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Bending Moment Stochastic Study of Concrete Plates under Sinusoidal Distributed Loading

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Abstract: In this paper, the Fourier method and the Levy's solution are used to obtain solutions to the bending moment stochastic response of concrete plates with uncertain parameters. Up to now, the geometric and material parameters were main uncertain parameters for the analysis of response variability for sinusoidal distributed loaded plates. However, since the load is another parameter influencing the behavior of concrete plates, the independent evaluation of response variability due to the randomness in loading is also given. Furthermore, if the concern is studying the uncertainty in geometric parameters, the influence of the uncertainty in all geometric parameters, not only thickness, have to be investigated. A numerical application generating random values has been used to obtain stochastic analysis which has the advantage of the simplicity. Through the results, it becomes possible to deal with all uncertain parameters in concrete plates. Results show that there are qualitatively similarity between stochastic and deterministic responses. Also, dispersion coefficients have been investigated. It is clear from results that even though there is qualitative likeness in both deterministic and stochastic bending moment distribution but, there are differences between the changes intensity in minimum, mean and maximum bending moment distribution. For example, the changes intensity for mean value in x direction is 3200 for each meter. While, it is 2933 for minimum bending moment distribution.

Key words: Stochastic bending moment, Concrete plate, Fourier, Levy, Sinusoidal

INTRODUCTION

When engineering systems are taken into account within the framework of numerical tools, the assumption that these systems have deterministic parameters is implicitly made. Thus, the system parameters assume as constant values over the system domain. However, in real plate structures, the properties have several uncertainties.

Due to advances in computational mechanics, numerical and computational methods there are tremendous developments in the modeling of the structural behaviors with uncertainties as unavoidable part of them.

Uncertainty in system parameters and system responses have been evaluated by strong efforts to develop computational methods (Schuëller, 1997). The incorporation of uncertainty in structural analysis has been advocated by Freudenthal and others in sixties (Freudenthal *et al.*, 1996). Transformation or fast probability integration methods have been used to analyze probabilistic structural (Cruse *et al.*, 1988). Many earlier works were based on stochastic finite element methods and perturbation approach, which were applied to plate problems (Hisada and Nakagiri, 1981; Vanmarke *et al*., 1986; Liu *et al*., 1986; Lawrence, 1987).

Up to now, many researches about geometrical parameters (Nieuwenhof and Coyette, 2003; Choi and Noh, 1996; Choi and Noh, 2000; Graham and Deodatis, 2001 and Stefanou and Papadrakakis, 2004) and temporal uncertainties (Choi and Noh, 1996; Falsone and Impollonia, 2002) have been performed. However, the uncertainties have been mainly focused on characteristic constants of material, such as elasticity modulus (Choi and Noh, 1993; Graham and Deodatis, 1998; Zhu *et al*., 2001; Vanmarcke and Grigoriu, 1983; Butcher and Shinozuka, 1988; Deodatis and Shinozuka, 1989; Impollonia and Sofi, 2003; Shinozuka and Deodatis, 1988) and Poisson's ratio (Chun, 2004).

Even though some research works consider these parameters but, this is attributed to the fact that loading is one of the most important parameters which it's uncertainty is not really negligible. Therefore, considering the effect of randomness in loading is also required.

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Practically, one of the most crude and simple methodologies in load modeling is the Fourier method, which is also used by Navier's solution. As accepted generally, Navier's solution is useful for solving different kinds of loading conditions and analyzing the response of plate structures.

Considerable safety of plate structures such as buildings floors and vessels (Khan *et al*., 2010; Shariati *et al*., 2008; Sezar *et al*., 2010) adds the study of risky. In the present study, along uncertainty in geometrical and material properties, uncertainty in loading conditions has been also considered and analytical findings for stochastic bending moment response are presented.

MATERIALS AND METHODS

Bending of Plates:

In the plate structures, the plate response can be obtained by solving the Lagrange differential equation 1.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D} \tag{1}$$

where, $q(x, y)$ and D denote loading function and flexural rigidity, respectively. Flexural rigidity for a rectangular plate is given by Eq. 2.

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{2}$$

where, E is the elasticity modulus, t is the plate thickness and υ is the Poisson's ratio.

Bending and twisting moments can be also given by Eq.3, 4 and 5.

$$M_x = -D(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}) \tag{3}$$

$$M_y = -D(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}) \tag{4}$$

$$M_{xy} = -D(1-\nu)(\frac{\partial^2 w}{\partial x \partial y}) \tag{5}$$

In the plate structures, if the attention is finding the plate response using Levy's solution, the total bending moment of the plate structure can be obtained from Eq. 6.

$$W = W_h + W_p \tag{6}$$

in which W_h can be given by Eq. 7.

$$W_h = \sum_{m=1}^{\infty}\left(A_m \sinh\frac{m\pi y}{a} + B_m \cosh\frac{m\pi y}{a} + C_m y \sinh\frac{m\pi y}{a} + D_m y \cosh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \tag{7}$$

where, A_m, B_m, C_m and D_m can be achieved using boundary conditions of the plate.

In Eq. 6, W_p can be given by Eq. 8.

$$W_p = \sum_{m=1}^{\infty} k_m(y)\sin\frac{m\pi x}{a} \tag{8}$$

in which $k_m(y)$ can be obtained from the solving the differential equation giving by Eq. 9.

$$\frac{d^4 k_m}{dy^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 k_m}{dy^2} + \left(\frac{m\pi}{a}\right)^4 k_m = \frac{q_m(y)}{D} \tag{9}$$

where, $q_m(y)$ depends on the loading function and is given by Eq. 10.

$$p_m(y) = \frac{2}{a}\int_0^a q(x,y)\sin\frac{m\pi x}{a}dx \tag{10}$$

In this study, a plate structure under sinusoidal distributed loading has been considered. Figure 1 shows the plate model, which is a plate under sinusoidal distributed loading.

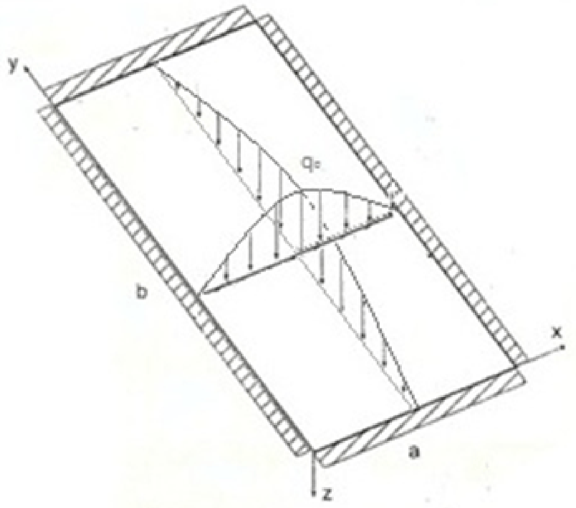

Fig. 1: Model of plate structure

Stochastic Numerical Application:

Variation in geometric parameters, material properties and loading affect the uncertainty in response of plate structures. In order to study the stochastic response of plate structures, the elasticity modulus E, Poisson's ratio υ, plate thickness t, plate dimensions and loading condition $q(x, y)$ were modeled as random variables. Each random variable is modeled as Eq. 11.

$$Z = \mu_z \left(1 + \nu_z \alpha_z\right) \tag{11}$$

where, μ_z is the mean value, α_z is a set of random numbers with a zero mean and υ_z is the coefficient of variation for the random variable.

In the stochastic calculation of the response resulting from the variability of the plate structure variables, the coefficients of variation, υ_z, were assumed to be equal to 0.025 for the plate dimensions and the load. However, for the reason that the other variables including elasticity modulus, Poisson's ratio and thickness are more uncertain, the coefficients of variation for these variables were assumed to be equal to 0.05.

Even though this numerical stochastic method requires a large number of randomly generated values for the parameters but, the method has the advantage of the simplicity. Moreover, using this method it is possible to study the outputs statistically and also, mean values, standard deviation and other statistical parameters can be easily obtained.

Plates under Sinusoidal Loading:

If the sinusoidal load distribution is given by Eq. 12:

$$q = q_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{12}$$

where, m and n are integer numbers, we shall obtain for the bending moment the following expression.

$$W\left(x, y\right) = \frac{q_0}{\pi^4 D\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{13}$$

from which the expressions for bending moments, twisting moments and shearing forces can be obtained by Eq. 14 to Eq. 18.

$$M_x = \frac{q_0}{\pi^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2}\left(\frac{1}{a^2} + \frac{\nu}{b^2}\right) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{14}$$

$$M_y = \frac{q_0}{\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}\left(\frac{\nu}{a^2}+\frac{1}{b^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (15)$$

$$M_{xy} = \frac{q_0\left(1-\nu\right)}{\pi^2 ab\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \qquad (16)$$

$$Q_x = \frac{q_0}{\pi a\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (17)$$

$$Q_y = \frac{q_0}{\pi a\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \qquad (18)$$

The value set of the variables for the model are taken in Table 1.

Table 1: Set of parameters

Parameters (random variables)	Mean values
Elasticity modulus, *E (GPa)*	15, 20, 25
Poisson's ratio, *υ*	0.15, 0.17
Plate width, *a (m)*	3, 4
Plate length, *b (m)*	3, 4
Plate thickness, *t (cm)*	10, 15, 20
Load, *q (kN)*	10, 15, 25

RESULTS AND DISCUSION

Some stochastic responses for the concrete plate under sinusoidal distributed load are shown in Fig. 2 to Fig.4.

Fig. 2 to Fig. 4 indicate that stochastic and deterministic responses are qualitatively similar. However, the intensity changes are different for mean and dispersion values. Table 2 shows the coefficients of intensity for different responses when sinusoidal distributed loading is considered.

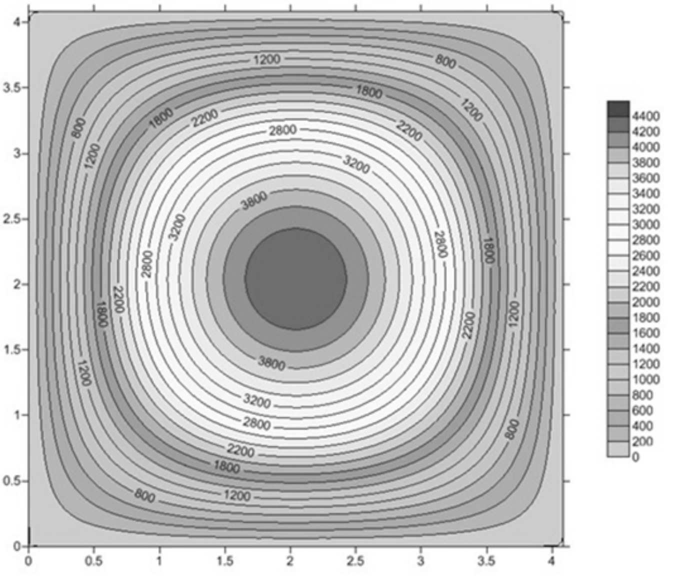

Fig. 2: Minimum dispersion of the bending moment along y axis, (E=15 GPa, t=20 cm, q=10 kN, υ=0.17).

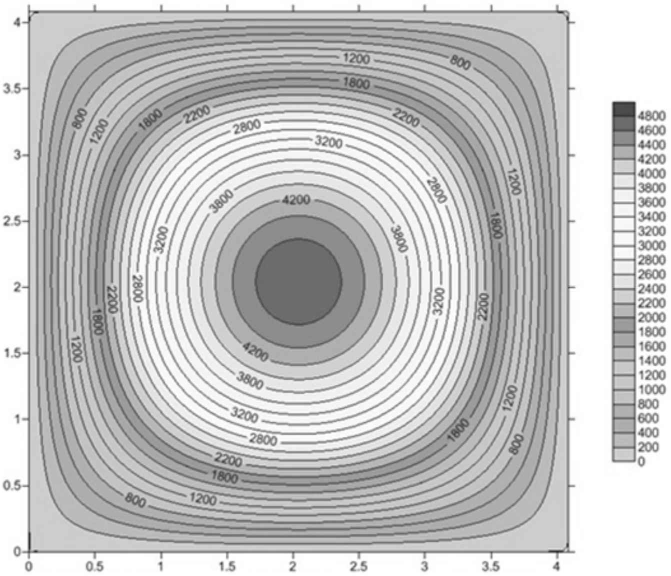

Fig. 3: Means value of the bending moment along y axis, (E=15 GPa, t=20 cm, q=10 kN, υ=0.17).

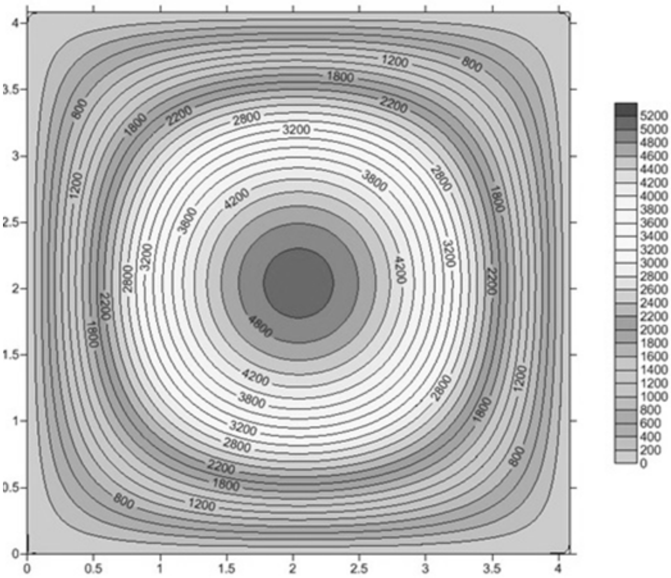

Fig. 4: Maximum dispersion of the bending moment along y axis, (E=15 GPa, t=20 cm, q=10 kN, υ=0.17).

It is clear from Table 2 that for a plate under sinusoidal distributed load the dispersion coefficients for bending moment is equal to 0.083. Table 2 shows that bending moment is sensitive to the uncertainty in parameters and it is vary from (*mean-0.083*mean*) to (*mean+0.083*mean*).

Fig.2 to 4 indicate that there are similarity between stochastic and deterministic bending moment, qualitatively. For example, the plate bending moment reaches the maximum value at the center point of the plate and also, the total bending moment is symmetrically distributed.

Table 2: Changes intensity

Item	Intensity direction	Changes intensity			Proportion to the mean intensity		
		Min	Mean	Max	Min	Mean	Max
M_y	Both x, y	2933	3200	3467	0.916	1	1.083

It is clear from figures that even though there is qualitative likeness in both deterministic and stochastic bending moment distribution but, there are differences between the changes intensity in minimum, mean and maximum bending moment distribution. For example, the changes intensity for mean value in x direction is 3200 for each meter. While, it is 2933 for minimum bending moment distribution.

In general, the changes intensities take the values of 2933, 3200 and 3467 for minimum, mean and maximum bending moment, respectively.

According to Figures, the stochastic responses are qualitatively similar to the deterministic responses. However, the changes intensities are different for results. The intensity of the changes are taken in Table 2.

Conclusions:

The Levy's solution is appropriate for computing the structural response when the Fourier loading series is available. In this paper, the Fourier method and the Levy's solution are used to obtain solutions to the stochastic bending moment response of concrete plates with uncertain parameters for sinusoidal distributed loading.

Since the load is a parameter influencing the behavior of concrete plates, along the geometrical and material characteristics, the independent evaluation of response variability due to the randomness in loading is also given.

A numerical application generating random values has been used to obtain stochastic analysis which has the advantage of the simplicity.

Through the results, there are qualitatively similarity between stochastic and deterministic bending moment responses. In addition, changes intensity for different sets of plate properties were obtained and presented. The results show that the intensity of changes are different for mean values and dispersions for different responses.

It is clear from results that the bending moment is sensitive to the uncertainty in parameters and it is vary from (*mean-0.083*mean*) to (*mean+0.083*mean*) for a plate structure under sinusoidal distributed load.

In general, the results show that for the plate structures the responses can be extremely different due to uncertainty in parameters.

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