Australian Journal of Basic and Applied Sciences, 5(8): 140-146, 2011
ISSN 1991-8178

Bending Moment Stochastic Study of Concrete Plates under Uniform Loading

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Abstract: In this paper, the Fourier method and the Navier's solution are used to obtain solutions to the bending moments stochastic response of concrete plates with uncertain parameters. Up to now, the geometric and material parameters were main uncertain parameters for the analysis of response variability for uniform loaded plates. However, since the load is another parameter influencing the behavior of concrete plates, the independent evaluation of response variability due to the randomness in loading is also given. Furthermore, if the concern is studying the uncertainty in geometric parameters, the influence of the uncertainty in all geometric parameters, not only thickness, have to be investigated. A numerical application generating random values has been used to obtain stochastic analysis, which has the advantage of the simplicity. Through the results, it becomes possible to deal with all uncertain parameters in concrete plates. Results show that there are qualitatively similarity between stochastic and deterministic responses. Also, dispersion coefficients have been investigated. As it can be seen from results, the bending moments is almost vary from (*mean value-0.11*mean value*) to (*mean value+0.11*mean value*). It is also clear that even though there is qualitative likeness in both deterministic and stochastic bending moments distribution but, there are differences between the changes intensity in minimum, mean and maximum bending moments distribution.

Key words: Stochastic bending moments, Plate, Fourier, Navier, Uniform

INTRODUCTION

When engineering systems are taken into account within the framework of numerical tools, the assumption that these systems have deterministic parameters is implicitly made. Thus, the system parameters assume as constant values over the system domain. However, in real plate structures, the properties have several uncertainties.

Due to advances in computational mechanics, numerical and computational methods there are tremendous developments in the modeling of the structural behaviors with uncertainties as unavoidable part of them.

Uncertainty in system parameters and system responses have been evaluated by strong efforts to develop computational methods (Schuëller, 1997). The incorporation of uncertainty in structural analysis has been advocated by Freudenthal and others in sixties (Freudenthal *et al.*, 1996). Transformation or fast probability integration methods have been used to analyze probabilistic structural (Cruse *et al.*, 1988). Many earlier works were based on stochastic finite element methods and perturbation approach, which were applied to plate problems (Hisada and Nakagiri, 1981; Vanmarke *et al*., 1986; Liu *et al*., 1986; Lawrence, 1987).

Up to now, many researches about geometrical parameters (Nieuwenhof and Coyette, 2003; Choi and Noh, 1996; Choi and Noh, 2000; Graham and Deodatis, 2001 and Stefanou and Papadrakakis, 2004) and temporal uncertainties (Choi and Noh, 1996; Falsone and Impollonia, 2002) have been performed. However, the uncertainties have been mainly focused on characteristic constants of material, such as elasticity modulus (Choi and Noh, 1993; Graham and Deodatis, 1998; Zhu *et al*., 2001; Vanmarcke and Grigoriu, 1983; Butcher and Shinozuka, 1988; Deodatis and Shinozuka, 1989; Impollonia and Sofi, 2003; Shinozuka and Deodatis, 1988) and Poisson's ratio (Chun, 2004).

Even though some research works consider these parameters but, this is attributed to the fact that loading is one of the most important parameters which it's uncertainty is not really negligible. Therefore, considering the effect of randomness in loading is also required.

Practically, one of the most crude and simple methodologies in load modeling is the Fourier method, which is also used by Navier's solution. As accepted generally, Navier's solution is useful for solving different kinds of loading conditions and analyzing the response of plate structures.

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Considerable safety of plate structures such as buildings floors and vessels (Khan *et al*., 2010; Shariati *et al*., 2008; Sezar *et al*., 2010) adds the study of risky. In the present study, along uncertainty in geometrical and material properties, uncertainty in loading conditions has been also considered and analytical findings for stochastic bending moments response are presented.

MATERIALS AND METHODS

Bending of Plates:

In the plate structures, the plate response can be obtained by solving the Lagrange differential equation1.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D} \tag{1}$$

where, $q(x, y)$ and D denote loading function and flexural rigidity, respectively. Flexural rigidity for a rectangular plate is given by Eq. 2.

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{2}$$

where, E is the elasticity modulus, t is the plate thickness and υ is the Poisson's ratio.

Bending and twisting moments can be also given by Eq.3, 4 and 5.

$$M_x = -D(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}) \tag{3}$$

$$M_y = -D(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}) \tag{4}$$

$$M_{xy} = -D(1-\nu)(\frac{\partial^2 w}{\partial x \partial y}) \tag{5}$$

In the plate structures, if the attention is finding the plate response using Navier's solution, the loading function $q(x, y)$ and the plate total deflection can be represented in the form of a double trigonometric series using Eq. 6 and 7.

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{6}$$

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{7}$$

in which a and b are the dimensions of the plate along x and y axis, respectively. In addition, q_{mn} and c_{mn} are given by Eq. 8 and 9.

$$q_{mn} = \frac{4}{ab}\int_0^b\int_0^a q(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dxdy \tag{7}$$

$$c_{mn} = \frac{1}{\pi^4 D}\frac{q_{mn}}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \tag{9}$$

In this study, a plate structure under uniform loading has been considered. Figure 1 shows the plate model, which is a plate under uniform loading.

Stochastic Numerical Application:

Variation in geometric parameters, material properties and loading affect the uncertainty in response of plate structures. In order to study the stochastic response of plate structures, the elasticity modulus E, Poisson's ratio υ, plate thickness t, plate dimensions and loading condition $q(x, y)$ were modeled as random variables. Each random variable is modeled as Eq. 10.

$$Z = \mu_z \left(1 + \nu_z \alpha_z\right) \tag{10}$$

where, μ_z is the mean value, α_z is a set of random numbers with a zero mean and υ_z is the coefficient of variation for the random variable.

In the stochastic calculation of the response resulted from the variability of the plate structure variables, the coefficients of variation, υ_z, were assumed to be equal to 0.025 for the plate dimensions and the load. However, for the reason that the other variables including elasticity modulus, Poisson's ratio and thickness are more uncertain, the coefficients of variation for these variables were assumed to be equal to 0.05.

Even though this numerical stochastic method requires a large number of randomly generated values for the parameters but, the method has the advantage of the simplicity. Moreover, using this method it is possible to study the outputs statistically and also, mean values, standard deviation and other statistical parameters can be easily obtained.

Plates under Uniform Loading:

If the uniform load distribution is given by Eq. 11:

$$q\left(x, y\right) = q_0 \tag{11}$$

we can proceed using Navier's solution, and we should obtain the Eq. 12 for deflection.

$$W\left(x, y\right) = \frac{16 q_0}{\pi^6 D} \sum_{m=1,3,...}^{\infty} \sum_{n=1,3,...}^{\infty} \frac{\sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)}{mn\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \tag{12}$$

substituting Eq. 12 in Eq. 3, 4 and 5, these follow that:

$$M_x = \frac{16 q_0}{\pi^4} \sum_{m=1,3,...}^{\infty} \sum_{n=1,3,...}^{\infty} \frac{\left(\frac{m}{a}\right)^2 + \nu\left(\frac{n}{b}\right)^2}{mn\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{13}$$

$$M_y = \frac{16 q_0}{\pi^4} \sum_{m=1,3,...}^{\infty} \sum_{n=1,3,...}^{\infty} \frac{\nu\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}{mn\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{14}$$

$$M_{xy} = -\frac{16\left(1-\nu\right)}{\pi^4 \left(ab\right)} \sum_{m=1,3,...}^{\infty} \sum_{n=1,3,...}^{\infty} \frac{1}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{15}$$

Table 1 shows the value set of the variables for the response calculation of the plate structure.

Table 1: Set of parameters

Parameters (random variables)	Mean values
Elasticity modulus, *E (GPa)*	15, 20, 25
Poisson's ratio, υ	0.15, 0.17
Plate width, *a (m)*	2, 3
Plate length, *b (m)*	4, 6
Plate thickness, *t (cm)*	10, 15
Load, *q* (kN/m^2)	10, 15, 20

RESULTS AND DISCUSION

Some of the stochastic responses for the model are taken in Fig. 2 to Fig. 4. Figures show the mean value and dispersions, mean value plus/minus deviation, of total bending moments when *E*, *t*, *q* and υ take 20 *GPa*, 10 *cm*, 20 *kN/m*^{2} and 0.15, respectively. Fig.2 indicates that there are similarity between stochastic and deterministic bending moments, qualitatively. For example, the plate bending moments reaches the maximum value at the center point of the plate and also, the total deflection is symmetrically distributed.

As it can be seen from Fig. 2, the bending moment is almost vary from (*mean value-0.11*mean value*) to (*mean value+0.11*mean value*).

It is clear from Fig. 2 that even though there is qualitative likeness in both deterministic and stochastic bending moments distribution but, there are differences between the changes intensity in minimum, mean and maximum bending moments distribution. For example, the changes intensity for mean value in x direction is 12000 for each meter. While, it is 13333 for maximum bending moments distribution.

In general, the changes intensities take the values of 11333 and 5667 for minimum bending moments distribution along x and y direction, respectively. Also, they are 12000 and 6000 for mean bending moments distribution. While, they are 13333 and 6667 for maximum bending moments distribution.

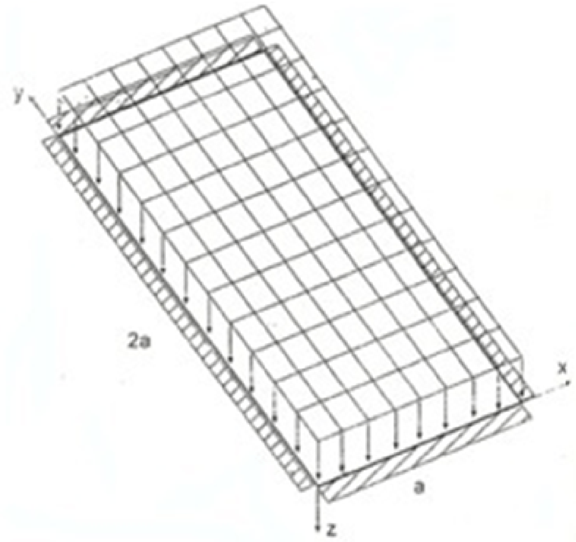

Fig. 1: Model of plate structure

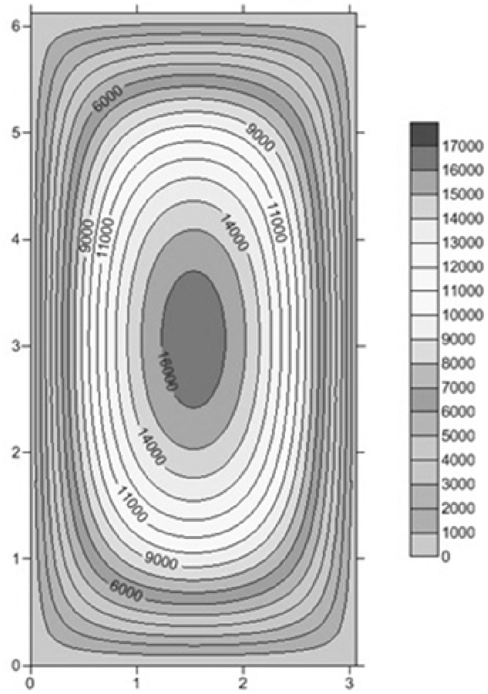

Fig. 2: Minimum dispersion of the bending moment along x axis, (E=20 GPa, t=10 cm, q=20 kN/m^2, υ=0.15)

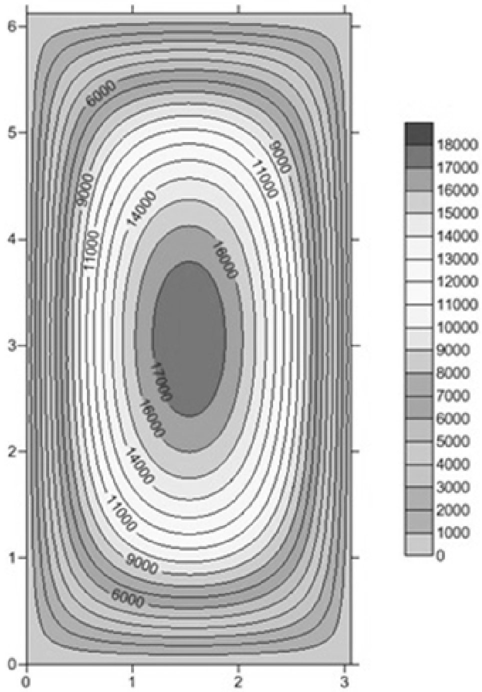

Fig. 3: Means value of the bending moment along x axis, (E=20 GPa, t=10 cm, q=20 kN/m^2, υ=0.15)

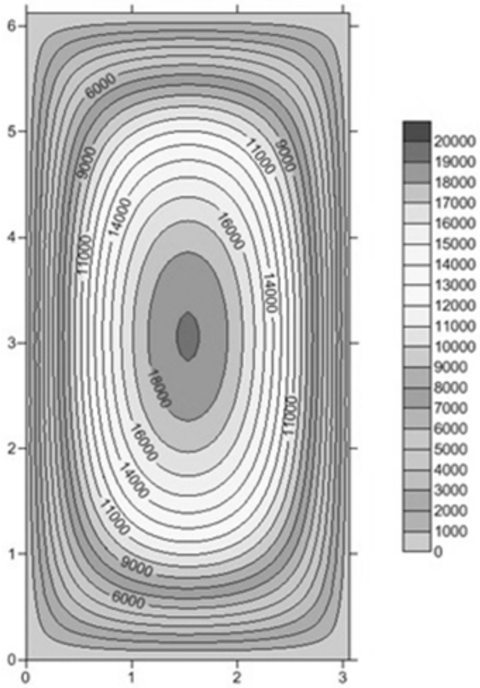

Fig. 4: Maximum dispersion of the bending moment along x axis, (E=20 GPa, t=10 cm, q=20 kN/m^2, υ=0.15)

According to Figures, the stochastic responses are qualitatively similar to the deterministic responses. However, the changes intensities are different for results. The intensity of the changes are taken in Table 2.

According to Table 2 and referring to the proportion to the mean intensity, the dispersion coefficients for bending moments is almost 0.07 to 0.11. That means the dispersion of these two responses can be vary from (*mean-0.07 to 0.11*mean*) to (*mean+0.07 to 0.11*mean*).

Table 2: Changes intensity

Item	Intensity direction	Changes intensity			Proportion to the mean intensity		
		Min	Mean	Max	Min	Mean	Max
M_x	x	11333	12000	13333	0.944	1	1.11
M_x	y	5667	6000	6667	0.944	1	1.11
M_y	x	867	933	1000	0.930	1	1.07
M_y	y	433	467	500	0.927	1	1.071

Conclusions:

The Navier's solution is appropriate for computing the structural response when the Fourier loading series is available. In this paper, the Fourier method and the Navier's solutions are used to obtain solutions to the stochastic bending moments response of concrete plates with uncertain parameters.

Since the load is a parameter influencing the behavior of concrete plates, along the geometrical and material characteristics, the independent evaluation of response variability due to the randomness in loading is also given.

A numerical application generating random values has been used to obtain stochastic analysis which has the advantage of the simplicity.

Through the results, there are qualitatively similarity between stochastic and deterministic responses. In addition, changes intensity for different sets of plate properties were obtained and presented. The results show that the intensity of changes are different for mean values and dispersions for different responses.

As it can be seen from results, the bending moments is almost vary from (*mean value-0.07 to 0.11*mean value*) to (*mean value+0.07 to 0.11*mean value*). It is also clear that even though there is qualitative likeness in both deterministic and stochastic bending moments distribution but, there are differences between the changes intensity in minimum, mean and maximum bending moments distribution.

In general, the results show that for plate structures the responses can be extremely different due to uncertainty in parameters.

REFERENCES

Butcher, C.G. and M. Shinozuka, 1988. Structural response variability II. Journal of Engineering Mechanics, 114(12): 2035-54.

Choi, C.K. and H.C. Noh, 1993. Stochastic finite element analysis by using quadrilateral elements. KSCE Journal of Civil Engineering, 13(5): 29-37.

Choi, C.K. and H.C. Noh, 1996. Stochastic finite element analysis with direct integration method. 4th International Conference on Civil Engineering, pp: 522-531.

Choi, C.K. and H.C. Noh, 1996. Stochastic finite element analysis of plate structures by weighted integral method. Structural Engineering & Mechanics, 4(6): 703-715.

Choi, C.K. and H.C. Noh, 2000. Weighted integral SFEM including higher order terms. Journal of Engineering Mechanics, 126(8): 59-866.

Chun. N.H., 2004. A formulation for stochastic finite element analysis of plate structures with uncertain Poisson’s ratio. Computer Methods in Applied Mechanics and Engineering, 193(45): 4857-4873.

Cruse, T.A., O.H. Burnside, Y.T. Wu, E.Z. Polch and J.B. Dias, 1988. Probabilistic structural analysis methods for select space propulsion system structural components. Computers & Structures, 29(5): 891-901.

Deodatis. G. and M. Shinozuka, 1989. Bounds on response variability of stochastic systems. Journal of Engineering Mechanics, 115(11): 2543-2563.

Falsone, G. and N. Impollonia, 2002. A new approach for the stochastic analysis of finite element modeled structures with uncertain parameters. Computer Methods in Applied Mechanics and Engineering, 191: 5067-5085.

Freudenthal, A.M., J.M. Garrelts and M. Shinozuka, 1966. The analysis of structural safety. Journal of Structural Division, 92: 267-325.

Graham, L. and G. Deodatis, 1998. Variability response functions for stochastic plate bending problems. Structural Safety, 20: 167-88.

Graham, L.L. and G. Deodatis, 2001. Response and eigenvalue analysis of stochastic finite element systems with multiple correlated material and geometric properties. Probabilistic Engineering Mechanics, 16: 11-29.

Hisada, T. and S. Nakagiri, 1981. Stochastic finite element method developed for structural safety and reliability. In proceedings of the 3rd international conference on structural safety and reliability, pp: 395-408.

Impollonia, N. and A. Sofi, 2003. A response surface approach for the static analysis of stochastic structures with geometrical nonlinearities. Computer Methods in Applied Mechanics and Engineering, 192: 4109-4129.

Khan, A., S. Islam, S. Iqbal, and M. Elahi, 2010. Guided Waves in a Fluid-loaded Orthotropic Plate. World Applied Sciences Journal, 8(7): 804-808.

Lawrence, M.A., 1987. Random variables in finite element analysis. International Journal for Numerical Methods in Engineering, 24:1849-63.

Liu WK, T. Belytschko and A. Mani, 1986. Random field finite elements. International Journal for Numerical Methods in Engineering, 23: 1831-45.

Nieuwenhof, B.V.D. and J.P. Coyette, 2003. Modal approaches for the stochastic finite element analysis of structures with material and geometric uncertainties. Computer Methods in Applied Mechanics and Engineering, 192: 3705-3729.

Schuëller, G.A., 1997. State of the art report on computational stochastic mechanics. Probabilistic Engineering Mechanics, 12(4): 197-321.

Sezar, F.S., S. Dilmac and A. Guner, 2010. Experimental Study on the Effect of Wall Materials on the Thermal Behavior of Mezzanine Floor. World Applied Sciences Journal, 8(3): 374-381.

Shariati, M., M.H. Keyhani and I. Shalchian, 2008. Effect of Stacking Sequence on the Temperature Distribution in a Composite Multi-ply Laminates Vessel. World Applied Sciences Journal, 3(5): 833-840.

Shinozuka, M. and G. Deodatis, 1988. Response variability of stochastic finite element systems. Journal of Engineering Mechanics, 114(3): 499-519.

Stefanou, G. and M. Papadrakakis, 2004. Stochastic finite element analysis of shells with combined random material and geometric properties. Computer Methods in Applied Mechanics and Engineering, 193(1): 139-160.

Vanmarcke, E. and M. Grigoriu, 1983. Stochastic finite element analysis of simple beams. Journal of Engineering Mechanics, 109(5): 1203-14.

Vanmarke, E., M. Shinozuka, S. Nakagiri, G.I. Schueller and M. Grigoriu, 1986. Random fields and stochastic finite elements. Structural Safety, 3:143-66.

Zhu, W.Q., Y.J. Ren and W.Q. Wu. Stochastic FEM, based on local averages of random vector fields. Journal of Engineering Mechanics, 118(3): 496-511.