Stochastic Deflection Study of Concrete Plates under Uniform Loading

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Abstract: In this paper, the Fourier method and the Navier's solution are used to obtain solutions to the deflection stochastic response of concrete plates with uncertain parameters. Up to now, the geometric and material parameters were main uncertain parameters for the analysis of response variability for uniform loaded plates. However, since the load is another parameter influencing the behavior of concrete plates, the independent evaluation of response variability due to the randomness in loading is also given. Furthermore, if the concern is studying the uncertainty in geometric parameters, the influence of the uncertainty in all geometric parameters, not only thickness, have to be investigated. A numerical application generating random values has been used to obtain stochastic analysis, which has the advantage of the simplicity. Through the results, it becomes possible to deal with all uncertain parameters in concrete plates. Results show that there are qualitatively similarity between stochastic and deterministic responses. Also, dispersion coefficients have been investigated. As it can be seen from results, the deflection is almost vary from (*mean value-0.07*mean value*) to (*mean value+0.07*mean value*). It is also clear that even though there is qualitative likeness in both deterministic and stochastic deflection distribution but, there are differences between the changes intensity in minimum, mean and maximum deflection distribution.

Key words: Stochastic deflection, Plate, Fourier, Navier, Uniform

INTRODUCTION

When engineering systems are taken into account within the framework of numerical tools, the assumption that these systems have deterministic parameters is implicitly made. Thus, the system parameters assume as constant values over the system domain. However, in real plate structures, the properties have several uncertainties.

Due to advances in computational mechanics, numerical and computational methods there are tremendous developments in the modeling of the structural behaviors with uncertainties as unavoidable part of them.

Uncertainty in system parameters and system responses have been evaluated by strong efforts to develop computational methods (Schuëller, 1997). The incorporation of uncertainty in structural analysis has been advocated by Freudenthal and others in sixties (Freudenthal *et al.*, 1996). Transformation or fast probability integration methods have been used to analyze probabilistic structural (Cruse *et al.*, 1988). Many earlier works were based on stochastic finite element methods and perturbation approach, which were applied to plate problems (Hisada and Nakagiri, 1981; Vanmarke *et al.*, 1986; Liu *et al.*, 1986; Lawrence, 1987).

Up to now, many researches about geometrical parameters (Nieuwenhof and Coyette, 2003; Choi and Noh, 1996; Choi and Noh, 2000; Graham and Deodatis, 2001 and Stefanou and Papadrakakis, 2004) and temporal uncertainties (Choi and Noh, 1996; Falsone and Impollonia, 2002) have been performed. However, the uncertainties have been mainly focused on characteristic constants of material, such as elasticity modulus (Choi and Noh, 1993; Graham and Deodatis, 1998; Zhu *et al.*, 2001; Vanmarcke and Grigoriu, 1983; Butcher and Shinozuka, 1988; Deodatis and Shinozuka, 1989; Impollonia and Sofi, 2003; Shinozuka and Deodatis, 1988) and Poisson's ratio (Chun, 2004).

Even though some research works consider these parameters but, this is attributed to the fact that loading is one of the most important parameters which it's uncertainty is not really negligible. Therefore, considering the effect of randomness in loading is also required.

Practically, one of the most crude and simple methodologies in load modeling is the Fourier method, which is also used by Navier's solution. As accepted generally, Navier's solution is useful for solving different

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kinds of loading conditions and analyzing the response of plate structures.

Considerable safety of plate structures such as buildings floors and vessels (Khan *et al.*, 2010; Shariati *et al.*, 2008; Sezar *et al.*, 2010) adds the study of risky. In the present study, along uncertainty in geometrical and material properties, uncertainty in loading conditions has been also considered and analytical findings for stochastic deflection response are presented.

MATERIALS AND METHODS

Bending of plates:

In the plate structures, the plate response can be obtained by solving the Lagrange differential equation1.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D} \tag{1}$$

where, $q(x, y)$ and D denote loading function and flexural rigidity, respectively. Flexural rigidity for a rectangular plate is given by Eq. 2.

$$D = \frac{Et^3}{12(1-v^2)} \tag{2}$$

where, E is the elasticity modulus, t is the plate thickness and v is the Poisson's ratio.

Bending and twisting moments can be also given by Eq.3, 4 and 5.

$$M_x = -D(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}) \tag{3}$$

$$M_y = -D(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}) \tag{4}$$

$$M_{xy} = -D(1-v)(\frac{\partial^2 w}{\partial x \partial y}) \tag{5}$$

In the plate structures, if the attention is finding the plate response using Navier's solution, the loading function $q(x, y)$ and the plate total deflection can be represented in the form of a double trigonometric series using Eq. 6 and 7.

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{6}$$

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{7}$$

in which a and b are the dimensions of the plate along x and y axis, respectively. In addition, q_{mn} and c_{mn} are given by Eq. 8 and 9.

$$q_{mn} = \frac{4}{ab}\int_0^b\int_0^a q(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy \tag{8}$$

$$c_{mn} = \frac{1}{\pi^4 D}\frac{q_{mn}}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \tag{9}$$

In this study, a plate structure under uniform loading has been considered. Figure 1 shows the plate model, which is a plate under uniform loading.

Stochastic Numerical Application:

Variation in geometric parameters, material properties and loading affect the uncertainty in response of plate structures. In order to study the stochastic response of plate structures, the elasticity modulus E, Poisson's ratio v, plate thickness t, plate dimensions and loading condition $q(x, y)$ were modeled as random variables. Each random variable is modeled as Eq. 10.

$$Z = \mu_z \left(1 + \nu_z \alpha_z \right) \tag{10}$$

where, μ_z is the mean value, α_z is a set of random numbers with a zero mean and ν_z is the coefficient of variation for the random variable.

In the stochastic calculation of the response resulted from the variability of the plate structure variables, the coefficients of variation, ν_z, were assumed to be equal to 0.025 for the plate dimensions and the load. However, for the reason that the other variables including elasticity modulus, Poisson's ratio and thickness are more uncertain, the coefficients of variation for these variables were assumed to be equal to 0.05.

Even though this numerical stochastic method requires a large number of randomly generated values for the parameters but, the method has the advantage of the simplicity. Moreover, using this method it is possible to study the outputs statistically and also, mean values, standard deviation and other statistical parameters can be easily obtained.

Plates under Uniform Loading:
If the uniform load distribution is given by Eq. 11:

$$q\left(x, y \right) = q_0 \tag{11}$$

we can proceed using Navier's solution, and we should obtain the Eq. 12 for deflection.

$$W\left(x, y \right) = \frac{16 q_0}{\pi^6 D} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{\sin\left(\frac{m\pi x}{a} \right) \sin\left(\frac{n\pi y}{b} \right)}{mn \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2} \tag{12}$$

substituting Eq. 12 in Eq. 3, 4 and 5, these follow that:

$$M_x = \frac{16 q_0}{\pi^4} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{\left(\frac{m}{a} \right)^2 + \nu \left(\frac{n}{b} \right)^2}{mn \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{13}$$

$$M_y = \frac{16 q_0}{\pi^4} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{\nu \left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}{mn \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{14}$$

$$M_{xy} = -\frac{16\left(1-\nu\right)}{\pi^4 \left(ab\right)} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{\left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right]^2} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{15}$$

Table 1 shows the value set of the variables for the response calculation of the plate structure.

Table 1: Set of parameters

Parameters (random variables)	Mean values
Elasticity modulus, E *(GPa)*	15, 20, 25
Poisson's ratio, ν	0.15, 0.17
Plate width, a *(m)*	2, 3
Plate length, b *(m)*	4, 6
Plate thickness, t *(cm)*	10, 15
Load, q *(kN/m²)*	10, 15, 20

RESULTS AND DISCUSION

Some of the stochastic responses for the model are taken in Fig. 2 to Fig. 4. Figures show the mean value and dispersions, mean value plus/minus deviation, of total deflection when *E*, *t*, *q* and *υ* take 25 *GPa*, 10 *cm*, 20 *kN/m²* and 0.15, respectively. Fig.2 indicates that there are similarity between stochastic and deterministic deflection, qualitatively. For example, the plate deflection reaches the maximum value at the center point of the plate and also, the total deflection is symmetrically distributed.

As it can be seen from Fig. 2, the deflection is almost vary from (*mean value-0.07*mean value*) to (*mean value+0.07*mean value*).
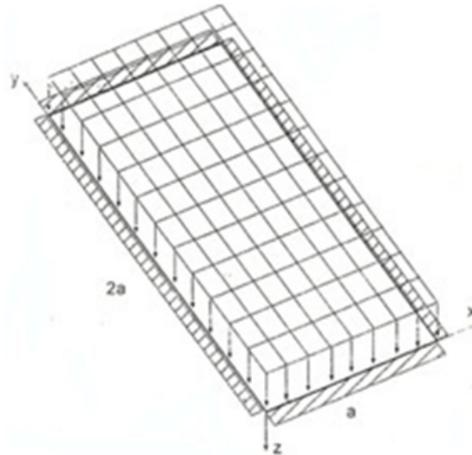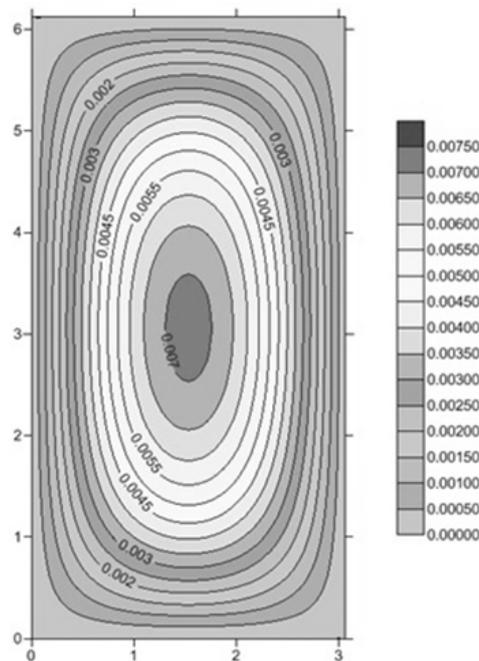

Fig. 1: Model of plate structure

Fig. 2: Minimum dispersion of deflection, (E=25 GPa, t=10 cm, q=20 kN/m², υ=0.15)

It is clear from Fig. 2 that even though there is qualitative likeness in both deterministic and stochastic deflection distribution but, there are differences between the changes intensity in minimum, mean and maximum deflection distribution. For example, the changes intensity for mean value in x direction is 0.0053 for each meter. While, it is 0.0057 for maximum deflection distribution.

In general, the changes intensities take the values of 0.005 and 0.0025 for minimum deflection distribution along x and y direction, respectively. Also, they are 0.0053 and 0.0027 for mean deflection distribution. While, they are 0.0057 and 0.0028 for maximum deflection distribution.
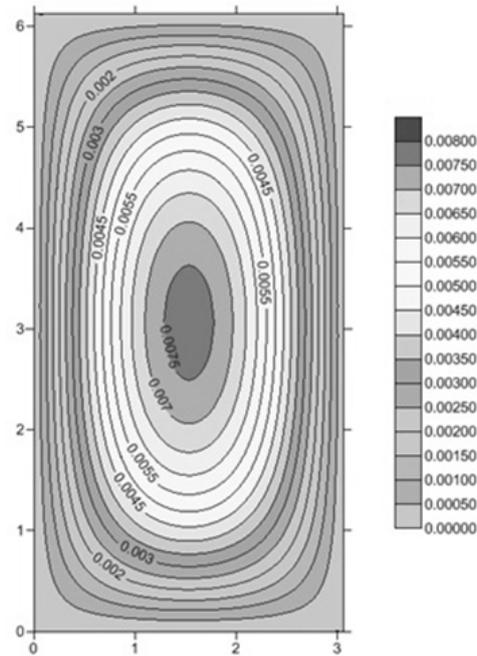
Fig. 3: Mean value of deflection, (E=25 GPa, t=10 cm, q=20 kN/m^2, υ=0.15)
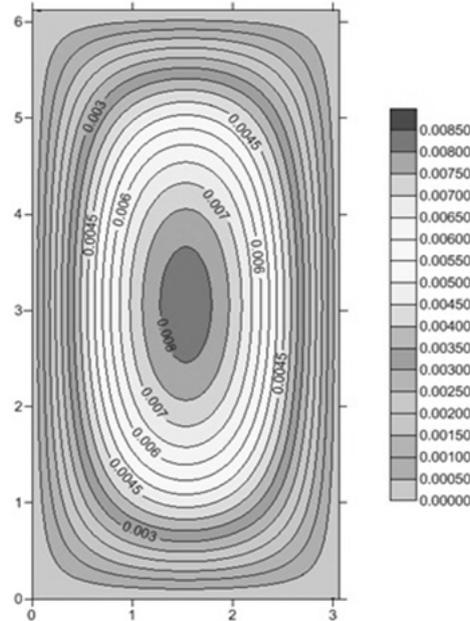

Fig. 4: Maximum dispersion of deflection, (E=25 GPa, t=10 cm, q=20 kN/m^2, υ=0.15)

According to Figures, the stochastic responses are qualitatively similar to the deterministic responses. However, the changes intensities are different for results. The intensity of the changes are taken in Table 2.

According to Table 2 and referring to the proportion to the mean intensity, the dispersion coefficients for deflection is almost 0.07. That means the dispersion of these two responses can be vary from (*mean-0.07*mean*) to (*mean+0.07*mean*).

Conclusions

The Navier's solution is appropriate for computing the structural response when the Fourier loading series is available. In this paper, the Fourier method and the Navier's solutions are used to obtain solutions to the stochastic deflection response of concrete plates with uncertain parameters.

Table 2: Changes intensity

Item	Intensity direction	Changes intensity			Proportion to the mean intensity		
		Min	Mean	Max	Min	Mean	Max
W	x	0.0050	0.0053	0.00567	0.943	1	1.07
W	y	0.00250	0.00267	0.00283	0.936	1	1.06

Since the load is a parameter influencing the behavior of concrete plates, along the geometrical and material characteristics, the independent evaluation of response variability due to the randomness in loading is also given.

A numerical application generating random values has been used to obtain stochastic analysis which has the advantage of the simplicity.

Through the results, there are qualitatively similarity between stochastic and deterministic responses. In addition, changes intensity for different sets of plate properties were obtained and presented. The results show that the intensity of changes are different for mean values and dispersions for different responses.

As it can be seen from results, the deflection is almost vary from (*mean value-0.07*mean value*) to (*mean value+0.07*mean value*). It is also clear that even though there is qualitative likeness in both deterministic and stochastic deflection distribution but, there are differences between the changes intensity in minimum, mean and maximum deflection distribution.

In general, the results show that for plate structures the responses can be extremely different due to uncertainty in parameters.

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