

Modeling of Abrasive Waterjet Turning

¹Mehdi Zohoor, ²Iman Zohourkari

¹Assistant Professor, Faculty of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran.

²PhD Student, Faculty of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran.

Abstract: In this article, an erosion-based model presented for abrasive waterjet (AWJ) turning process. Hashish erosion model was modified and applied to estimate the volume of material removed by abrasive particles. In addition to that, the material removal rate, the current variation of diameter and final diameter of workpiece were calculated. In the proposed model, the following variable parameters were considered: The continuous change in local impact angle due to change in workpiece diameter, axial traverse speed of the nozzle, the abrasive particle roundness and density. Finally, the theoretical results have been compared with the experimental results and found a good correlation between them.

Key words: Waterjet turning - Erosion model - Material removal rate - Abrasive particle - Flow stress.

INTRODUCTION

Abrasive waterjet (AWJ) is a well-recognized technology for cutting variety of materials such as composites and super alloys (Wang and Guo, 2002; Arola and Ramulu, 1993). Recently, AWJ technique has been used in milling (Ojmertz, 1997) and turning operations (Hashish, 1987). In turning operation, the workpiece rotates while the water jet is traversing in axial and radial directions to produce the required geometry. Some reports have been published about the estimation of volume removal rate (Ansari, A.I. and Hashish) surface finish control (Hashish, 2001), flow visualization study (Ansari, 1992), and modeling (Manu, 2009) of the AWJ turning process.

Unlike conventional turning, AWJ turning is less sensitive to the workpiece shape. This process is involved with low cutting forces, so it is independent of length-to-diameter ratio of the workpiece and therefore enables the machining process to turn long parts with small diameter and close tolerances. This process is ideally suitable for machining materials with low machinability such as ceramics, composites, glass, etc. (Kovacevic, 1997). Useful studies have been done based on experimental investigations. From a visualization study, Hashish reported that the material removal takes place on the face of the workpiece rather than on the circumference (Ansari, 1992). Ansari and Hashish conducted experimental investigations to study various parameters on the volume of material removed in AWJ turning (Ansari, 1991). The results show that the volume of material removed in AWJ turning is similar to that achieved in AWJ cutting. Zhong and Han (Zhong, 2002) studied the influence of variation in process parameters on turning of glass with abrasive waterjet. They reported that lower traverse speed of jet and higher rotational speed of workpiece resulted in lower waviness and surface roughness for turned specimen. Many attempts have been conducted to model AWJ cutting of ductile metallic materials and brittle ceramic materials. However, attempts on modeling of AWJ turning process are very much limited. Zeng *et al.*, (1994) presented a semi-empirical model to predict radius reduction in turning using a regression model. Based on an empirical approach presented by Henning (Henning, 1999), the material removal in AWJ turning process is assumed to be the accumulation of the volume removed by a single particle impacts on the surface of the workpiece.

Empirical models do not explain the mechanics of the process. In addition, to determine the exponents and the coefficients of the empirical models, the regression analysis should be undergone. An analytical model was suggested by Ansari and Hashish (ASME) that relates the volume sweep rate to material removal rate. This model could predict the final diameter of specimen in various set of AWJ turning process parameters.

Corresponding Author: Mehdi Zohoor, Assistant Professor, Faculty of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran.
E-mail: mzohoor@kntu.ac.ir

Hashish modified his linear AWJ cutting model for AWJ turning (Hashish, 1984). He considered that material is removed from the surface of the rotating workpiece and assumed that the total depth of cut consists of cutting-wear depth and deformation-wear depth. The cutting-wear depth for shallow impact angle zone was estimated by Finnie's theory of erosion (Finnie, 1960) and the deformation-wear depth was estimated by Bitter's theory of erosion (Bitter, 1963). In spite of continuous change of impact angle during the diameter reduction of the workpiece, Hashish's analytical model does not consider the changes in impact angle. A different approach considering the varying local impact angle presented to predict the final diameter by Manu and Babu (2009). They applied Finnie's theory of erosion to model AWJ turning of ductile materials. However, their model is not able to predict accurate final diameter in various traverse speeds (Zohourkari, 2010). Moreover, at angles near to zero (when the impact angle is very low) their model predicts higher volume of removed material. By applying Hashish erosion model, Zohourkari and Zohoor presented a qualitative study of modeling of AWJ turning (Zohourkari, 2010). The objective of the present work is to develop a comprehensive model for AWJ turning of cylindrical parts subjected to various traverse speeds which can predict the volume of material removed, the material removal rate, the current variation of diameter and final diameter of workpiece. The distinctively proposed model considers the continuous change in local impact angle due to change in workpiece diameter, axial traverse speed of the jet, the abrasive particle roundness and density.

Mechanism of Awj Turning:

In abrasive waterjet turning, it is assumed that the waterjet with a velocity of V_w strikes the surface of the rotating workpiece with an initial diameter of D at a speed of N revolutions per minute. Then the erosion is started and the material removal takes place on the circumference of the rotating workpiece. The material removal is extended by axial travelling of jet (Figure 1). The distance between jet centerline and the specimen centerline is termed as the radial position of jet and is denoted by x . The angle α is the local impact angle that the jet makes with the tangent of surface at point of impact (Figure 2). Where, α can be computed as:

$$\alpha = \cos^{-1} \left(\frac{2x}{D} \right) \tag{1}$$

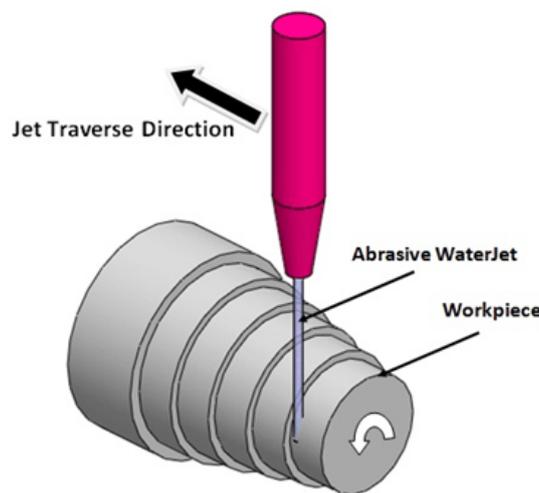


Fig. 1: Process of abrasive waterjet turning.

It can be assumed that AWJ turning is approximately equivalent to the impact of an inclined jet to a flat surface which moving with a velocity equal to the surface linear velocity of the rotating workpiece. The methodology of AWJ turning involves estimating the volume of material removed by employing suitable erosion model. The scope of the presented work is limited to AWJ turning of ductile materials using modified Hashish erosion model. The workpiece material considered is aluminum 6063-T6.

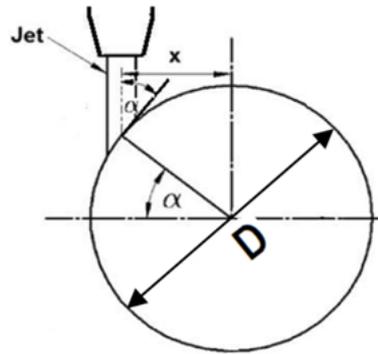


Fig. 2: Jet impact angle (α) on circumference of workpiece.

Modeling of Awj Turning:

Calculation of Pure Waterjet Velocity:

When high pressurized water exits through an orifice, due to the conversion of the hydraulic energy into kinetic energy, a high speed waterjet is generated. According to Bernouli's law, the following relationship is obtained (Zohoor, 2009):

$$P_{atm} + \frac{\rho_w}{2} V_{th}^2 + \rho_w g h_1 = P + \frac{\rho_w}{2} V_{pipe}^2 + \rho_w g h_2 \quad (2)$$

Where, P_{atm} is the atmospheric pressure, ρ_w is the water density which is taken as 1000 kg/m^3 , V_{pipe} is the velocity before exiting the orifice, V_{th} is the theoretical velocity of the water after exiting the orifice, P is the water pressure before the orifice, g is the gravity acceleration, h_1 and h_2 are the height of two points after and before the orifice respectively.

Assume: $h_1 - h_2 \approx 0$, $P \gg P_{atm}$ and $V_{th} \gg V_{pipe}$.

The approximate velocity of the exited waterjet is:

$$V_{th} = \sqrt{\frac{2P}{\rho_w}} \quad (3)$$

Momentum losses occur due to three parameters which are: (I) wall friction, (II) fluid flow disturbances, and (III) water compressibility. To modify Eq. (3), a factor C_v is added to the equation. Therefore, the output water velocity " V_w " becomes:

$$V_w = C_v \sqrt{\frac{2P}{\rho_w}} \quad (4)$$

Calculation of Abrasive Particles Velocity:

The abrasive particle acceleration in an abrasive waterjet is a matter of momentum transfer from the high velocity water to the abrasive particles injected at low velocities which sucks air into the mixing chamber. Using a momentum balance expression (Momber, 1998) as follows:

$$\dot{m}_a V_{a0} + \dot{m}_w V_w + \dot{m}_L V_L = (\dot{m}_a + \dot{m}_L + \dot{m}_w) V_a \quad (5)$$

Where, \dot{m}_a , \dot{m}_w and \dot{m}_L are the mass flow rates for the abrasives, water and air respectively. V_{a0} and V_L are the input velocities of abrasives and air respectively. V_a , is the output velocity of the abrasive waterjet.

Neglecting the amount of air ($\dot{m}_L \approx 0$) and considering $V_{\omega} \ll V_w$, Eq. (5), is simplified as follows:

$$\dot{m}_w V_w = (\dot{m}_a + \dot{m}_w) V_a \quad (6)$$

A moment transfer efficiency denoted by φ is added for the losses encountered during the process. Therefore, the velocity of abrasive particles is given by:

$$V_a = \varphi \frac{V_w}{[1 + (\dot{m}_a / \dot{m}_w)]} \quad (7)$$

The mass flow rate of water \dot{m}_w is estimated by using the expression relating the diameter of waterjet orifice (d_o), waterjet velocity (V_w), water density (ρ_w) and velocity coefficient of orifice (C_d) as (Momber, 1998):

$$\dot{m}_w = C_d \frac{\pi}{4} d_o^2 V_w \rho \quad (8)$$

The typical values of C_v , C_d and φ are found to be 0.98, 0.7 and 0.8, respectively (Momber, 1998).

Determination of Workpiece Diameter after Each Revolution:

The volume of material removed during each revolution, can be estimated from the the volume, determined from the geometry of the removed chip in the form of rectangular strip. That is, the product of three dimensions of a rectangular strip which are: current workpiece circumference (πD_k), jet diameter (d_j) and radial depth of penetration (Δr_k) during one revolution.

$$V_{rectangle} = \Delta r_k \pi D_k d_j \quad (9)$$

Where, D_k is the workpiece diameter at the beginning of the K^{th} revolution.

The volume of material removed at K^{th} revolution (Q_k) which can be obtained by an erosion model (e.g. Hashish erosion model (Hashish, 1984), is equal to the volume determined by Eq. (9). Therefore, the depth of cut (radial depth of penetration, (Δr_k) for the K^{th} revolution is:

$$\Delta r_k = \frac{Q_k}{\pi D_k d_j} \quad (10)$$

Finally, The Workpiece diameter after K^{th} revolution (D_{k+1}) can be obtained as:

$$D_{k+1} = D_k - 2\Delta r_k \quad (11)$$

The above procedure (Eqs. (1)-(11)) should be repeated until the impact angle tends to zero. This happens when the final diameter is achieved at each point on the workpiece periphery. The maximum value of k (K_{max}) is equal to the number of revolutions required to get the final diameters which is denoted by ($n_{p-erosion}$).

Erosion Models:

Finnie's Theory of Erosion:

Finnie was the first scientist to derive a single-particle erosive cutting model. The model assumes a hard particle with velocity V_a impacting a surface at an angle α . The material of the surface is assumed to be rigid - plastic. The final expression and boundary conditions for the volume of material removed from the workpiece due to the impact of a single particle can be obtained from Eq. (12) (Finnie, 1960).

$$Q = \begin{cases} \frac{mV^2}{\psi p K} \left[\sin(2\alpha) - \frac{6}{K} \sin^2(\alpha) \right], & \tan\alpha \leq \frac{K}{6} \\ \frac{mV^2}{\psi p K} \left[\frac{K \cos^2(\alpha)}{6} \right], & \tan\alpha > \frac{K}{6} \end{cases} \quad (12)$$

Where, m is mass of a single particle, α is the impact angle, K is the ratio of vertical to horizontal force components, and ψ is the ratio of the depth of contact l to the depth of the cut y_t as shown in Figure 3. p is the flow stress of the eroded workpiece material and Q is the total volume of target material removed. The total volume removed by multiple particles having a total mass M can be obtained from Eq. (13) (Finnie, 1960).

$$Q = \begin{cases} c \frac{MV_a^2}{\psi p K} \left[\sin(2\alpha) - \frac{6}{K} \sin^2(\alpha) \right], & \tan\alpha \leq \frac{K}{6} \\ c \frac{MV_a^2}{\psi p K} \left[\frac{k \cos^2(\alpha)}{6} \right], & \tan\alpha > \frac{K}{6} \end{cases} \quad (13)$$

The constant c is used to compensate for the particles that do not follow the ideal model (some particles impact with each other, or fracture during erosion). Finnie model (Finnie, 1960) for erosion is only valid for ductile materials, and does not include any brittle fracture behavior of the material.

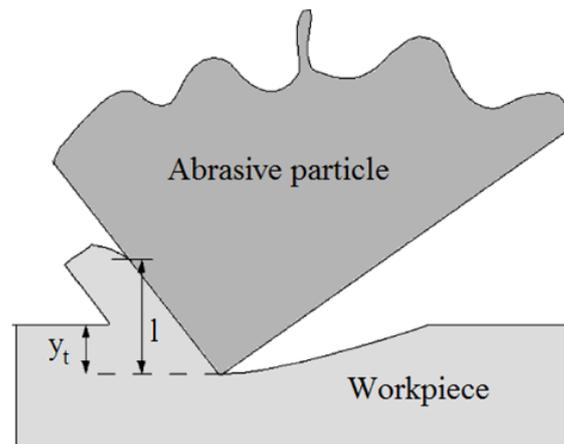


Fig. 3: Depth of cut and length of contact.

Modification of Hashish Model for Erosion:

Hashish (1984) presented an erosion model to include the effect of the particle shape. He also modified the velocity exponent predicted by Finnie as shown in Eq. (14).

$$Q = \frac{7}{\pi} \frac{m}{\rho_p} \left(\frac{V_a}{C_k} \right)^{2.5} \sin(2\alpha) \sqrt{\sin\alpha} \quad (14)$$

The value of C_k can be computed from Eq. (15):

$$C_k = \sqrt{\frac{3pR_f^{3/5}}{\rho_p}} \quad (15)$$

Where, R_f is the particle roundness factor and ρ_p is the abrasive particle density given in table 2.

To determine Q, when multiple particles applied with mass M in turning process, Eq. (14) must be modified as expression 16:

$$Q = \frac{7}{\pi} q \frac{M}{\rho_p} \left(\frac{V_a}{C_k} \right)^{2.5} \sin(2\alpha) \sqrt{\sin\alpha} \quad (16)$$

Where, q is a constant determined by experiment (here assumed equal to 0.2).

Number of Revolutions to Achieve Desired Diameter at Each Point:

The jet moves along the axial direction of the part so as to extend the cutting action along the length of the part. To prevent thread cutting, the axial distance moved by the jet during one revolution of the workpiece (feed rate) must be a fraction of the jet diameter (less than or equal to jet diameter). This causes in the workpiece surface being subjected to a definite number of cutting passes at each point during the turning operation.

Consider a waterjet of diameter d_j passing over strip of infinitesimal width (dx) with the traverse speed of u . The jet has to travel an axial distance of $(d_j + dx)$ to pass the strip width completely. Hence the time taken to cover this distance is given by Δt and is calculated as follows:

$$\Delta t = \frac{d_j + dx}{u} \quad (17)$$

Number of workpiece revolutions (n_p) at the time of travelling waterjet (Δt) a distance equal to $(d_j + dx)$ is evaluated by Eq. (18):

$$n_p = N \Delta t = N \left(\frac{d_j + dx}{u} \right) \quad (18)$$

Therefore, number of cutting pass at each point (number of jet contact at each point) (n_p) can be calculated as follows (Manu and Babu, 2009):

$$n_p = N \frac{d_j}{u}, \quad (\text{when } dx \rightarrow 0) \quad (19)$$

In order to increase the efficiency of the turning process, it is necessary that, the value of (n_p) obtained from Eq. (19), must be equal to ($n_{p-erosion}$) which was suggested to get final diameter in section 3.3.

Theoretical and Experimental Parameters:

In order to check the accuracy of the proposed model, aluminum cylindrical stepped bar (6063-T6) as shown in Figure 4 was considered as desired shape for the workpiece. The Process parameters and abrasive properties used for the proposed model are listed in Tables 1 and 2.

Two experimental setups for process parameters were selected. Experimental Setup1 was used to evaluate the accuracy of prediction of final diameter and experimental setup 2 for checking the accuracy of prediction of current variation of workpiece diameter.

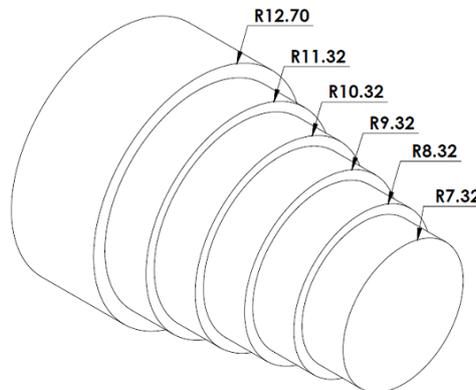


Fig. 4: Final geometry of desired workpiece.

Table 1: AWJ turning parameters.

Experimental Parameter	Level	
	Setup 1	Setup 2
Pressure,(MPa)	250	275
Mixing tube diameter, (mm)	0.76	1.11
Orifice diameter, (mm)	0.25	0.33
Abrasive mass flow rate, g/s	5	7.7
Rotational speed, rpm	200	106.66
Axial traverse speed, mm/min	2, 2.5, 10, 20	3.81

Table 2: Abrasive particle properties.

Abrasive	Garnet
Mesh size	80, 60
Density (kg/m ³)	4000
Roundness factor	0.4, 0.42

Theoretical and Experimental Results and Discussion:

The flow stress “p” is an important parameter which was determined using volumes of removed material from 12 tests (Manu and Babu, 2009) based on the Finnie erosion model (Eq. (13)) and modified Hashish erosion model (Eq. (16)). These results are given in Table 3. Then, the average flow stress for each model was calculated.

Table 3: Determination of flow stress.

Nozzle Diameter (mm)	Surface speed of workpiece, mm/min	Removed volume, mm ³	Jet contact time, s	Finnie's prediction of Flow stress, Mpa	Modified Hashish 's prediction of Flow stress, MPa
1.6	1000	587.91	21.38	1136.68	7567.91
	2000	474.04	11.3	744.67	5397.96
	3000	340.76	7.02	643.44	4803.19
	4000	186.64	3.93	657.7	4887.97
1.2	1000	661.49	17.85	843.09	5960.88
	2000	385.39	9.39	761.57	5493.27
	3000	288.08	5.37	582.85	4433.92
	4000	95.67	3.28	1071.42	7219.33
0.76	1000	673.36	27.69	1285.18	8359.51
	2000	454.79	12.4	852.04	6010.45
	3000	360.72	11.54	999.32	6830.42
	4000	290.74	8.28	889.98	6223.56
Average				874	6099

Experimental setup 1 was used to validate the proposed model, the results (final diameters) obtained by the proposed model, Manu model and experiment (Manu and Babu, 2009) were compared as shown in tables 4-8. Comparison of the quantitative results (tables 4-7) and graphical results (table 8) indicated a good correlation between the presented model and experimental results. Therefore, it can be concluded that, the presented model can predicts the final diameters in various traverse speed successfully.

Table 4: Prediction of final diameter for traverse speed u=2 mm/s.

Initial diameter	Target diameter	Manu model	Presented model	Experiment(Manu and Babu, 2009)
25.40	22.640	22.631	22.639	23.636
25.40	20.640	20.557	20.639	21.590
25.40	18.640	18.527	18.639	19.318
25.40	16.640	16.331	16.639	17.045
25.40	14.640	13.977	14.637	14.772

Table 5: Prediction of final diameter for traverse speed u=2.5 mm/s.

Initial diameter	Target diameter	Manu model	Presented model	Experiment (Manu and Babu, 2009)
25.40	22.640	22.631	22.639	23.234
25.40	20.640	20.557	20.639	21.265
25.40	18.640	18.527	18.639	18.902
25.40	16.640	16.331	16.639	16.933
25.40	14.640	13.977	14.637	14.570

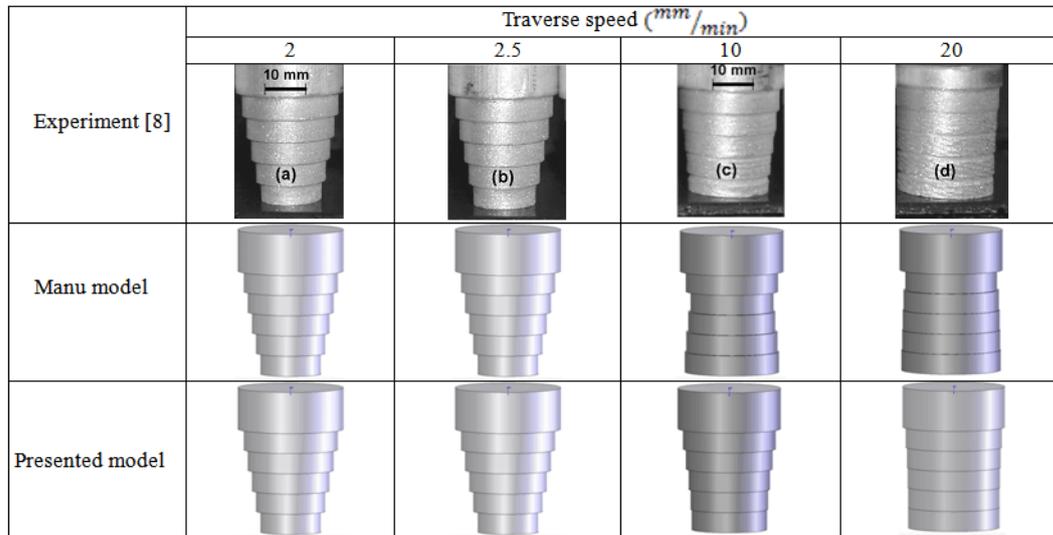
Table 6: Prediction of final diameter for traverse speed $u=10$ mm/s.

Initial diameter	Target diameter	Manu model	Presented model	Experiment (Manu and Babu, 2009)
25.40	22.640	22.631	22.903	23.470
25.40	20.640	20.557	21.367	21.936
25.40	18.640	21.313	20.066	20.781
25.40	16.640	22.319	19.099	19.627
25.40	14.640	23.118	18.600	18.857

Table 7: Prediction of final diameter for traverse speed $u=20$ mm/s.

Initial diameter	Target diameter	Manu model	Presented model	Experiment (Manu and Babu, 2009)
25.40	22.640	22.631	23.329	24.630
25.40	20.640	23.519	23.329	24.053
25.40	18.640	23.894	22.924	23.475
25.40	16.640	24.219	22.719	23.090
25.40	14.640	24.498	22.691	22.706

Table 8: Comparison of the theoretical and experimental results.



Experimental setup 2 was used to determine the current variation of workpiece diameter. The Presented model, Manu model and experimental results, are demonstrated in figure 5.

It shows that at the start of turning, Manu model results are closer to the experimental data. But, after passing the time (17 seconds and farther), the presented model predicts the current variation of diameter better than Manu model.

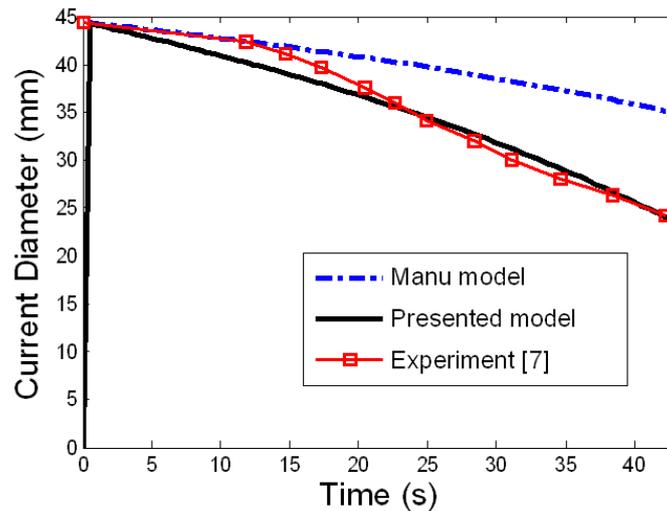


Fig. 5: Prediction of diameter change during the process.

The workpiece diameter varies during the turning process and the impact angle changes due to the variation of workpiece diameter. So that, the material removal rate (MRR) would not remain constant during the process.

In order to calculate the average of “MRR” in duration called cycle time equal to 40 second. First, “MRR” was determined in 8 time steps by using Eq. (20) (each time step was assumed 5 seconds). Then the average of “MRR” was determined based on these 8 steps as shown in table 9.

$$MRR = \frac{\pi}{4} (D_{i-1}^2 - D_i^2) \times u \quad , \quad (i = 1 - 9) \quad (20)$$

Where, “u” is the traverse speed of working-head along the length of the workpiece.

Table 9: Comparison of material removal rate estimated by the models.

Time (s)	Current diameter “ Experiment (Ansari and Hashish, 1992)” (mm)	Current diameter “Manu model” (mm)	Current diameter “Proposed model” (mm)
0	44.35	44.35	44.35
5	43.93	43.61	42.72
10	42.87	42.72	40.87
15	40.9	41.78	38.86
20	37.93	40.76	36.68
25	34.02	39.66	34.56
30	30.79	38.61	31.61
35	27.81	37.15	28.96
40	25.46	35.67	25.24
Average MRR (mm ³ /s)	8.2210	4.3300	8.2905
Traverse speed = u = 3.81 mm/min.			

Comparison of the theoretical and experimental results in table 9, shows a good agreement between them. Therefore, the presented model has a good accuracy for estimation of “MRR” when compared to the experimental results.

Conclusions:

This paper proposed a mathematical model for abrasive waterjet turning. The theoretical results have been compared with the experimental results and found a good correlation between them. In contrast with the other studies in this field, the following important points would be extracted from the results:

1. The proposed model can predict the current variation of diameter of the specimen.
2. The presented model can calculate the final geometry of the near-net-shape workpiece, in various traverse speeds, successfully.
3. Different flow stresses obtained by Finnie and modified Hashish erosion models were used as an empirical constant to calculate the values of material removal volume. By applying the material removal volume obtained by Finnie and modified Hashish erosion models, the values of material removal rate were calculated.

REFERENCES

Ansari, A.I., 1991. "A study on turning with abrasive waterjets," Ph.D. Dissertation, Michigan technological University.

Arola, D. and M. Ramulu, 1993. "Mechanism of material removal in abrasive waterjet machining of common aerospace materials", *Proceeding of 7th American Water Jet Conf.*, Seattle, USA.

Ansari, A.I. and M. Hashish, "Effect of abrasive waterjet parameters on volume removal trends in turning," *ASME Journal of Engineering for Industry*, 117(4): 475-484.

Ansari, A.I., M. Hashish and M.M. Ohadi, 1992, "Flow visualization study of the macromechanics of abrasive waterjet turning," *Experimental Mechanics*, 32(4): 358-364.

Bitter, J.G.A., 1963. "A study of erosion phenomena- part I," *Wear*, 6(1): 5-21.

Finnie, I., 1960. "Erosion of surfaces by solid particles," *Wear*, 3(2): 87-103.

Hashish, M., 1987. "Turning with abrasive waterjets—a first investigation," *ASME Journal of Engineering for Industry.*, 109(4): 281-290.

Henning, A., 1999. "Modeling of turning operation for abrasive waterjets" *Proceeding of 10th American Waterjet Conference*, Houston, Texas.

Hashish, M., 1984. "A modeling study of metal cutting with abrasive waterjets," *ASME Journal of Engineering Materials and Technology*, 106(1): 88-100.

Hashish, M., 2001. "Macro characteristics of AWJ turned surfaces, *Proceeding of 2001 WJTA American Waterjet Conf.*, Minneapolis, Minnesota.

Kovacevic, R., M. Hashish, R. Mohan, M. Ramulu, T.J. Kim and E.S. Geskin, 1997. "State of the art of research and development in abrasive waterjet machining," *ASME Journal of manufacturing Science. and Engineering*, 119: 776-785.

Momber, A.W., R. Kovacevic, 1998. *Principles of Abrasive WaterJet Machining*, London, Springer-Verlag.

Manu, R. and N.R. Babu, 2009. "An erosion-based model for abrasive waterjet turning of ductile materials," *Wear*, 266: 1091-1097.

Ojmertz, C., 1997. "A study on abrasive waterjet milling", Ph.D. Dissertation, Chalmers University of Technology.

Wang, J. and D.M. Guo, 2002, "A predictive depth of penetration model for abrasive waterjet cutting of polymer matrix composites," *Journal of Materials Processing Technology*, 121(2-3): 390-394.

Zhong, Z.W. and Z.Z. Han, 2002. "Turning of glass with abrasive waterjet," *Material and Manufacturing Processes*, 17(3): 330-349.

Zeng, J., S. Wu and Kim, 1994. "Development of a parameter prediction model for abrasive waterjet turning," *Proceeding of 12th International Conference on Jet Cutting Technology*, Rouen, France, 1994.

Zohourkari, I. and M. Zohoor, 2010. "An erosion-based modeling of abrasive waterjet turning," *Proceedings of the ASME 2010 10th Biennial Conference on Engineering Systems Design and Analysis, ESDA 2010*, July 12-14, Istanbul, Turkey.

Zohoor, M., 2009. *Automation and Manufacturing Processes*, Tehran, K.N. Toosi University of Technology Publisher. (In Persian).