Australian Journal of Basic and Applied Sciences, 5(8): 268-275, 2011
ISSN 1991-8178

Dynamic Behavior of Double Layer Cylindrical Space Truss Roofs

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Abstract: Recently, there are a lot of studies on dynamic behaviors of double layer cylindrical space trusses. In some case, they used H/S (Rise to Span) or α (initial angle) as a main geometrical characteristic of structure and tried to depend the dynamic behavior of these structures to them (H/S, α). Moreover on those papers, they classify their models base on these mentioned parameters. But in this paper, the effects of each geometrical parameter such as Rise, Span, length, H/S (Rise to Span),H*S*L(Rise multiplied by Span multiplied by length) on dynamic behavior of vault trusses are studied. For this purpose, the main period time of structure as a dynamic characteristic that includes mass and stiffness matrix property is chosen, and the effective of structural geometry on the main period are studied.

Key words: Barrel vaults, Double layer, Dynamic study, Space truss.

INTRODUCTION

Space trusses are one of the lightest steel structures with three-dimensional and complex structural behaviors made of thousands of steel tubular bars connected together by nodes. The very high degree of indeterminacy, their multiple redundancies and their appropriate three-dimensional geometrical forms provide additional margins of safety to prevent them from sudden collapse in the case of accidental local failure of one or more elements, when the overall loading is below the service load. They have become widely popular as large span roof structures, particularly in areas such as sport centers, exhibition halls and airport hangars. The main advantages of these structures are that they are light in weight, have a high degree of indeterminacy and great stiffness, simple production and fast assembly, are totally prefabricated, do not need site welding, are easily formed into various attractive geometrical surfaces, have the ability to cover large areas with widely spaced column supports, have generally good response against earthquakes and are cost effective.

With their low weight and great stiffness, space trusses are believed to attract low forces during seismic activities and can be considered to be amongst the least likely to suffer damage, when compared to other large span roofs (Marsh, 2000) Despite the earlier assumptions, recent studies show that where strong earthquakes are probable, these structures are vulnerable to seismic failures, especially when roofs are covered with snow. Kawaguchi (1997) reported damages due to earthquake. M.Rezaeei *el al*. (2011) presented a new method to calculate the viscous fictitious damping for dynamic relaxation on some structures such as space truss. K. Koohestani and A. Kaveh (2010) used a new method for vibration analysis of single and double layer shallow dome. J.G. Cai *el al*. (2008) studied on the seismic performance of space beam string structure. Coan and Plaut (Coan, 1983) determined the dynamic response of lattice dome. Sadeghi (2004) investigated the dynamic behavior of double layer barrel vaults; and showed that they are vulnerable to earthquakes and have a brittle behavior. Zhang and Lan (2000) have reviewed research findings on dynamic characteristics of space trusses. Further studies on seismic behavior of space structures by Ishikawa *et al* (2000). Kuneida *et al.* (2011) studied the vibrational characteristics of some existing structures and Ishikawa. The above studyshowed the importance and the need for carrying out dynamic analysis in design of space trusses. But it must regarded that dynamic analysis of a space truss is, in some manner,a cumbersome procedure due to extreme complexity of structural configuration and numerous degree of freedoms, as well as a node to node distribution of mass which would consequently results in a complicated dynamic response. So some engineers refuse to perform the dynamic analysis of space truss in their designs. The main aim of this paper is to simplify the dynamic study of this type of structures.

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50 double layer cylindrical space truss (DLCST) are modeled and their dynamic behavior are studied and the effect of structural geometry on their behavior is investigated.

2. First Natural Period of Structure as a Dynamic Characteristic:

The free vibration dynamic equilibrium equation for a space frame with multi-degree of freedom (MDF) without viscous damping is as follows.

$$[M][\ddot{v}(t)]+[K][v(t)]=0 \tag{1}$$

Where, [M] and [K] are respectively the structure mass and stiffness matrices. [*v*] and [*v(t)*]are the acceleration and displacement vectors of structures.

In order to calculate the natural j^{th} mode shape of the free vibration with frequency 'ω_j' it can be assumed that:

$$v_j(t)=\varphi_j \sin(\omega t-\alpha) \tag{2}$$

And in a vector form:

$$[v] \;=[\varphi]\sin(\omega t-\alpha) \tag{3}$$

With replacing Eqn. 3 into Eqn. 1:

$$\{[M][\varphi](-\omega^2)+[K][\varphi]\}\sin(\omega t-\alpha)=0 \tag{4}$$

Or

$$\Big[[K]-\omega^2[M][\varphi]\Big]=0 \tag{5}$$

This is the equation called "characteristic or frequency equation" that used for a MDF system with 'N' as the number of freedom. Calculation of natural mode shapes of a Multi Degree Freedom structure is resulted to finding the eigenvalues of The Eq. 5. As soon as 'ω_j' , j=1 to N, are calculated, by using the Eqn. 6 the natural period of vibration can be obtained from natural circular frequency of vibration:

$$T=\frac{2\pi}{\omega} \tag{6}$$

As it is shown, the duration period of structure in each mode is depended on the mass and stiffness matrix. The dynamic behavior of the structure is a function of these matrixes too (Eq. 1). So in this paper, the main period of structure is chosen as a dynamic characteristic of space frames. A comprehensive study on the effect of structural configuration and geometry on main vibration mode is carried out.

2.1. Effect of the Damping on the Main Period Time:

The free vibration dynamic equilibrium equation for single-degree of freedom (SDF) with viscous damping is as follows.

$$m\ddot{u}+c\dot{u}+ku=0 \tag{7}$$

$$\ddot{u}+\frac{c}{m}\dot{u}+\frac{k}{m}u=0 \tag{8}$$

Where, m and k are respectively the structure mass and stiffness and c is a damping factor that represented the energy dissipation in a cycle of amplitude or a period of forced harmonic vibration.
If we consider:

$$2\xi\omega_n = \frac{c}{m} \qquad \omega_n^2 = \frac{k}{m}$$

Then the equation (8) will be:

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = 0 \tag{9}$$

Where ξ is called, damping ratio and it's depend to mass and stiffness of the system. In most mention structure such as buildings, bridges, dams,…ξ is less than 0.1. By solving (Eq. 9) for the system with $\xi < 1$, displacment function will be arrived as bellow:

$$u(t) = e^{-\xi\omega_n t} \times \left(u(0)\cos\omega_D t + \left(\frac{\xi\omega_n u(0) + \dot{u}(0)}{\omega_D} \right) \sin\omega_D t \right) \tag{10}$$

Where $\omega_D = \omega_n\sqrt{1-\xi^2}$ that shows the natural frequency for the system with damping factor is related to the natural frequency for the system without damping. Moreover, damping detracted natural frequency from ω_n to the ω_D and increased period time from T_n to the T_D but for the system with $\xi < 20\%$, it's effect on the ω and T is neglected (Chopra, 1995). As the damping ratio for the common structure are located in this range, so ω_D and T_D are approximately equal to the ω_n and T_n.

2.2. Calculating the Mass and Stiffness Matrix for Space Trusses (Weaver, 1987):

An element of a truss that is hinged in the joint j and k is shown in the Fig1. In this study, the connections are considered in the ideal manner so the rotations in the each end of the element are neglected. The two main flexural surfaces are defined by the surfaces that are built from *y'* and *z'* with *x'* (local axis). In each end, the translation in direction *x',y' and z'* are shown with three numbered arrow. The stiffness matrix (6×6) for the prismatic element in the local direction is represented as follow:

$$k' = \begin{bmatrix} k'_{jj} & k'_{jk} \\ k'_{kj} & k'_{kk} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & & & & & \\ 0 & 0 & & & Sym & \\ 0 & 0 & 0 & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

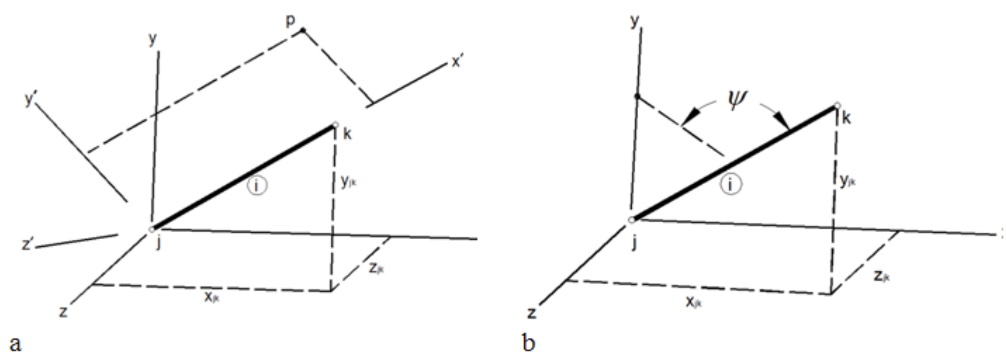


Fig. 1: An element of a truss.

Whereas, there isn't any stiffness due to connection at the joints in the perpendicular direction to the truss axis, most of the terms in the *k'* will be zero. As the same way, for the local axis, the mass matrix will be shown as follow:

$$M' = \begin{bmatrix} M'_{jj} & M'_{jk} \\ M'_{kj} & M'_{kk} \end{bmatrix} = \frac{\rho AL}{6} \begin{bmatrix} 2 & & & & & \\ 0 & 2 & & & Sym & \\ 0 & 0 & 2 & & & \\ 1 & 0 & 0 & 2 & & \\ 0 & 1 & 0 & 0 & 2 & \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}$$

To form the rotation matrix, third point such as P (in addition of J and K) is used for defining flexural surfaces. This point is located in *x'-y'* surface but do not lie on the *x'* axis. If possible, this point must be considered as another joint of the structure that its coordinate are determined. The sentences of the matrix will be driven by considering the properties of vector multiplication.

$$e_{z'} = \frac{e_{x'} \times e_{jp}}{\left| e_{x'} \times e_{jp} \right|} \tag{11}$$

$$e_{y'} = e_{z'} \times e_{x'} \tag{12}$$

Where (e) is a vector in its index direction. For example $e_{x'}$ is equal to:

$$e_{x'} = \begin{bmatrix} c_x & c_y & c_z \end{bmatrix} \tag{13}$$

$$c_x = \frac{x_{jk}}{L} \qquad c_y = \frac{y_{jk}}{L} \qquad c_z = \frac{z_{jk}}{L} \qquad L = \sqrt{x_{jk}^2 + y_{jk}^2 + z_{jk}^2} \tag{14}$$

The same description can be presented for the unite vector by using the coordinate of j and p. If the rotational matrix is used for this three unite vectors, then:

$$R = \begin{bmatrix} e_{x'} \\ e_{y'} \\ e_{z'} \end{bmatrix} = \begin{bmatrix} c_x & c_y & c_z \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix}$$

The *R* operator with 6*6 is used to transfer the stiffness matrix into the structural direction:

$$\hat{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

By using the above operator, the stiffness matrix will be presented in the structural direction as follow:

$$K = \hat{R}^T K' \hat{R} = \frac{EA}{L} \begin{bmatrix} c_x^2 & & & & & \\ c_x c_y & c_y^2 & & & & \\ c_x c_z & c_y c_z & c_z^2 & & & \\ -c_x^2 & -c_x c_y & -c_x c_z & c_x^2 & & \\ -c_x c_y & -c_y^2 & -c_y c_z & c_x c_y & c_y^2 & \\ -c_x c_z & -c_y c_z & -c_z^2 & c_x c_z & c_y c_z & c_z^2 \end{bmatrix}$$

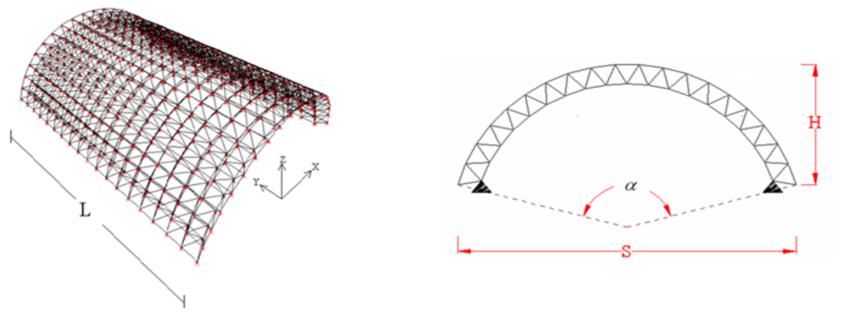
Noted that there are a lot of parameter that influence in the stiffness matrix such as how the structure is connected to the earth or another structure as a support and how the element are connected together, but when the stiffness matrix is written, the effect of these parameter are considered. So any variation on these parameters made a change in the stiffness matrix and finally on the period time. Consequently, the stiffness matrix is affected by the type of connection that will be mentioned in section 3.
As the same way, for the mass matrix:

$$M = \hat{R}^T M' \hat{R}$$

3. Method of Study:

In this article 50 double layer cylindrical space truss roofs (DLCST) are modeled. The geometrical specifications of these models are given in table 1 and Fig. 2.

All end nodes of inner layers are hinged to the rigid supports. They have three rotational degrees of freedom, but their transitional degrees are restrained. All inner nodes used for the connection of structural members have three transitional degrees of freedom.



a: 3D view b:Elavation

Fig. 2: General geometrical properties of models.

3.1. Mechanical Properties of Materials:

It has been assumed that the same material is used for construction of all models. Mild steel material, with the Young's modulus of 206 GPa, Poisson's ratio of 0.3 and Yield stress of 243.5 MPa selected for all members in of all models. The material behavior is proposed to be elastic perfectly. However in none of models the nonlinear behavior is allowed and only linear part of material behavior is contributed in analysis.

Table 1: Geometrical properties of models.

Model name	Span (S) m	Rise (H) m	Length (L) m	Angl (α) degree	Model name	Span (S) m	Rise (H) m	Length (L) m	Angl (α) degree
140	80	14.00	40	M26	90	40	4.14	20	M1
150	80	15.35	40	M27	100	40	4.66	20	M2
160	80	16.78	40	M28	110	40	5.21	20	M3
170	80	18.33	40	M29	120	40	5.77	20	M4
180	80	20.00	40	M30	130	40	6.37	20	M5
90	100	10.366	50	M31	140	40	7.00	20	M6
100	100	11.66	50	M32	150	40	7.67	20	M7
110	100	13.01	50	M33	160	40	8.39	20	M8
120	100	14.43	50	M34	170	40	9.16	20	M9
130	100	15.93	50	M35	180	40	10.00	20	M10
140	100	17.51	50	M36	90	60	6.21	30	M11
150	100	19.18	50	M37	100	60	6.99	30	M12
160	100	20.98	50	M38	110	60	7.81	30	M13
170	100	22.91	50	M39	120	60	8.66	30	M14
180	100	25.00	50	M40	130	60	9.56	30	M15
90	60	12.43	60	M41	140	60	10.50	30	M16
100	60	13.99	60	M42	150	60	11.51	30	M17
110	60	15.62	60	M43	160	60	12.59	30	M18
120	60	17.32	60	M44	170	60	13.74	30	M19
130	60	19.11	60	M45	180	60	15.00	30	M20
140	60	21.01	60	M46	90	80	8.28	40	M21
150	60	23.02	60	M47	100	80	9.33	40	M22
160	60	25.17	60	M48	110	80	10.41	40	M23
170	60	27.49	60	M49	120	80	11.55	40	M24
180	60	30	60	M50	130	80	12.74	40	M25

3.2. Loading Condition:

One of the most significant loads on space structure is the snow load. In space structures, the ratio of snow to dead load is considerably greater than the one in ordinary building. In regular buildings the probability of coincidence of snow and earthquake loads does not play a significant role, because the snow loads are usually a negligible fraction of the total seismic weight. On the contrary, in space frames snow loads can easily reach 2 or 3 times the self-weight of a space structures. Therefore, even a very small probability of experiencing a strong seismic event when having heavy snow on the roof can lead to a severe consequences such as collapse of roof. Hence, it is essential to consider the combination of snow and earthquakes in design (Moghaddam, 2000). In the analysis conducted in this paper the seismic weight is assumed to be included with the whole gravity weight of structure which in a horizontal projection is assumed to be 490 Pa plus 20 percent of live load which is considered to be totally due to a snow load of 1370 kPa. Above condition belongs to a weather zone with extreme cold winters. The effect of milder situation in terms of less amount of snow is considered to have negligible effect on the dynamic behavior of space frame and is not included in analysis presented in this paper.

3.3. Method of Analysis:

The mass and stiffness matrix are calculated for any DLCST by using of the equation that presented in section 2.To evaluate the effect of each geometrical characteristicon the mass and stiffness matrix -the main parameter that defined the dynamic behavior of each structure- the natural period of vibration (T_0) is calculated by equations 5 and 6. And also and numerical analysis are fulfilled by finite element software- SAP2000.

Table 2: Main Period Time for Each Model.

Model name	P.Time (T) sec	Model name	P.Time (T) sec	Model name	P.Time (T) sec	Model name	P.Time (T) sec
M1	0.134	M14	0.392	M27	0.845	M40	1.548
M2	0.158	M15	0.476	M28	0.944	M41	0.670
M3	0.185	M16	0.526	M29	1.079	M42	0.784
M4	0.194	M17	0.567	M30	1.181	M43	0.921
M5	0.213	M18	0.634	M31	0.530	M44	1.055
M6	0.247	M19	0.751	M32	0.610	M45	1.163
M7	0.276	M20	0.777	M33	0.698	M46	1.284
M8	0.312	M21	0.397	M34	0.824	M47	1.381
M9	0.382	M22	0.443	M35	0.935	M48	1.536
M10	0.482	M23	0.532	M36	0.998	M49	1.746
M11	0.254	M24	0.586	M37	1.133	M50	1.954
M12	0.283	M25	0.679	M38	1.263		
M13	0.324	M26	0.755	M39	1.387		

4. Results:

For any of DLCST, the natural period time are given in Tab.2. Comparing the results from analytical analysis for each DLCST with numerical analysis that performed by finite element software SAP2000 shows the accuracy of the calculation-their difference is less than 0.001 sec.

5. Discussion:

The correlation coefficient is calculated for each selected geometrical parameter to evaluate their effect on the dynamic behavior and also period time of the structure. The correlation is a mathematic coefficient that determines the relation between two parameters. Two parameters are correlated together when their value change uniformly that means while one of the parameter is increase or decrease the other one is increase or decrease, too that their relation can be define by an equation . The correlation coefficient will be positive while these two parameters move in a same direction, otherwise if they moved in contrary direction its value must be negative.

In this paper the equation that presented by CARL PIRSON are used:

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\left[\sum(x-\bar{x})^2(y-\bar{y})^2\right]^{1/2}} \tag{15}$$

According to table 3, dependence between the main period time as a main characteristic of the structure's dynamic behavior and the Rise of the structure is more than other characteristic, although the effect of 'Span *Rise* Length' and 'Span *Rise' is noticeable. In table 3 it is also noticeable that the influence of $\frac{H}{S}$ on the period time is not significant but some of researchers tried to categorize their model based on the (Moghaddam, 2000). It seems the dependence of the dynamic behavior of structure to 'Rise to Span' $\frac{H}{S}$ is not reasonable and it's better to study the variety of dynamic behavior based on the Rise. Moreover, according to table 3, it is not reasonable to relate the dynamic property to α.

Table 3: dependence between the main time period and geometrical parameter. Correlations

	H	S	L	α	H/S	H/L	S*H	S×H×
Pearson Correlation	0.998	0.792	0.432	0.559	0.562	0.811	0.971	0.922

5.1. Proposed an Equation to Calculate Period Time for DLCST:

If a direct line is drown between the sporadic point where the summation of the square deviation parallel to Y axis is minimum and then this line is called regression of Y to X.

If the equation of this line is then the value of a and b can be calculated by principle of minimum square, as follow:

$$\sum y = \sum a + b\sum x = an + b\sum x \tag{16}$$

$$\sum xy = a\sum x + b\sum x^2 \tag{17}$$

Where n is the number of the points and by solving equation 16 and 17, simultaneously, the value of a and b will be derived.

The summation of the square regression is defined as follow which is a criterion of sporadic for the predicted Y value (derived from the regression line) from the average Y's value. if this value are near to 100% then the trust to the regression line will be increase.

For arriving the approximate equation to calculate the main period time of vault truss (DLCST), three statistic models are used. The accuracy of each model is evaluated based on 'R square'.

Table 4: "T" is a function of (S*H*L), (S*H) and H.

Model	a	b	c
Independent parameter	S*H*L,S*H ,H	S*H , H	H
Dependent parameter	T	T	T
R Square	0.998	0.998	0.977

According to table 4, we can see that the value of 'R square' in model 'c' is smaller than model 'a' and 'b' , but it is equal in model 'a' and 'b', so for simplifying, the model 'b' is selected to arrive an approximate equation, because it has fewer variation than model 'a'.

Based on calculated coefficient from equations 16 and 17, the following equation is suggested:

$$T = 0.061H+0.000143\ S*H-0.174 \tag{17}$$

The value of 'R square' in model 'c' has an appropriate accuracy too. So for more simplifying, the period time of these trusses are depended on the Rise of the structure only.
So:

$$T = 0.0707H-0.2189 \tag{18}$$

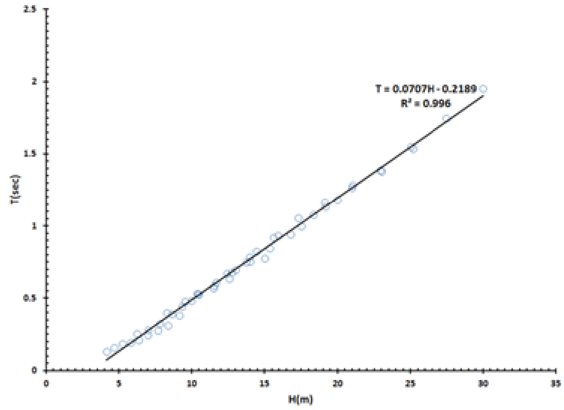


Fig. 3: Main Period Time's Line.

Conclusions:

There are a lot of parameters that influence in the main period time of the structure. For example in the building structures some parameter such as length , wide ,high ,mass, connection and… are affected in the main period time of this structure but most of the building code focus on the high as a principal parameter that almost can predict the dynamic behavior of these structures. In this paper, according to the statistical studies, the main period time and consequently the dynamic behavior of vault truss (DLCST) is depended to the rise of the structure (H). So for dynamic study it seems that it would be better to classify these trusses based on their rise and it's not reasonable to estimate their dynamic behavior base on $\frac{H}{S}$ or α .

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