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Using the Frequency Shifts for Damage Detection in Beam Type Structures

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Abstract: A new method for detection of damage and its magnitude in beam type structures using the natural frequency shifts is presented. The method is an iteration based approach in which a nonlinear equation is solved by using successive approximations. The beam is discretized into the finite elements and it is assumed that the damage induces stiffness reductions in each element as a result of changes in material properties. A damage index corresponding to each element and proportional to the intensity of damage of the element is defined. The frequency shifts due to the damage existence is taken into account and the mathematical relation of a parameter reflecting the frequency shifts in terms of the damage indexes of the structural elements is obtained which has incremental form. The incremental equation is nonlinear in terms of the damage indexes and is solved by applying successive approximations which results in the damage index corresponding to each element. In order to examine the capability of the method some testing examples of beams with various boundary conditions and damage states are studied. The results confirm the robustness of the proposed method and also it is found that the method is able to identify the damage location accurately by applying a fine mesh along the beam.

Key words: Finite Element; Natural Frequency; Frequency Shift; Damage; Beam.

INTRODUCTION

Structural health monitoring and damage detection are of interest in civil and mechanical engineering and also in space and aircraft industries. Engineers tend to identify the damage and its intensity during the service life of the structures. The damage occurrence during the service life is inevitable. Natural events and unpredicted scenario of loadings can cause damage in structural systems. Damage detection at the earliest possible stage is an important issue in structural reliability and also in reducing the maintenance cost. This is due to the fact that the early damage detection gives time to repair or remove the damaged structural elements that might lead to catastrophic events. Investigation on the damage identification has been an active research subject since last decades. Damage detection using the static and dynamic response characteristics of the structure has been received a growing attention in recent years. Hjelmstad and shin (1997), Sanayei and Scamolpi (1991), Sanayi and Onipede (1991) and also Sanayei *et al*. (1997) presented algorithms based on using the static test data to detect the damage and assess the structure. Bakhtiari-Nejad *et al*. (2005) presented a static based damage detection algorithm in which the change in the measured static displacement is expressed in terms of the location and intensity of damage. It is viable that displacing a real structure at some degrees of freedom needs large magnitude of static loads that might be difficult to arrange practically for a structure under the service conditions. On the other hand, the dynamic damage detection methods comprise of different techniques in which even ambient induced vibrations can be used for damage detection purposes. It is well known that the modal parameters like modal frequencies, mode shapes and modal damping are dependent on the structural physical properties. Therefore it is expected that changes in the physical properties, which rise from the presence of damage in structural components, can cause detectable changes in the modal parameters. This fact motivated the researchers to develop the vibration based damage detection techniques. Using the frequency shifts caused by damage existence is one of the vibration based damage identification methods. Salawu (1997) performed a complete review of the techniques in which frequency shifts are used for damage identification. The work of (West, 1984) is possibly one of the early works in which the mode shape information of the finite element model is utilized for finding the damage location. The author used the modal assurance criterion (MAC) for finding the correlation of the test data obtained from the undamaged space

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shuttle and modal information obtained from the specimen under the acoustic loading. Penny *et al*. (1993) demonstrated that for the finite element model of a beam structure, using the absolute changes in mode shape curvatures provides good information of the existing damage. Stubbs *et al*. (1992) presented a modal strain energy based method in which the decrease in modal strain energy between two structural DOF is used for damage detection. Other works in (Chen, 1994; Dong, 1994; Yao, 1995) also present damage detection techniques from the changes in mode shape curvature or strain-based mode shapes. Work in (Liu, 1995) used modal data for damage detection in Trusses. Ahmadian *et al*. (1997) presented a damage detection procedure that utilizes measured displacements of a structure. The method is based on this fact that damage existence in substructures or a small region of a large structure causes changes in the substructure's mode shapes where the modes of other parts of the structure will be unaffected. Doubling *et al*. (1998) presented a review of the vibration based damage identification methods. Escobar *et al*. (2005) presented the transformation matrix method in which the changes in the dynamic parameters of the structure are used to identify the damages. The authors mentioned that the computational work is independent of the number of damaged elements and also the method is exact as long as all mode shapes are available. Sahin and Shenoi (2003) and also Zapico and Gonzalez (2006) applied artificial neural networks for damage identification purposes. The former focuses on quantification and localization of the damage in the beam like structures and the latter work deals with the identification of damages induced by earthquakes in buildings. Vakil-Baghmisheh *et al*. (2008) applied genetic algorithms for finding the possible changes in the natural frequency of the beam type structures for crack detection purposes. Rodriguez *et al*. (2009) presented a method to identify the location and the assessment of damage in structural elements. The intensity of damage at element level is introduced by using a damage index which is calculated by solving an eigenvalue problem. The found eigenvalues are the damage index of the structural elements. In the present work an iteration based method is presented in which the damaged zone and the intensity of damages are found by applying a direct iterative procedure. In the present approach the stiffness matrix of a damaged structure is found from the difference of the stiffness matrix of the undamaged structure and the loss of stiffness of the whole structure. The latter one is calculated from the contribution of the intact stiffness matrix of each element multiplied by a value indicating the degree of damage extent of the element. The amount of stiffness loss in each element is represented by a scalar so-called as damage index which is equal to zero for an intact element and 1 for the whole loss of stiffness of the element. The frequency ratio of damaged structure and intact structure is defined and the mathematical relation of this ratio with the damage indexes of the structural elements is obtained in an incremental form. This incremental equation is nonlinear in terms of the damage indexes and is solved by applying successive approximations which results in the damage index corresponding to each element. The proposed method is then evaluated by using some test problems with known damaged scenarios.

2. Formulation:

In the present study a new method is developed which uses the natural frequency of the damaged structure for identifying the damaged element and finding the damage percentage of the element. The following assumptions have been made in the present method:

All the natural frequency of the damaged structure exists.

The mass matrix of the structure remains intact after the damage occurs in structure.

In this study the natural frequency ratios are used in the formulation which provides faster convergence relative to using the natural frequency itself. The frequency ratio is defined by dividing the natural frequency of the damaged structure to the corresponding natural frequency of the undamaged structure. The stiffness matrix of a damaged structure can be found from the difference of the stiffness matrix of the undamaged structure and the loss of stiffness of the whole structure. The latter one can be calculated from the contribution of the intact stiffness matrix of each element multiplied by a value indicating the degree of damage extent of the element. The amount of stiffness loss in each element is represented by x_i which is equal to zero for an intact element and 1 for the whole loss of stiffness in an element. So the relation for calculating the stiffness matrix of damaged structure, $[K_d]$, can be written as follows:

$$[K_d] = [K] - \sum_{i=1}^{ne} x_i [K_i] \tag{1}$$

where ne is the total number of finite elements, $[K]$ is stiffness matrix of the intact structure, $[K_i]$ is the intact stiffness matrix of the element i and x_i is the damage index corresponding to the element i. The natural frequency of the undamaged structure can be calculated as follows:

$$\left\{\left[K\right]-\omega^{2}\left[M\right]\right\}\left[\varphi\right]=0 \tag{2}$$

where ω is the natural frequency of the intact structure, [φ] is the matrix of mode shapes of the structure, [*M*] is the system mass matrix. By replacing [*K*] with [K_d] in Eq. 2, we have:

$$\left\{\left[K_{d}\right]-\omega_{d}^{2}\left[M\right]\right\}\left[\varphi_{d}\right]=0 \tag{3}$$

In Eq. 3, ω_d and [φ_d] are the natural frequency and the mode shape matrix of the damaged structure, respectively.

By defining the frequency ratio of undamaged and damaged states as $\lambda_i=\frac{(\omega_i)_d}{\omega_i}$ one can calculate its changes due to the damage occurrence in structural elements. The changes of the frequency ratio in terms of the damage index x_i can be obtained by using the following relations:

$$\begin{cases} d\lambda_1=\frac{\partial\lambda_1}{\partial x_1}.dx_1+\frac{\partial\lambda_1}{\partial x_2}.dx_2+\cdots+\frac{\partial\lambda_1}{\partial x_{ne}}.dx_{ne} \\ d\lambda_2=\frac{\partial\lambda_2}{\partial x_1}.dx_1+\frac{\partial\lambda_2}{\partial x_2}.dx_2+\cdots+\frac{\partial\lambda_2}{\partial x_{ne}}.dx_{ne} \\ \vdots \\ d\lambda_{dof}=\frac{\partial\lambda_{dof}}{\partial x_1}.dx_1+\frac{\partial\lambda_{dof}}{\partial x_2}.dx_2+\cdots+\frac{\partial\lambda_{dof}}{\partial x_{ne}}.dx_{ne} \end{cases} \tag{4}$$

where *dof* is the total degree of freedoms of the model. Eqs. 4 can be re-written in the following form:

$$\begin{Bmatrix} d\lambda_1 \\ d\lambda_2 \\ \vdots \\ d\lambda_{dof} \end{Bmatrix}_{dof\times 1}=\begin{bmatrix} \frac{\partial\lambda_1}{\partial x_1} & \frac{\partial\lambda_1}{\partial x_2} & \cdots & \frac{\partial\lambda_1}{\partial x_{ne}} \\ \frac{\partial\lambda_2}{\partial x_1} & \frac{\partial\lambda_2}{\partial x_1} & \cdots & \frac{\partial\lambda_2}{\partial x_1} \\ \vdots & \ddots & & \\ \frac{\partial\lambda_{dof}}{\partial x_1} & \frac{\partial\lambda_{dof}}{\partial x_1} & \cdots & \frac{\partial\lambda_{dof}}{\partial x_1} \end{bmatrix}_{dof\times ne}\begin{Bmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_{ne} \end{Bmatrix}_{ne\times 1} \tag{5a}$$

Or in the compact form as:

$$\left\{d\lambda\right\}=[J]\left\{dx\right\} \tag{5b}$$

So the vector [*dx*] containing the damage indexes can be calculated as follows:

$$\left\{dx\right\}=[J]^{*}\left\{d\lambda\right\} \tag{6}$$

where [*J*] with the asterisk represents the generalized inverse of [*J*] due to the fact that [*J*] is not a square matrix. *dof* in Eq. (5a) represents the total degrees of freedom of the structure.

Considering the above equations, we can now develop an algorithm for damage detection in structures as follows:
First, assume the initial damage value x_i for each element.

Then [K_d] is derived by using Eq.1.

By using Eqs. 2 and 3 and taking in mind that $\lambda = \frac{\omega_d}{\omega}$, the values of λ_i can be calculated. Matrix of [J] is calculated.

Using Eq. 6, vector {dx} can be calculated and then the x_i would be updated as $x_i=x_i+dx_i$.

The obtained values x_i in step 5 are assumed as the new x_i and the steps 2 to 5 are repeated until the convergence is achieved.

In order to enhance the robustness of the proposed procedure the following points should have been taken into account:

The following equation is used for the numerical calculation of $\frac{\partial \lambda_i}{\partial x_i}$:

$$\frac{\partial \lambda_i}{\partial x_i} = \frac{\lambda_i\left(x_1, x_2, \ldots, x_i + \varepsilon, \ldots, x_{ne}\right) - \lambda_i\left(x_1, x_2, \ldots, x_i, \ldots, x_{ne}\right)}{\varepsilon} \tag{7}$$

in which ε is a small value, say $\varepsilon = 10^{-6}$.

In order to ensure to achieve the convergence in the iterations process, the obtained values of x_i is relaxed by dividing to 4 during the first four iterations.

3. Evaluation of the Method for the Damage Detection in Beam Type Structures:

In order to evaluate the capability of the proposed method a computer program has been developed using the formulation presented in the previous sections. Several examples are studied and the results have been obtained and presented in graphs to demonstrate the performance of the method. Some damage scenarios have been considered for the analysis. By assuming a percentage of stiffness degradation in some elements due to the damage presence, the natural frequencies of the damaged beam are obtained using Eq. (3). It is assumed that these frequencies of the damaged beam are from the measurements. The goal is to find the global damage extent corresponding to each element in percent which causes the observed frequency shifts. The geometry of the beam which is under study is as follows:

A continuous concrete beam with the following mechanical properties is used for the analysis.

$$\mathrm{E} = 24.8 \quad Gpa$$

$$\rho = 2450 \quad Kg/m^3$$

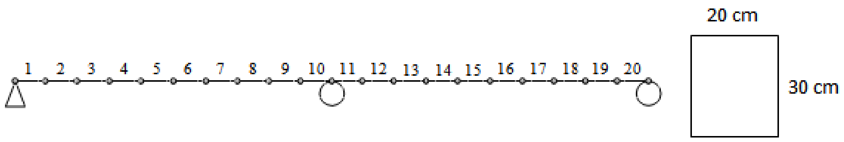

Fig. 1: A model of double span simply supported beam divided into 20 elements.

For the first example it is assumed that the beam has two equal spans of length 4 (m). The beam is divided into 20 elements as shown in Fig.1. The plane beam type element with two degrees of freedom for each node has been used in discretization. In order to study the effects of the number of damaged elements on the algorithm performance, two separate damage scenarios have been considered. Table 1 shows the first and the second damage scenarios.

As can be seen in Table 1, in the first scenario, the assumed damaged elements are next to the supports. Fig.2 shows the results for the first damage case applied for the two-span beam. The results of the algorithm during the iterations are shown which reveals that by increasing the number of iterations the results will converge to the exact damage values (assumed damage percentage). Also for the undamaged elements like element 9, the method predicts intact status. The results after 15 iterations are the same as exact damage percentages which are shown in Fig. 3. Fig.4 shows the results for the second scenario applied for the model of the beam with two spans. In this scenario more elements have been assumed to be damaged in addition of those which considered in the first scenario. As Fig. 4 shows, the results converge to the exact values by increasing the number of iterations and after 15 iterations the results are the same as the exact damage values,

Fig.5. The results of these two tests reveal that the present method is stable in the damage analysis and able to identify the damaged and intact zones along the beam.

Table 1: Simulated damage scenarios for two span beam.

Scenarion No.	Damaged elements	Simulated damage (%)
1	1	20
	10	30
	20	35
2	1	20
	3	30
	5	22
	10	30
	12	10
	16	20
	19	10
	20	35

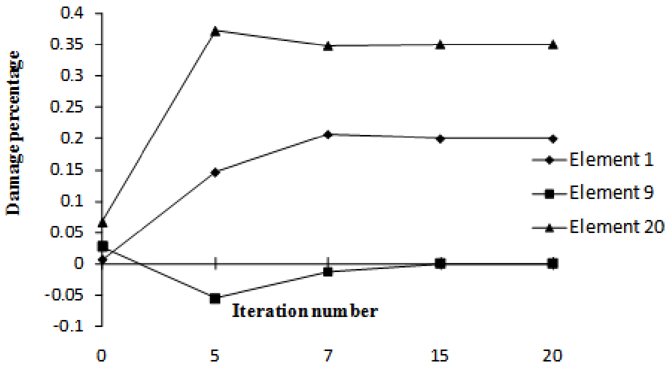

Fig. 2: The convergence behavior of the method for the first scenario applied on the double span beam.

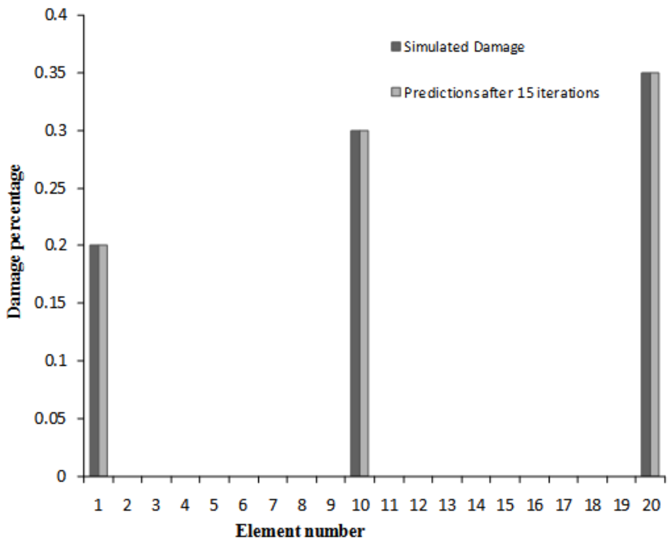

Fig. 3: Damage predictions of the method after 15 iterations for the first scenario.

In order to study the effects of the presence of the mid-supports on the algorithms' performance, the number of mid-supports is increased, as shown in Fig. 6. Two beam elements are considered in each span and planar beam elements similar to those used in the previous tests are used for the discretization. The simulated damage is shown in Table 2 in which eight elements are assumed as damaged elements.

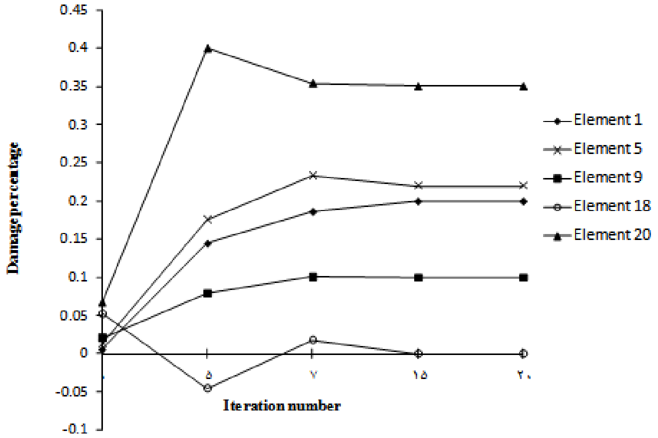

Fig. 4: The convergence behavior of the method for the second scenario applied on the double span beam.

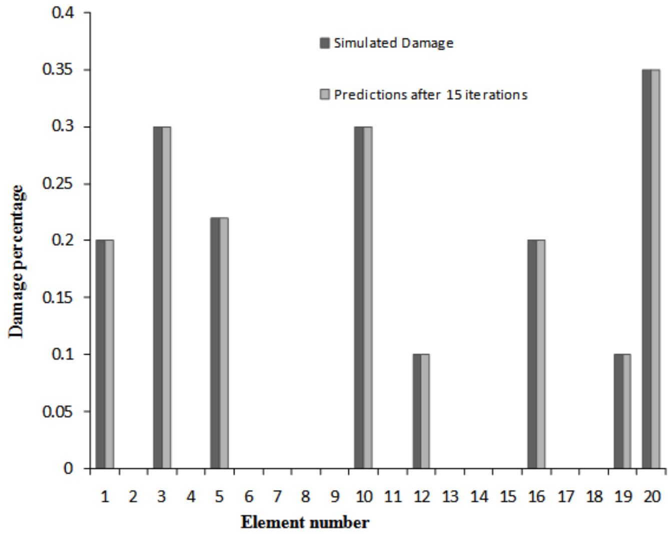

Fig. 5: Damage predictions of the method after 15 iterations for the second scenario.

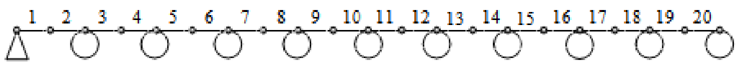

Fig. 6: A model of multi span continuous beam divided into 20 elements.

Table 2: Simulated damage scenarios for the multi-span beam.

Scenarion No.	Damaged elements	Simulated damage (%)
3	1	0.2
	3	0.3
	5	0.1
	10	0.2
	12	0.1
	16	0.2
	19	0.1
	20	0.35

In Fig.7 the results of the third damage scenario applied for the model of the multi-span beam are shown. As it is seen the proposed procedure can identify the damaged elements with the exact damage values. This testing reveals the instability of the procedure in which the multi-span damaged beams can be analyzed.

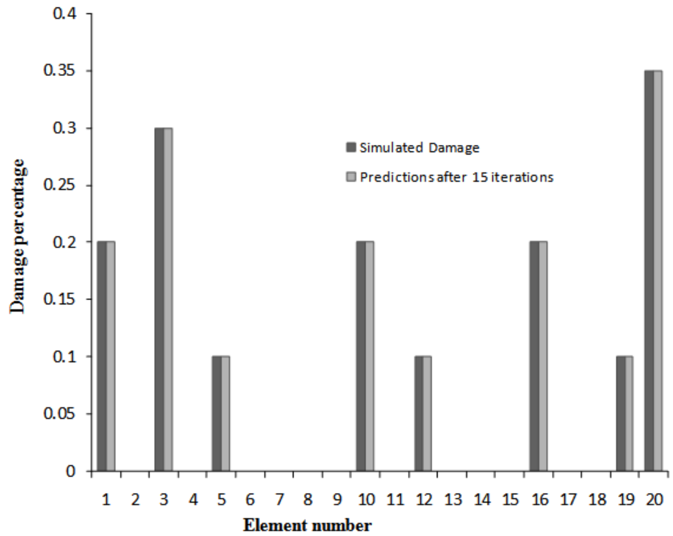

Fig. 7: The results for the third scenario applyied on the multi-span beam.

4. Conclusion:

An iterative method has been presented for damage detection on beam type structures using the natural frequency shifts. The natural frequency shifts are used for detecting the damaged elements and the global damage extent in each element. It is assumed that the damage in each element results in degradation of the element stiffness which affects the overall stiffness of the structure. The stiffness degradation of each element, actually as an element level damage index, is updated during the proposed iterative procedure. Several examples containing different damage scenarios for two-span and multi-span continuous beams have been studied. Based on the results obtained the following conclusions may be drawn:

The procedure is stable and can analyze different damage cases for continuous beams with arbitrary number of spans.

The procedure can predict the damaged and intact elements exactly. The latter feature is far from the capability of some of the existing damage detection methods.

By increasing the number of spans and the complexity of the boundary conditions, more iteration may be needed but the converged results are achieved.

Interestingly, by using more elements along the beam, it is possible to detect the damage locations precisely.

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