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Fault Analysis in Unbalanced and Unsymmetrical Distribution Systems

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Abstract: The short-circuit currents are very important quantity affecting the design of bus systems, grounding systems, circuit breakers, substation apparatus, rotating machines and in fact, almost all the aspects of distribution system design. Among those applications, a robust and efficient short-circuit-analysis program is very important for planning and operation of systems. In this paper, a fast and easy programmable short circuit analysis method for unbalanced distribution systems is proposed. Two relationship matrices, the bus-current-injection-to-branch-current matrix are used to represent the special topological characteristics of distribution networks. The proposed short-circuit analysis method is developed from these two matrices and can be used to solve the various types of unsymmetrical faults. Therefore, a computer program by Matlab software has been developed to calculate the short circuit current of unbalanced distribution networks. Simulation results obtained using the proposed technique and a fictitious network (similar to Iran distribution networks) will be presented.

Key word: Unbalanced distribution networks, Unsymmetrical faults, Faults currents, Overhead lines impedance.

INTRODUCTION

In recent years, by expanding distribution networks gradually, it is inevitable that, the occurrence possibility of short circuit event, picked up moderately with defined consuming costs.

Identifying the reason of short-circuit events and intensity of plausible(likely)currents, at least will help us to select protective devices and different network elements which are much more invulnerable across undesirable effects of short circuit current. In order to, reducing losses, balancing loads and supplying the network, network recovering applied incessantly to the distributed systems. By any variation occurred in network structure, protective devices required re-adjusting assistance with determining the short-circuit current of new network.

In the past, by the means of calculating the distributed networks fault currents for both symmetrical and unsymmetrical faults, single-phased and multi employed [*Roy. L., 1979*]-[*Brandwajn V, 1985*]. Regarding to unbalancing loads and unsymmetrical in distribution networks, applying these methods is undesirable and bring exaggerate errors and tolerances.

Recently, in addition to employing compensation method, load distribution problem deal with calculating the short-circuit current is convenient [*Chen, T.-H. 1992*]-[Gross, G., 1982]. Nevertheless, the speed of above method will increased by the assistance of corresponded loads and network utilities [*Karen N. Miu*.]. In reference[*X. Zhang, 1995*], the determination of short circuit current with using hybrid-compensation method displayed and it is used for computing the fault current in more than three-phase networks [*R.M. Ciric, 2005*].

Mr.Teng, introduced two new relationship matrices, by the name of BIBC (Bus current Injection to Branch Current)and BCBV(Branch Current to Bus Voltage) using in calculations of both load distribution and short-circuit event, and that is basis of the method used in this paper[J. Teng, 2005]- [*Teng, J.H., 2003*]. Initially, in this paper virtual line model regarding to Carson equations presented, include the consideration of the types and different formation lines for estimating network fault current. Then, more applicable and simple algorithm, in order to develop two matrices, introduced. The next section, focused on application of the matrices in network currents. Although, the three-phase faults towards each other relinquished in variety of papers, in this section, it is the considerable faults. Finally, by considering assumed the network coincided to internal distribution networks (ground line, line formation and the quality), the current of unsymmetrical faults in variation network points are investigating as this paper shown.

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Three-phase model in Distribution Network with Earth Return (DNER):

In order to survey about the virtual three- phase model in DNER, in addition to self-coupling, mutual-coupling between three phases and the effect of ground wire should consider. Figure 1 demonstrates a sample of three-phase line with the line-to-ground.

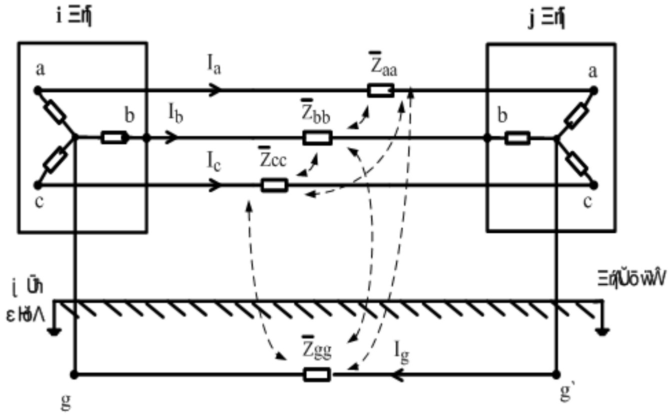

Fig. 1: The formation of three-phase line with the line-to-ground.

This paper used Carson equations to survey about impedances of the DNER three-phase circuit. To assess these equations, first the single-phase mode is designates and by enhance it, the impedance of three-phase circuit will described [*Rade M. 2004*].

Figure 2 shows the model of single-phase line including Carson equations. In this model the line Carson equation for one meter, side by side to ground surface and with I_a is displays.

The return of this line is the assuming line (*g-g*`) side by side to air-line. Crossing current of assuming line is equal to crossing current of line with inverse direction. Also GMR of assumption line is one meter and the distance between air-line (D_{ag}) and it calculate, based on meter. Therefore, according to voltage low and circuit in figure 2 we have:

$$\begin{bmatrix} V_{aa'} \\ V_{gg'} \end{bmatrix} = \begin{bmatrix} V_a - V_{a'} \\ V_g - V_{g'} \end{bmatrix} = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ag} \\ \bar{Z}_{ag} & \bar{Z}_{gg} \end{bmatrix} \begin{bmatrix} I_a \\ -I_a \end{bmatrix} \tag{1}$$

The value of $V_{g'}, V_g, V_{a'}, V_a$ is accord to ground as the reference (v_g=0). By placing this two formulates:

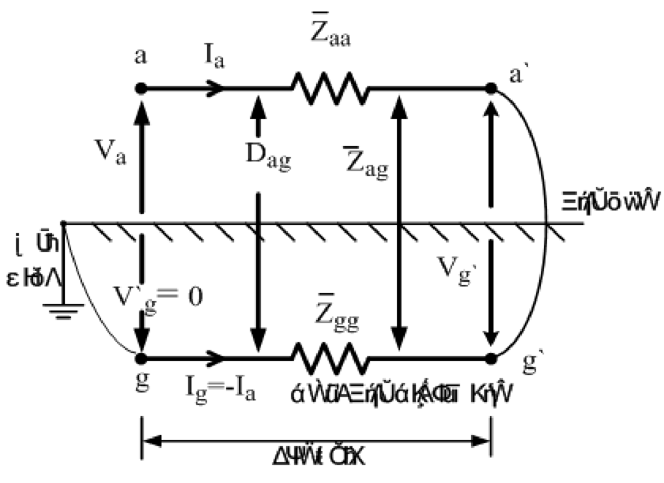

Fig. 2: Carson Line Model:

$$V_a = \left(\bar{Z}_{aa} + \bar{Z}_{gg} - 2\bar{Z}_{ag}\right) I_a = Z_{aa} I_a \tag{2}$$

Therefore we have:

$$Z_{aa} \cong \overline{Z}_{aa} + \overline{Z}_{gg} - 2\overline{Z}_{ag} \qquad (3)$$

In this formulate $\overline{Z}_{aa}$ is the line self-impedance and $\left(Z_{gg} - 2\overline{Z}_{ag}\right)$ is the effect of ground on it, which defined as correction section.

Regarding to designated assumptions for ground or neutral wire, the correction part formulated as bellow:

$$\overline{Z}_{gg} - 2\overline{Z}_{ag} = \pi^2 \times 10^{-4} f - j0.0386.8\pi \times 10^{-4}.f + j4\pi \times 10^{-4}.f \times Ln\frac{2}{5.6198 \times 10^{-3}} + j4\pi f \times 10^{-4} Ln\frac{\sqrt{\rho / f}}{h_a} \qquad (4)$$

Which it has:
f = Power System Frequency.
h_a = the height of *a-a*` from ground.
ρ = The special resistance of ground (usually equal to 100 ohm-meter).

Considering valid values of the right side of equation 4, it can be seen that all these equations dependent to frequency. Also, in this equation, first three parts of right side, related to self-impedance of ground and the other part contributed to mutual impedance of line and ground. Therefore, these impedances have:

$$\overline{Z}_{gg} = \pi^2 \times 10^{-4}.f - j0.0386.8\pi \times 10^{-4}.f + j4\pi \times 10^{-4} f.Ln\frac{2}{5.6198 \times 10^{-3}} \qquad (5)$$

$$\overline{Z}_{ag} = j2\pi f \times 10^{-4} Ln\frac{h_a}{\sqrt{\rho / f}} \qquad (6)$$

By expressed definitions, for a three-phase line with return earth wire, impedance matrix formulated as equation(7).

In this matrix, impedances related to ground are calculated by equations (5) and (6). Also the 3*3 sub matrix, consists of self and mutual impedances between lines a, b and c which achieved by (8) and (9).

$$\left[Z_{line}\right] = \begin{bmatrix} \overline{Z}_{aa} & \overline{Z}_{ab} & \overline{Z}_{ac} & \overline{Z}_{ag} \\ \overline{Z}_{ba} & \overline{Z}_{bb} & \overline{Z}_{bc} & \overline{Z}_{bg} \\ \overline{Z}_{ca} & \overline{Z}_{cb} & \overline{Z}_{cc} & \overline{Z}_{cg} \\ \overline{Z}_{ga} & \overline{Z}_{gb} & \overline{Z}_{gc} & \overline{Z}_{aa} \end{bmatrix} \qquad (7)$$

For self-impedances:

$$\overline{Z}_{aa} = r_a + j4\pi \times 10^{-4} f.Ln\left(\frac{2h_a}{GMR_a}\right) \Omega/km \qquad (8)$$

And mutual-impedance:

$$\overline{Z}_{ab} = j4\pi.10^{-4} f.Ln\left(\frac{\sqrt{d_{ab}{}^2 + \left(h_a + h_b\right)^2}}{\sqrt{d_{ab}{}^2 + \left(h_a - h_b\right)^2}}\right)\Omega/km \qquad (9)$$

That:

r_a = the resistance of phase wire a in a length unit $\left(\Omega/km\right)$.

h_b, h_a = the height of a and b wires based on meter.

d_{ab} = the horizontal distance between a and b wires based on meter.

As it shows, 4*4 matrix for three-phase line with return earth wire, completed. In practical mode, the initial matrix should reduce to 3*3 form of matrix. This 3*3 matrix, extracts from correction ground part in equation 3.

By considering described equations for line impedances, it can be extract that by applying required variations to the height of wire and vertical distances among them, the lines impedances according to different formations are calculated.

Introduction and Establishment of BIBC and BCBV Matrices:

In other past usual methods, networks were surveyed by impedance and admittance matrices. In contrast, the work required to establish an impedance matrix is much greater than that for the admittance matrix since the impedance matrix has more information. In this paper, regarding to radial or weakly meshed structures of distribution feeders, two relationship matrices, the bus-current-injection-to-branch-current matrix (BIBC) and branch-current-to-bus-voltage matrix (BCBV) were derived and used to observe and analysis the network. This section, firstly, defines and introduces these two matrices completely and then explains how they construct in the network.

Introduction to Matrices:

In order to have better understanding, construction and definitions of both matrices are displayed in the form of an example.

Figure 3. Demonstrates a single-phase mode distributed network with seven buses.

The injection of power to each bus, is converted to bus-current injection, by fallowing formula:

$$I_i = \left(\frac{p_i + jQ_i}{V_i}\right) \tag{10}$$

The relationship between bus-currents injection and branch currents can be determined by using above equation and Kirshhoff current law (KCL).

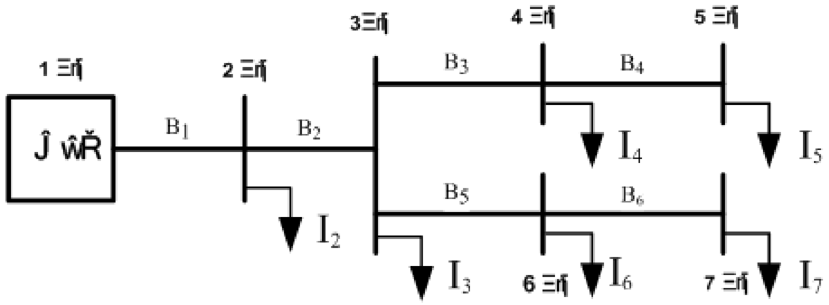

Fig. 3: The sample of a distributed network.

Thus, branch currents presents as function form of bus-currents injection. Considering, mentioned items for network in figure-3, branch currents B_2 and B_4 can be formulated base on bus-currents injection, as fallowing:

$$\begin{aligned} B_2 &= I_3 + I_4 + I_5 + I_6 + I_7 \\ B_4 &= I_5 \end{aligned} \tag{11}$$

By extending equation (11) for all buses, the equation of bus-currents injections and branch currents can be formulated as bellow:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} \tag{12}$$

And, totally as:

$$[B] = [BIBC][I] \tag{13}$$

Refers to equation (3), the constant matrix BIBC is an upper-triangular matrix consists of zero and 1 entries. The contribution of bus voltages and bus currents in figure-3 circuit for buses number 2, 3 and 4, is defined as:

$$V_4 = V_3 - B_3 Z_{34} \tag{14}$$

$$V_3 = V_2 - B_2 Z_{23} \tag{15}$$

$$V_4 = V_3 - B_3 Z_{34} \tag{16}$$

In above equations, V_i presents the voltage of bus number i and Z_{ij} presents the impedances between *i* and *j* buses. With applying equations (14) and (15) to equation (16), then equation (17) formulated as:

$$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34} \tag{17}$$

According to equation (15), it is extract that voltage of each bus can be formed as a function based on branch currents, line parameters and reference voltage. With applying this method for all buses in figure-3 circuit, the relationship between bus voltages and branch currents modified as:

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} & Z_{67} \end{bmatrix} \tag{18}$$

And totally written as:

$$[\Delta V] = [BCBV][B] \tag{19}$$

Where *[BCBV]* is the relating matrix among bus voltages and branch currents.

The Building Algorithm of BCBV and BIBC:

Before displaying the algorithm, it is noticeable to mention a note about numbering method of network buses, for calculating these two matrices. Here, original numbering method used for buses and lines. Numbering starts from reference bus.

According to equation (13), the building algorithm BIBC can be developed as fallowing algorithm:

Step 1: for a distribution system consists of *m* branch and *n* bus, the dimension of BIBC matrix is $(m+1)\times n$.
Step 2: the ith bus column of the BIBC matrix repeated in jth bus column of it, when line B_k is located between bus i and bus j.
Step 3: previous procedure is repeated continuously, until a complete form of BIBC is achieved.
Also, regarding to equation (19), the building algorithm of BCBV is developed as:
Step 1: for a distribution network with m branch and n bus, the dimension of BCBV matrix presented as $(m+1)\times n$.
Step 2: *the determined* Thevinin impedance seen from initial part of feeder, located in the first rows of the matrix.
Step 3: the ith row of the BIBC matrix repeated in jth row of it, when line B_k is located between bus i and bus j.
Step 4: previous procedure is repeated requisite, to complete the BIBC matrix.

Above algorithm, also can be used in multi-phase networks. Each phase is considered by the form of an independent network and these procedures are then applied. In a three-phase network, at first matrices construct for phase a, then b and at the end c, so BIBC and BCBV matrices will extracted from the combination of a, b and c. therefore, the numbering method for buses is similar to original numbering for single-phase mode and lines numbering form is line-to-line in extension of buses numbering.

Regarding to definition of BIBC, branch currents variations produced from network faults, can be calculated directly. Also considering BCBV matrix definition, it can be used for estimating bus voltage variations concerning branch current variations.

Unsymmetrical Fault Analysis:

The maximum faults exist in distribution networks, comes from unsymmetrical type of faults. Unsymmetrical faults consist of single-line-to-ground fault, double- line-to-ground fault, line-to-line fault and regarding to unbalanced load and Unsymmetrical parameters of distribution systems, three-phase fault and three-line-to-ground faults are mentioned in this group of faults. Among all faults, single-line-to-ground is the most convenient type of faults in network. Beside this, the power of switches is computed bases on three-line-to-ground results, which is the drastic type of fault and has the minimum probability of occurrence among them.

In the presented method, a combination of explained matrices in previous section boundary conditions, are used to survey about these faults. Figure 4 illustrates all types of line-to-ground faults and double-phase.

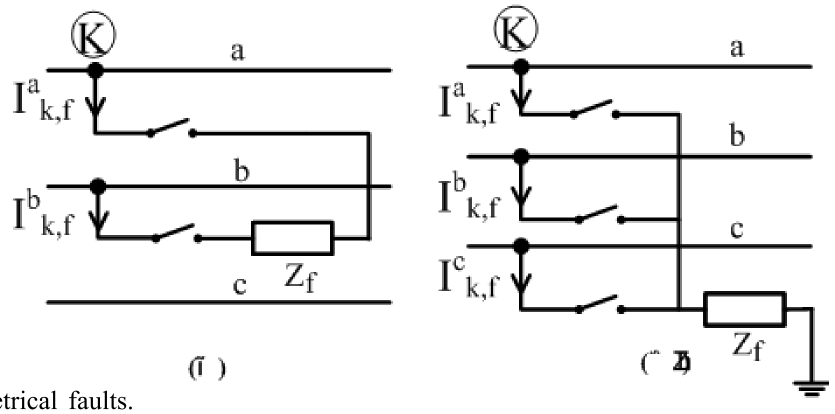

Fig. 4: Unsymmetrical faults.

Single-phase-to Ground:

Figure 4-a means that, when the faults with impedance of Z_f , occurs on phase a in *k* bus, then system has:

$$v_f^a = Z_{k,f} . I_{k,f}^a \tag{20}$$

$$v_f^a = Z_{k,f} . I_{k,f}^a \tag{21}$$

From above equations, $I_{k,f}^{c}$, $I_{k,f}^{b}$, $I_{k,f}^{a}$ are the phases current injection of *k*th bus and also I_{f}^{c}, I_{f}^{b}, I_{f}^{a} are the fault currents occurred in phases a, b and c, respectively. Refers to single phased fault and assumed worthless amount of loads current in comparison with fault current, the current only flows in phase a and other faults contain zero current. If the single fault occurs on phase a, the current $I_{k,f}^{a}$ of k bus flows directly to ground, where in conclusion reforms its voltage from $v_{k,0}^{a}$ into $v_{k,f}^{a}$. So for the recent variations in voltage, it should be:

$$\Delta v_{k,f}^{a} = v_{k,0}^{a} - Z_f I_{k,f}^{a} \tag{22}$$

Where $v_{k,f}^{a}, v_{k,0}^{a}$ are pre-fault and after-fault voltages, occurs on phase a and k bus. Then, by considering equations (13) and (20), it should be:

$$\left[B_f\right] = \left[BIBC\right]\left[\begin{matrix} 0 & ... & I_{k,f}^{a} & 0 & ... \end{matrix}\right]^T \tag{23}$$

Regarding to BIBC matrix, equation (23) van be rewritten as:

$$\left[B_f\right] = \left[BIBC_k^a\right] I_{k,f}^{a} \tag{24}$$

In this equation, $\left[BIBC_k^a\right]$ is a column vector of the BIBC matrix, based on bus and phase faults.

To compute the changes of bus voltages generated by the fault branch currents of the network and according to equation (19) and (24), it should be:

$$\left[\Delta v_f\right] = \left[BCBV\right]\left[BIBC_k^a\right] I_{k,f}^{a} \tag{25}$$

Thus, the variations of voltage of phase a, in bus where the fault occurs, can be expressed as:

$$\Delta v_{k,f}^{a} = \left[BCBV_k^a\right]\left[BIBC_k^a\right] I_{k,f}^{a} \tag{26}$$

Where, $\left[BCBV_k^a\right]$ is a row vector of $\left[BCBV\right]$ corresponding bus and phase faults. Considering the resulted voltage variations of the fault exists in equation (20), the fault current formulated as:

$$I_{k,f}^{a} = \left(\left[BCBv_k^a\right]\left[BIBC_k^a\right] + Z_f^{-1}\right)\left(v_{k,0}^{a}\right) = \left(\left[Z_{se}^{a}\right]\right)\left(v_{k,0}^{a}\right) \tag{27}$$

Where $\left[Z_{sc}^{a}\right]$, is a single dimension matrix (1*1) that extracts from multiplying BIBC and BCBV matrices, and related to phase and bus faults. This matrix is applicable for impedance seen from the fault point and achieving that, can simplify the calculations. Then, branch currents and bus voltage variations can be computed directly by equations (24) and (25).

The Line-to-Line Fault:

Corresponding figure-4-b, the boundary conditions describes as fallows while a line-to-line fault with impedance Z_f occurs between a and b phases of k bus:

$$v_{k,f}^{a} - v_{k,f}^{b} = Z_f I_{k,f}^{a} \tag{28}$$

$$v_{k,f}^{a} - v_{k,f}^{b} = Z_f I_{k,f}^{a} \tag{29}$$

Regarding to above equations, bus voltage variations after-fault can be expressed as:

$$\Delta v_{k,f}^{a} - \Delta v_{k,f}^{\ b} = \left(v_{k,0}^{a} - v_{k,0}^{b}\right) - Z_f I_{k,f}^{a} \tag{30}$$

Similar to pre-fault, the branch currents displays as:

$$\left[B\right] = \left[BIBC\right]\left[\ 0\ \ ...\ \ I_{k,f}^{a}\quad -I_{k,f}^{a}\quad 0\ \ ...\right]^{T} \tag{31}$$

And for calculating the bus voltage variations of the network, we have:

$$\left[\Delta v_f\right] = \left[BCBV\right]\left[BIBC_k^a BIBC_k^a\right]\begin{bmatrix} I_{k,f}^{a} \\ -I_{k,f}^{a} \end{bmatrix} \tag{32}$$

Thus, the equation of bus voltage variations, where involves the fault, can be written as:

$$\begin{bmatrix} \Delta v_{k,f}^{a} \\ \Delta v_{k,f}^{b} \end{bmatrix} = \begin{bmatrix} BCBv_k^a \\ BCBv_k^b \end{bmatrix}\begin{bmatrix} BIBC_k^a \\ BIBC_k^b \end{bmatrix}\begin{bmatrix} I_{k,f}^{a} \\ -I_{k,f}^{a} \end{bmatrix} = \begin{bmatrix} L_k^{aa}\ L_{\ k}^{ab} \\ L_k^{bb}\ L_k^{bb} \end{bmatrix}\begin{bmatrix} I_{k,f}^{a} \\ -I_{k,f}^{a} \end{bmatrix} \tag{33}$$

In which:

$$\begin{bmatrix} L_k^{aa} & L_k^{ab} \\ L_k^{ba} & L_k^{bb} \end{bmatrix} = \begin{bmatrix} BCBv_k^a BIBC_k^a & BCBv_k^a BIBC_k^b \\ BCBv_k^b BIBC_k^a & BCBv_k^b BIBC_k^b \end{bmatrix} \tag{34}$$

By considering that $L_k^{ba} = L_k^{ab}$ and substituting equation (33) into equation (27), the fault currents determined as fallowing:

$$I_{k,f}^{a} = \left[\left(L_k^{aa} + L_k^{bb} + ZL_k^{ab}\right)^{-1}\left|v_{i,0}^{a} - v_{i,0}^{b}\right|\right] \tag{35}$$

$$I_{k,f}^{a} = \left[\left(L_k^{aa} + L_k^{bb} + ZL_k^{ab}\right)^{-1}\left|v_{i,0}^{a} - v_{i,0}^{b}\right|\right] \tag{36}$$

After that, by applying equation (31) and (32), branch currents and bus voltages after-fault occurrence, can be computed easily.

The Double-line-to-ground Fault:

Regarding to, figure-4-a for a double-line-to-ground with impedance Z_f corresponding to a and b phases of k bus, we have:

$$v_{k,f}^{a} = v_{k,f}^{b} = Z_f\left(I_{k,f}^{a} + I_{k,f}^{b}\right) \tag{37}$$

$$v_{k,f}^{a} = v_{k,f}^{b} = Z_f\left(I_{k,f}^{a} + I_{k,f}^{b}\right) \tag{38}$$

The phase (line) voltage changes after fault, is expressed as:

$$\Delta v_{f,0}^{b} = v_{k,0}^{b} - Z_f \left(I_{k,f}^{a} + I_{k,f}^{b} \right) \tag{39}$$

$$\Delta v_{f,0}^{b} = v_{k,0}^{b} - Z_f \left(I_{k,f}^{a} + I_{k,f}^{b} \right) \tag{40}$$

Similar to previous modes, for branch currents:

$$\left[B_f \right] = \left[BIBC \right] \left[\begin{matrix} 0 & \ldots & I_{k,f}^{a} & I_{k,f}^{b} & 0 & \ldots \end{matrix} \right]^T \tag{41}$$

Also, in order to changing the bus voltage:

$$\left[\Delta v_f \right] = \left[BCBV \right] \left[BIBC_k^a BIBC_k^b \right] \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \end{bmatrix} \tag{42}$$

Then, by substituting equation (39) into both equations (40) and (42), fallowing equation extracts:

$$\begin{bmatrix} v_{k,0}^{a} - Z_f \left(I_{k,f}^{a} + I_{k,f}^{b} \right) \\ v_{k,0}^{b} - Z_f \left(I_{k,f}^{a} + I_{k,f}^{b} \right) \end{bmatrix} = \begin{bmatrix} BCBv_k^a \\ BCBv_k^b \end{bmatrix} \begin{bmatrix} BIBC_k^a \\ BIBC_k^b \end{bmatrix}^T \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \end{bmatrix} \tag{43}$$

Finally, fault current can be calculated as:

$$\begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \end{bmatrix} = \left(\begin{bmatrix} BCBv_k^a \\ BCBv_k^b \end{bmatrix} \begin{bmatrix} BIBC_k^a \\ BIBC_k^b \end{bmatrix}^T + \begin{bmatrix} Z_f & Z_f \\ Z_f & Z_f \end{bmatrix} \right)^{-1} \times \begin{bmatrix} v_{k,0}^{a} \\ v_{k,0}^{b} \end{bmatrix} = \left(\left[Z_{sc}^{ab} \right] \right)^{-1} \begin{bmatrix} \Delta v_{k,0}^{a} \\ \Delta v_{k,0}^{b} \end{bmatrix} \tag{44}$$

In this equation, [Z_{sc}] (short-circuit impedance) is matrix with 2*2 dimension.

After calculating the fault currents, by applying equations (41) and (42), branch currents and bus voltages after the fault occurrence, can be computed easily.

The Three-line-to-ground Fault:

Also, in order to do short circuiting of the three-line-to-ground, regarding to figure-4-a of bus k:

$$I_k^a = I_{k,f}^{a}, \quad I_k^b = I_{k,f}^{b}, \quad I_k^c = I_{k,f}^{c} \tag{45}$$

And, in order to calculate the phase voltages of bus k:

$$v_{k,f}^{a} = v_{k,f}^{b} = v_{k,f}^{c} = Z_f \left(I_{k,f}^{a} + I_{k,f}^{b} + I_{k,f}^{c} \right) \tag{46}$$

Although The method of the fault current calculations in this fault is similar to previous modes, the size of matrices are one unit greater.
Therefore, fault currents, in this mode, can be determined as:

$$\begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} = \left(\begin{bmatrix} BCBV_f^a \\ BCBV_f^b \\ BCBV_f^c \end{bmatrix} \begin{bmatrix} BIBC_k^a \\ BIBC_k^b \\ BIBC_k^c \end{bmatrix}^T + \begin{bmatrix} Z_f & Z_f & Z_f \\ Z_f & Z_f & Z_f \\ Z_f & Z_f & Z_f \end{bmatrix} \right)^{-1} \times \begin{bmatrix} v_{k,0}^{a} \\ v_{k,0}^{b} \\ v_{k,0}^{c} \end{bmatrix} = \left(\left[Z_{sc}^{abc} \right]^{-1} \right) \begin{bmatrix} v_{k,0}^{a} \\ v_{k,0}^{b} \\ v_{k,0}^{c} \end{bmatrix} \tag{47}$$

In above equation, $\left[Z_{sc}^{abc} \right]$ is a 3*3 size matrix. Branch currents and bus voltages after the fault occurrence are defined by equations (48) and (49):

$$\left[\Delta v_f\right] = \left[BCBV\right]\left[B_f\right] \tag{48}$$

$$\left[B_f\right] = \left[BIBC_k^a \;\; BIBC_k^b \;\; BIBC_k^c\right]\begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix} \tag{49}$$

Three-phase Faults with No Ground Connection:

Figure-5 shows a sample of three-phase fault with impedances between Z_{f3}, Z_{f2}, Z_{f1} . Regarding to above figure, we have:

$$I_k^a = I_{k,f}^a, \quad I_k^b = I_{k,f}^b, \quad I_k^c = I_{k,f}^c \tag{50}$$

And by considering Kirshhoff's current law in common point between three phases, we have:

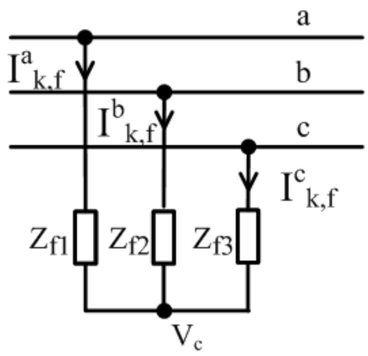

Fig. 5: Three-phase fault.

$$I_{k,f}^a + I_{k,f}^b + I_{k,f}^c = 0 \tag{51}$$

Nevertheless, phase voltages after the fault occurrence, regarding to the common point, can be written as:

$$\begin{aligned} V_{k,f}^a &= Z_{f1} I_{k,f}^a + V_c \\ V_{k,f}^b &= Z_{f2} I_{k,f}^a + V_c \\ V_{k,f}^c &= Z_{f3} I_{k,f}^a + V_c \end{aligned} \tag{52}$$

Thus, the voltage variations of phases in a mentioned bus can be expressed as:

$$\begin{aligned} \Delta V_{k,f}^a &= V_{k,0}^a - Z_{f1} I_{k,f}^a - V_c \\ \Delta V_{k,f}^b &= V_{k,0}^b - Z_{f2} I_{k,f}^b - V_c \\ \Delta V_{k,f}^c &= V_{k,0}^c - Z_{f3} I_{k,f}^c - V_c \end{aligned} \tag{53}$$

And in order to branch current variations, we have:

$$\left[B_f\right] = \left[BIBC\right]\left[\,0 \;\; \ldots \;\; I_{k,f}^a \;\; I_{k,f}^b \;\; I_{k,f}^c \;\; 0 \ldots\right]^T \tag{54}$$

Then, in order to calculate the bus voltage variations, the fallowing equation is used:

$$\begin{bmatrix} \Delta V_{k,f}^a \\ \Delta V_{k,f}^b \\ \Delta V_{k,f}^c \end{bmatrix} = \begin{bmatrix} BCBV_k^a \\ BCBV_k^b \\ BCBV_k^c \end{bmatrix}\begin{bmatrix} BIBC_k^a \\ BIBC_k^b \\ BIBC_k^c \end{bmatrix}^T \begin{bmatrix} I_{k,f}^a \\ I_{k,f}^b \\ I_{k,f}^c \end{bmatrix} \tag{55}$$

By substituting equation (49) into equation (51), the fallowing equation is extracted:

$$\begin{bmatrix} V_{k,0}^{a} - Z_{f1} I_{k,f}^{a} - V_{c} \\ V_{k,0}^{b} - Z_{f2} I_{k,f}^{b} - V_{c} \\ V_{k,0}^{c} - Z_{f3} I_{k,f}^{c} - V_{c} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} \tag{56}$$

By simplifying the equation (56), the fallowing equation is achieved:

$$\begin{bmatrix} V_{k,0}^{a} \\ V_{k,0}^{b} \\ V_{k,0}^{c} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} + \begin{bmatrix} Z_{f1} & 0 & 0 \\ 0 & Z_{f2} & 0 \\ 0 & 0 & Z_{f3} \end{bmatrix} \times \begin{bmatrix} I_{k,f}^{a} \\ I_{k,f}^{b} \\ I_{k,f}^{c} \end{bmatrix} + \begin{bmatrix} V_{c} \\ V_{c} \\ V_{c} \end{bmatrix} \tag{57}$$

By considering this note that, the voltage of common point in addition to the fault current is unknown, therefore, the above equation can be written by the assistant of equation (51).

Finally, the currents of three-phase fault and the voltage of common point of them can be displayed as equation (58) and (59).

$$\begin{bmatrix} I_{f}^{a} \\ I_{f}^{b} \\ I_{f}^{v} \\ V \end{bmatrix} = \left[Z_{sc}^{abc} \right]^{-1} \left[V_{k,0}^{a} \quad V_{k,0}^{b} \quad V_{k,0}^{c} \quad 0 \right]^{T} \tag{58}$$

$$\begin{bmatrix} I_{f}^{a} \\ I_{f}^{b} \\ I_{f}^{v} \\ V \end{bmatrix} = \left[Z_{sc}^{abc} \right]^{-1} \left[V_{k,0}^{a} \quad V_{k,0}^{b} \quad V_{k,0}^{c} \quad 0 \right]^{T} \tag{59}$$

Also, in order to compute the branch currents variations and bus voltages caused by this type of fault, equation (15) and (17), can be used.

RESULTS AND DISCUSSION

A program is provided and edited to calculate the short-circuit current in unsymmetrical distributed networks by using MATLAB software. This program is initially, computed the virtual impedance of networks lines with earth return wire, by applying the conductor characters and its formation. After that, regarding to displayed algorithms, the BCBV and the BIBC matrices constructed and then, by using load distribution results and mentioned equations in section 5, the fault currents according to any type of faults of each points of the network, can be founded. The voltage variations also can be computed easily, due to the calculated fault current in system.

In order to calculate the fault currents, a network with 20 KV appropriate for distribution networks of Iran (line type, line configuration and system with no earth line) is considered. Figure 6 indicates the mentioned network. The mentioned networks details will explain in the appendix. In order to survey the calculations of the system fault currents, two modes are considered:

A- Evaluation the Faults Effect on Voltage Profile of Network:

In this mode, the voltage profile in steady state mode evaluated after it occurred on phase a of bus number 6 figure 7 indicates the voltage of network in steady state mode. The unbalancing is obvious in the figure. If short-circuit event for single line-to-ground occurs through the bus number 6 and the phase b, the fault current determined as 628.077A flows in to ground directly and the results on the voltage profile is displayed in figure 8.

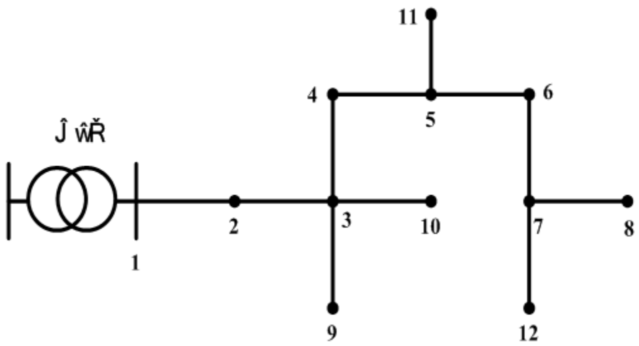

Fig. 6: The assumed analysis network.

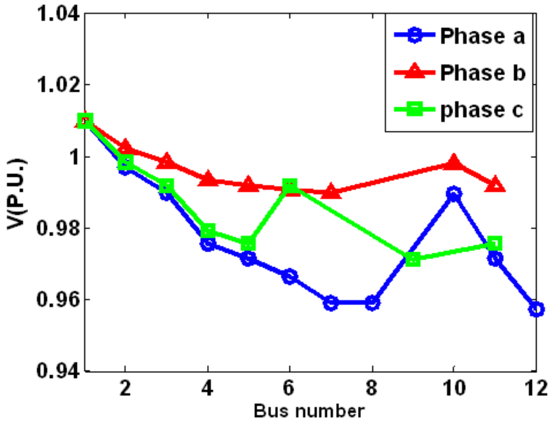

Fig. 7: The voltage profile of the network in steady state mode.

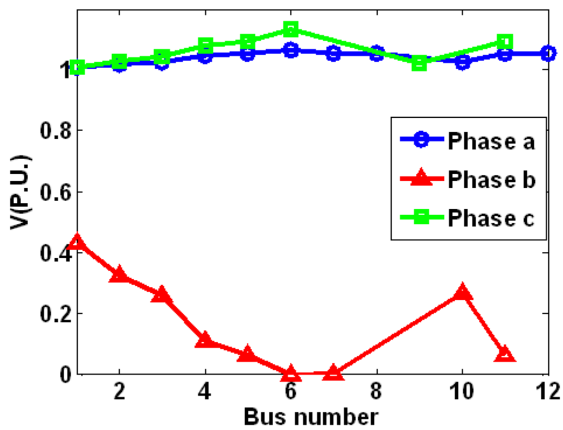

Fig. 8: The voltage profile, after the occurrence fault on bus number 6 and phase b.

Increasing the voltage of phase a and phase c is an obvious result of the network single line short-circuit, as it clearly shows in figure 8.

B- Analysis and Comparing Short-circuit Currents in Various Types:

The currents, caused by the variety of faults occurred on bus 6, owing to survey about this mode, is evaluated. Table 1 demonstrates results of the faults:

According to the above table and its results, it can be verified that, the maximum and minimum fault current relates to three-phase and three-line-to-ground faults (three phase summation), and single-phase fault, respectively. Also, the current summation of three phases based on their angels, in the three-phase fault is zero.

Conclusion:

Nevertheless, the currents and voltages under the fault conditions, may effect on the designing of a distribution system. In addition, the validation of these data is required to identify the power of the fault current for connecting or disconnecting process trough circuit-breakers, switches, fuses, selecting the type of distributed system equipments, the type of protective devices in any points of system and the setting span.

Table 1: Currents caused by different types of occurrence faults trough bus number 6.

Types of Faults	Phase Number	Ia (A)	Ib (A)	Ic (A)
	Phase a	628.0077	---	---
Single-Phase	Phase b	---	643.6006	---
	Phase c	---	---	844.3803
	Phase ab	805.9927	882.489	---
Double-Line-to-Ground	Phase ac	865.1994	---	820.125
	Phase bc	---	832.3644	889.43
	Phase ab	-809.0791	809.2805	---
Double-Phase	Phase ac	---	---	809.0791
	Phase bc	---	-822.746	822.746
Three-Phase	Phase abc	926.2236	945.1733	947.3557
Three-Line-to-Ground	Phase abc	922.6061	947.0862	948.9577

In this paper, considering the Crson line model owing to calculate the lines impedance and by applying two relationship matrices, BIBC and BCBV, a program is produced in order to evaluate short-circuit current based on the types of unsymmetrical faults in the network and the test results were estimated. In this program and proposed method these two matrices only used to evaluate all the faults. Thus, test results for proposed method achieve high accuracy with a lower memory requirement. Test results shown that, the maximum value of the fault current belongs to the three-line-to-ground that is required to select the power of switches in the discussed network.

The short-circuit current perform the exaggerate effects on voltage profile, where makes this problem more significant.

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Appendix:

The formation of well-known conductors in Iran distributed network:

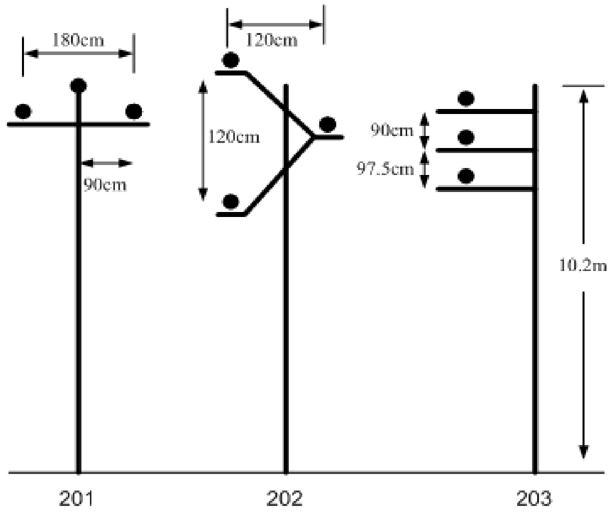

Fig. I: Different conductors configurations, 201- Cross arm, 202-Jenaghi, 203- Flag shaped of formation.

Vary line modes, regarding to quality and type of formation:

Table II: The length and formation of lines between buses in assumed network.

Line length (m)	Line formation	Destination bus	Source bus
3500	100	2	1
2000	101	3	2
5000	101	4	3
1500	101	5	4
2070	102	6	5
3670	103	7	6
150	106	8	7
1500	108	9	3
750	103	10	3
5000	100	11	5
930	106	12	7