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Influence of Earthquake Magnitude Variable on Response of Structures

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Abstract: The formulation of the response in the frequency domain is very efficient in the computation and evaluation of the stochastic response of structures. The effect of the variability in earthquake variables on the response is sufficiently important to require careful consideration in design. The response reliability is dependent on the variability of the excitation, ground motion. In this paper, the effects of earthquake magnitude on the structure response has been studied. Analytical results show that uncertainty in earthquake magnitude variable causes uncertainty in the response of models and this uncertainty in large earthquakes is important. It is also clear from the results that the increase of the earthquake magnitude causes increase in the coefficient of variation of response.

Key words: earthquake magnitude, modal analysis, frequency domain, seismological Fourier spectrum.

INTRODUCTION

Among all sources of uncertainty stemming from the material properties, the design assumptions, and the earthquake-induced ground motion, the later seems to be the most unpredictable (Kappos, 2002) and it has a significant effect on the variability of structural response (Padgett and Desroches, 2007). The problems of dynamic response analysis and reliability assessment of structures with uncertain system and excitation parameters have been the subject of extensive research during the last few years (Papadimitriou *et al.*, 1995; Sakurai *et al.*, 2001; Chaudhuri and Chakraborty, 2006; Yazdani and Komachi, 2009; Yazdani and Takada, 2009). The need for stochastic dynamic analysis of engineering systems stems from the fact that an important class of structural loads, which evolves with time, exhibits strong variability in both amplitude and frequency content (Manolis and Koliopoulos, 2001).

The simplified statistical structure model can be efficiently used in analytical random vibration studies and for mathematically studying the importance of the temporal non-stationary in both the amplitude and frequency content of ground motion on the response of structures. For linear structures and for Gaussian excitation, the response is also Gaussian (Solnes, 1997). Therefore, only the mean and the covariance response are needed to completely determine the joint probability density function of the response (Nigam, 1983).

Excitation processes display a wider power spectrum density function compared with the corresponding response function (Lin and Cai, 2004). The significance of this result is that it verifies the assumption of wide band-limited noise (or even white noise), which is often made for the input of structural dynamic systems. As the input process is filtered through the oscillating system, only a narrow band of frequencies around the oscillator's natural frequencies are transmitted, and thus the output displays a narrow band power spectrum. This is why the following study is restricted to the peak of the narrow band stochastic process. In random vibration theory, the probability density for the response of a system under Gaussian white noise or band-limited excitation can be calculated based on the frequency information of excitation (Sun, 2006). This information only requires the computation of the Fourier amplitude spectrum of excitation, which can be calculated based on the seismological method. For regions where recorded ground motion data are scarce, it becomes imperative to use physical models to represent the ground motion generation and propagation.

The formulation of the response in the frequency domain is very efficient in the computation and evaluation of the stochastic response spectrum, which the frequency domain approach is appropriate for probabilistic analyses. In the present work an attempt has been made to study the effect of the variability

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in earthquake magnitude variable on the stochastic response of different models. The presented frequency domain formulation for the modal response variability is the utilization of suitable explicit relationships between the modal characteristics and the uncertain earthquake ground motion variables.

MATERIALS AND METHOD

Computation of Response:

The mathematic description of excitation response characteristics of structural systems subjected to stochastic inputs, follows closely the standard theory of mechanical vibrations with some change in emphasis (Thomson, 1993). Physical systems whose behavior, characterized by an arbitrary response quantity can be represented by the equation 1:

$$L[X(t)] = Y(t) \tag{1}$$

where the differential operator $L[*]$ describes the main properties of the system, that is, acts as a mathematical model for the system. $X(t)$ is the response vector of the system or the system output when it is subjected to an excitation vector or input vector $Y(t)$, stochastic or deterministic. The operation performed on the input vector signal $Y(t)$ is the inverse operation $L^{-1}[*]$, that is, the output vector signal $X(t)$ is obtained by it. This inverse operation is referred to as filtering of the signal input $Y(t)$. The linear operator $L[*]$ will be time invariant and act as a linear filter. Based on this format, the equation of motion of proportionally damped linear elastic multi-degree-of-freedom (MDOF) subjected to stochastic uni-directional horizontal ground acceleration is obtained as

$$[m]\{\ddot{X}\} + [c]\{\dot{X}\} + [k]\{X\} = Y(t) \tag{2}$$

where $\{X\}$ is the vector relative displacement (output) under a stochastic excitation vector (input) and $[m]$, $[k]$ and $[c]$ are mass, stiffness and damping matrix, respectively. Since the input vector (ground motion) is a random process, the output vector $\{X\}$ will also be a random process (Elishakoff, 1999). It is convenient that the frequency response method is applied for representing the relationship between input and output, in which both input and output processes are represented by harmonic functions (Ordaz, *et al*., 2003; Takewaki, 2001). By taking the Fourier transform of both sides, in frequency domain the differential equation 1 is

$$\{X(\omega)\} = [H(\omega)]\{Y(\omega)\} \tag{3}$$

where $[H(w)]$, as inverse operation $L^{-1}[*]$, is the structural transfer function matrix between the displacements and applied loads. Where the vector $\{Y\}$ is the external excitations of loading and $\{X\}$is the system displacement vector in frequency domain. Direct calculation of $[H(w)]$ requires a matrix inversion for each frequency variation. This is computational cumbersome, therefore, approximate analysis can be applied in practice. The modal analysis technique is used for the response calculation of dynamically sensitive structures. It requires natural frequencies and modal shapes of structural system, which are calculated from the eigenvalue solution. Then, the structural transfer function matrix is stated as a summation of the contribution of each vibration mode as

$$[H(\omega)] = \sum_{n=1}^{N} H_n(\omega)\{\phi\}_n\{\phi\}_n^T \tag{4}$$

in which N is the total number of natural modes to be included, $\{f\}_n$ is the eigenvector of the mode n, $H_n(w)$ is the frequency dependent modal participation factor defined as

$$H_n(\omega) = \frac{1}{k_n}\frac{\omega_n^2}{(\omega_n^2 - \omega^2) + 2i\xi_n\omega_n\omega} \tag{5}$$

In this equation, k_n is the generalized stiffness, w_n is the natural frequency and ζ_n is the percentage-damping ratio for the mode n. Equation 3 indicates that the response of damped linear elastic MDOF systems depends explicitly on the dynamic properties of the systems and smoothed version of the amplitude spectrum of ground motion and does not depend on the phase information of ground motion. Actually this fact is well known based on previous studies (Lyon, 1975; Nakashima *et al*., 1996).

There is a vast amount of research aimed to predicting amplitude Fourier model, coming especially from the engineering seismology field. Such models have usually been developed in the context of the stochastic modeling approach and random vibration theory (Boore, 2003). The Fourier amplitude spectrum of motion, $Y(M_0, r,w)$, expected at an average site at distance r from average earthquake of seismic moment M_0, as

$$Y(M_0, r, \omega) = E(M_0, \omega).P(r, \omega).S(\omega) \tag{6}$$

where ω is the angular frequency. The function on the right-hand site represent the earthquake source radiation $E(M_0, w)$, propagation path effects $P(r,w)$, site response $S(w)$ (Lam *et al*., 2000; Halldorsson and Papageorgiou, 2005).

The acceleration source spectrum of point source is defined as

$$E(M_0, \omega) = \frac{CM_0\omega^2}{(1+(\omega/\omega_c)^2)} \tag{7}$$

where C frequency-independent scaling factor equal to $FR_{f\theta}V/(4\pi\rho\beta^3)$, where $R_{f\theta}$ represents the averaged radiation pattern (0.55 for S waves), F is the free surface amplification (a factor equal to 2), V represents the partition onto two horizontal components ($1/\sqrt{2}$), ρ and β are the material density and shear-wave speed, respectively, in the source region. In this equation, w_c is corner frequency and is related to the time taken to rupture the fault causing a permanent slip. Following Brune assumption, the corner frequency is given by the following equation (Brune, 1970).

$$\omega_c = (2\pi)\times 4.9\times 10^6 \beta(\Delta\sigma / M_0)^{1/3} \tag{8}$$

where $\Delta\sigma$, in bar, is the stress drop, β in km/s, and M_0 in dyne-cm. The seismic moment, M_0 is often expressed in terms of the moment magnitude M_w which is defined as follows (Kanamori, 1977).

$$M_w = \frac{2}{3}\log M_0 - 10.7 \tag{9}$$

The seismological model depends on distance through the path attenuation function. The path functions represent the effects of geometric spreading, non-elastic, the path independent loss of high-frequency, and scattering attenuation (Boore and Atkinson, 1987). The theoretical geometric distance attenuation is assumed to be $1/r^n$, where the exponent, n, depends on r.

In equation 6, $S(\omega)$ is the upper crust amplification factor and the quarter-wavelength method is used to model the amplification factor of site soil (Boore and Joyner, 1997). They have proposed the site-amplification factor $S(\omega)$, as a function of the average shear wave velocities $(\overline{V}_S)$, representing the soil conditions in the upper 30 m.

According to above description, the Fourier amplitude spectrum of motion can be obtained based on equation 6. By substituting this input into equation 3, the displacement of structures can be calculated.

RESULTS AND DISCUSSION

Numerical Application:

Stochastic study is one of the most important studies in all field of the sciences (Tijani and Aromolaran, 2009; Okereke and Bassey, 2010; Kadry *et al*., 2007). In this paper, in order to study the stochastic response of structural frame, the earthquake magnitude M_w, source-to-site distance r, and amplification factors $S(\omega)$ were modeled as random variables. Each random variable is modeled as

$$Z = \mu_Z(1+V_Z\alpha_Z) \tag{10}$$

where μ_z is the mean value, α_z is a random variable with a zero mean and V_z is the coefficient of variation of the random variable. The overall variance in response of the structural frame is affected by the variances in each of the random variables of excitation. Table 1 shows the mean value set of earthquake ground motion variables for calculation of the Fourier amplitude spectra of ground motion which is used for achieving horizontal displacement of structures.

Table 1: Set of parameters of earthquake ground motion.

Parameters (random variables)	mean value
Earthquake magnitude, M_w	5.5, 6, 6.5, 7, 7.5
Focal distance, R (km)	40
Density, ρ (gr/cm^3)	2.8
Shear-wave velocity, β (km/s)	3.5
Stress drop, $\Delta\sigma$ (bar)	100
Geometric Attenuation	1/r
Amplification factor, $S(\omega)$	NEHRP class D

Figure 1 shows two shear building models, denoted as case 1 and 2, which are three stories, and five stories in height. For the two models the floor masses, the story stiffness and damping have been illustrated. In the stochastic calculation of the response resulting from the variability of the earthquake ground motion variables, the coefficient of variation, COV, is assumed to be equal to 0.02 for earthquake magnitude and 0.10 for distance variables, and 0.20 for site amplification owing to the later variable is more uncertain than earthquake magnitude and distance (Atkinson and Silva, 1997;. Anderson *et al*., 1996; Franceschina *et al*., 2006).

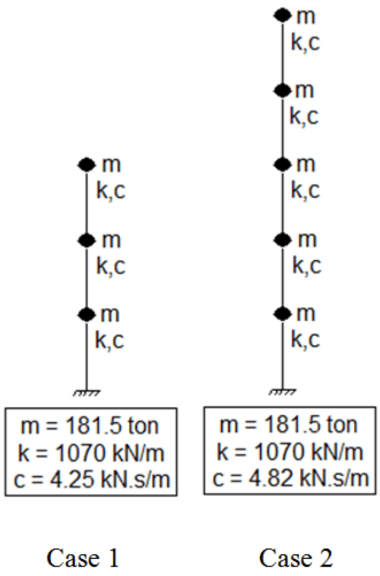

Fig. 1: Models of frames.

The horizontal displacement of the frame is evaluated based on the values of all structural and earthquake ground motion variables. The trials are created a large number of sets of randomly generated values for the uncertain parameters and compute the function for each set. The method has the advantage of conceptual simplicity, but it can require a large set of values of the function to obtain adequate accuracy. It is possible to study the output statistically and to obtain values of means, standard deviation, and other statistical parameters.

The overall horizontal displacement of the frame and inter-story drift at each floor is first evaluated using the values of all excitation variables. As an example in case 2, the regular five-story, the values of the overall horizontal displacement of the frame and inter-story drift at nodal point are illustrated in Figure 2 for different earthquake magnitude. The traces of this figure demonstrated the variation of nodal displacements and their dispersion when earthquake magnitude of site changes.

Table 2 compares the coefficient of variation in the maximum horizontal displacement and inter-story drift ratio by variation of earthquake magnitude. The earthquake magnitude influences the response of structure. The mean values and the dispersion of inter-story drift are illustrated in Figure 2 for different earthquake magnitudes. Table 2 indicated that the increase of the earthquake magnitude causes increase in the coefficient of variation of response.

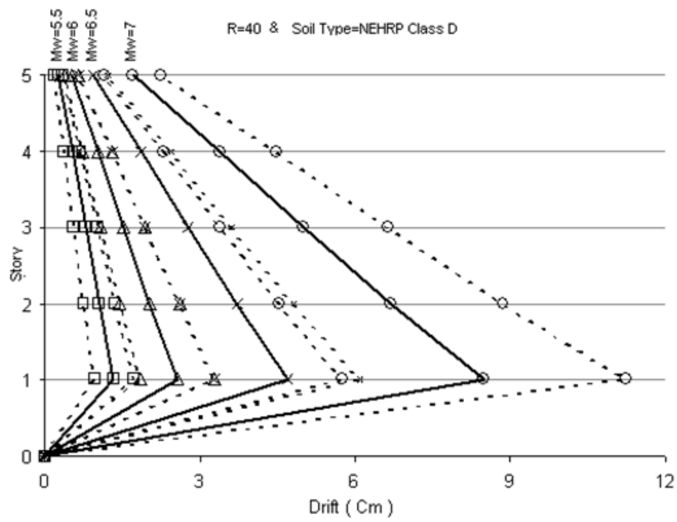

Fig. 2: Means value and mean plus/minus one standard deviation of overall horizontal displacement in 5DOF (Distance of 40 km, soil condition of NEHRP class D).

Table 2: Coefficient of variation of the stochastic response in 5DOF in the case of *R* 40, NEHRP class D, and different earthquake magnitude

	M_w5.5	M_w6.0	M_w6.5	M_w7.0	M_w7.5
Horizontal displacement at top of frame	0.272	0.285	0.304	0.322	0.349
Maximum inter-story drift ratio	0.272	0.285	0.304	0.322	0.349

Conclusions:

The formulation in the frequency domain is appropriate for computing the structural response when the Fourier acceleration amplitude spectrum is available. This formulation requires only the Fourier amplitude spectrum of input ground motion and the real part of transfer function. One of the essential characteristics of the seismological method is that it distills what is known about the various factors affecting ground motions into different functional forms. The presented expression in this study provides an important basis for a wider use of seismological theory in the understanding of the relation between seismological and earthquake magnitude variable. Dispersion in the drift demand for a given intensity measure of excitation is important in calculating the probability of exceeding a structural limit state. The results reveal the coefficient of variation and dispersion in the maximum horizontal displacement and inter-story drift ratio increases by increase of earthquake magnitude.

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