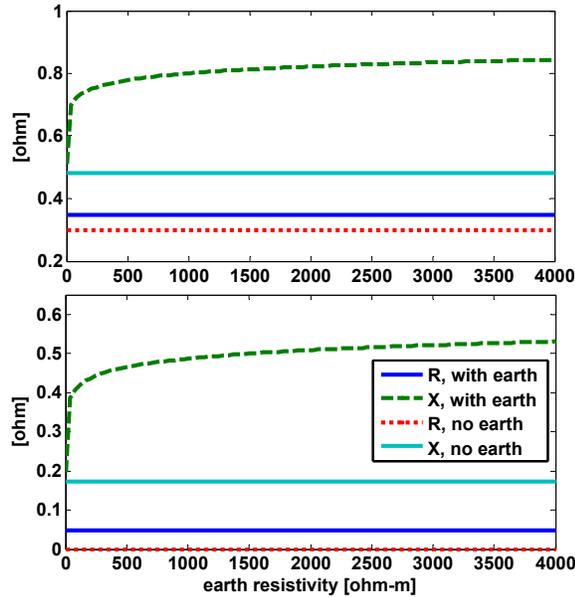
After the conductor type founds, formation used in this paper, correctness of the results, the impedance manner in terms of variations of its effective factor is evaluated. The study results are displayed in part 1 to part 4. It is considerable that in all modes the impedance calculates for phase-a in terms of 1 kilometer, distance to three-phase concerning the Hyna conductor.

In these sections, the 2-meter cross-arm (horizontal) distinguished as the conventional form and the lines recognized as the three wires (without the neutral conductor).

**1- The Earth Return and Its Resistivity Effects:**

Intending the earth return in calculations of impedances of lines, considering the equation (9), maybe caused its value variations, which is directly related to the earth resistivity ( $\rho$ ).

The specific earth resistance depends on the soil quality, and its range is variable from 2 ohm-m in coastal grounds to the 4000 ohm-m in rocky grounds (Safarian, A., 2006). Thus, in order to have the better evaluation, the analysis results in both parts of self-impedance and mutual-impedance are shown gradually and respectively in the top and downward parts of the figure-3. Each of these impedances itself is the combination of the resistance and reactance.

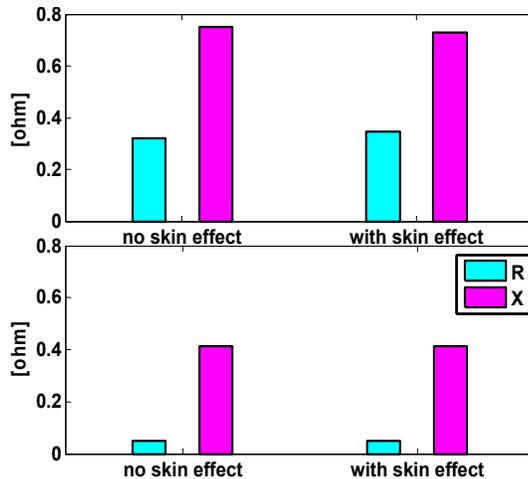


**Fig 3.** The the earth effect and its resistivity variations on the lines impedance.

According to above figure, it can be seen clearly that intending the earth effect, can increase the self-impedance and inducted-impedances. Increasing these values in resistors regarding to equation (10), remains steady in comparison with the growth of self and mutual reactance, which is depend on the earth resistivity. This augmentation for the low resistivity (zero to about 100 ohm-meter) has considerable and dramatic changes, hence for the high resistivity, it has marginal changes.

**2. Considering Skin Effects On Power Frequency:**

Generally, the skin effects causes resistivity increase and inductance decries. **Fig-4** displays the self-impedance (on up-side) and the mutual-impedance (on down-side) changes according to applying skin effects on power frequency of 50 Hz.



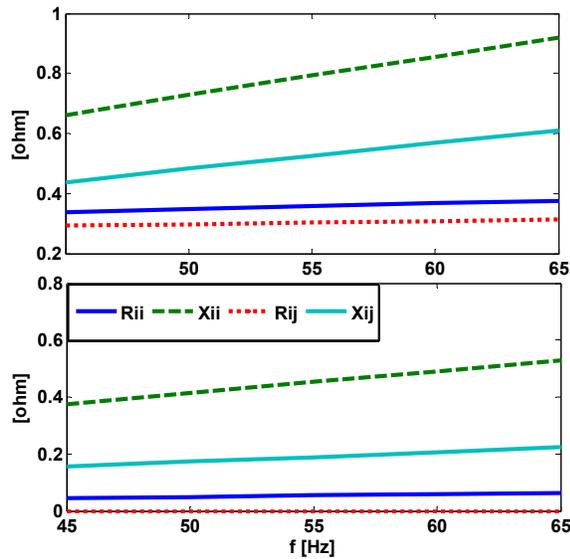
**Fig 4.** Applying the skin effect on impedance rate.

Above figure shows, the skin effect appliance, which is a little affected on self-impedance and then the mutual-impedance remains unchanged.

**3. Frequency Variations Effect:**

Owing to the relation between reactance and frequency, the frequency variation directly effected on the lines reactance. Also, when the earth applied in the impedance calculations ( $\rho \neq 0$ ), considering equation (10), these variation will increased. Figure 5, shows the frequency changes in impedances of lines with distinguishing the earth (up-side) and without (down-side).

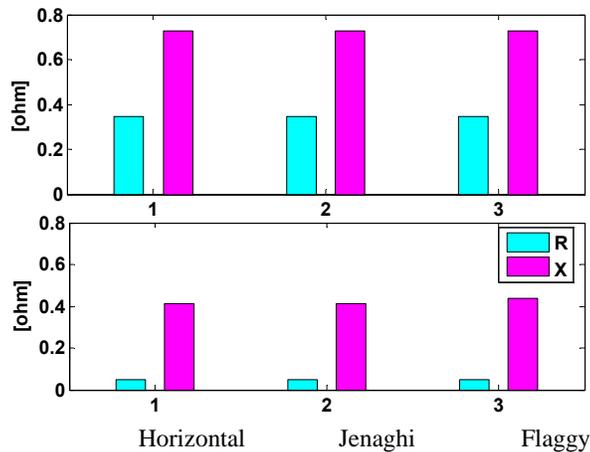
According to figure-5, enhancing frequency only effects on the reactance part, which has the maximum span variations for the self-reactance, distinguishing the earth effect.



**Fig 5.** Frequency variation effects on the impedances of linses (with and without applying the earth).

**4. The Lines Form Variation Effect:**

In this section, the impedances of lines corresponding formations on figure-2, were calculated and displays as figure 6.



**Fig 6.** Impedances of different formations shown in figure-2.

Corresponding different formations, measurements of the line self-impedance has non-considerable changes. The mutual-impedance value has the worthless increase for flaggy form in comparison with other forms.

**Conclusion:**

In this paper, a program is used in order to calculate the overhead distribution lines, by using Carson equations.

Then regarding to the program capabilities, items that effects on the impedances of lines, consists of considering the earth for return current wire, applying skin effect, the frequency changes and the neutral wire, were surveyed and the results were evaluated.

According to the results, applying the earth into the impedances of lines determinations may cause the noticeable reactance, resistance and impedance growth. Nevertheless, the frequency variations affect dramatically on the impedances of lines, according to the direct relation that the line reactance and the network frequency may have. Thus, notice to the frequency changes is one of the most significant items, in these computations.

In distribution networks, changing their formations were non-noticeable, because of the little lines distances and it can be worthless in computations. Also, according to the low nominal frequency, the skin effect is worthless.

#### **REFERENCES**

- Dommel, W., 1986. EMTP Theory Book. BPA.
- Kersting, W.H., 2002. Distribution system modeling and analysis. CRC Press, New York. Overhead line standard- line configuration standard of Iran.
- Rade M. Ciric, Luis F.Ochoa, Antonio Padilha, 2004. Power flow in distribution networks with earth return Elsevier, *Electrical Power and Energy Systems*, 26: 373-380.
- Stevenson, D. William, Grainger, J. John, 1994. Power system analysis. McGraw-Hill.
- Safarian, A. Koohsari, Sh., V. Majedi Asl, 2006. Evaluation of the effective parameters on zero impedance of underground cables in an industrial unit. 21th international power electrical conference, 98-F-PDS-745.
- Teng, J., 2005. Systematic short-circuit-analysis method for unbalanced distribution systems. *IEE Proc. Gener. Transm. Distrib.*, 152(4): 549-555.