Comparison of Fuzzy AHP and Fuzzy TOPSIS Methods for Plant Species Selection
(Case study: Reclamation Plan of Sungun Copper Mine; Iran)

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Abstract: Plant species selection is a multi-criteria evaluation decision and has a strategic importance for many companies. The conventional methods for Plant species selection are inadequate for dealing with the imprecise or vague nature of linguistic assessment. To overcome this difficulty, fuzzy multi-criteria decision-making methods are proposed. The aim of this study is to use fuzzy analytic hierarchy process (F.AHP) and the fuzzy technique for order preference by similarity to ideal solution (F.TOPSIS) methods for the selection of Plant Species in mine reclamation plan. The proposed methods have been applied to a Plant species selection problem of a surface mine land in Iran. After determining the criteria that affect the Plant species decisions, fuzzy AHP and fuzzy TOPSIS methods are applied to the problem and results are presented. The similarities and differences of two methods are also discussed.

Key words: Plant species selection, Multi-criteria evaluation decision, Fuzzy AHP, Fuzzy TOPSIS.

INTRODUCTION

In real life, the evaluation data of Plant species selection suitability for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms. And also, to efficiently resolve the ambiguity frequently arising in available information and do more justice to the essential fuzziness in human judgment and preference, the fuzzy set theory has been used to establish an ill-defined multiple criteria decision-making problems (Liang., 1999). Thus in this paper, fuzzy AHP and fuzzy TOPSIS methods are proposed for Plant species selection, where the ratings of various alternative under various subjective criteria and the weights of all criteria are represented by fuzzy numbers. Mine reclamation means the preparation of the mined ground for post mining land use, which is introduced at Surface Mines. From the early stages of mining operations, mine Reclamation should be considered as inseparable part of the whole plan of the mine. By reclamation, in addition to environment preservation, lands to production cycle can also return (Osanloo and Parsaei, 2004). Mine Reclamation, is a necessary step, in order to post mining land use, planting and create green space for the region. akbari et al (Akbari, Osanloo and Hamidian, 2006) have selected post mining land use through AHP method in sungun Copper Mine. Soltanmohammadi et al (Soltanmohammadi, Osanloo, & Aghajani, 2010) have used combination of group versions of AHP and TOPSIS techniques to determine a preference ranking list for possible post-mining land uses of a hypothetical mined land based on the Mined Land Suitability Analysis framework. Plant Species selecting, is one of the major steps in achieving to the goals of mine reclamation project. superior Plant Specie Selection in every reclamation program, have the many benefits which contains: health protection and environment restoring, preparing suitable perspective for the region, economic benefits, the welfare for local people, pollution reduction of soil and water and air, protection of underground water supply, prevention of soil erosion. Bangian and Osanloo (Bangian & Osanloo, 2008) have selected Plant Species for sungun Copper Mine by traditional AHP method. Alavi et al., have selected, proper Plant types for Sarcheshmeh Copper Mine Reclamation plan by fuzzy AHP method (Alavi, Akbari, & Parsaei, 2011). In this reasearch, According to the fuzzy AHP, the best alternative is A1 (maple) that the same as fuzzy TOPSIS.

2. Fuzzy Sets:

In order to deal with vagueness of human thought, Zadeh first introduced the fuzzy set theory(Zadeh, 1965). A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one (Zadeh, 1965). A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set (Ertağrul & Karakaşoğlu, 2006). Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for commonsense reasoning in decision-making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the

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The goal is to find a good approximate solution (Bojadziev, G & Bojadziev, M, 1998). Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world (Ertuğrul & Tuş, 2007). Modeling using fuzzy sets has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise (Zimmermann, 1992).

2.1 Linguistic Variable:
A linguistic variable is a variable whose values are words or sentences in a natural or artificial language (Zadeh, 1975). As an illustration, age is a linguistic variable if its values are assumed to be the fuzzy variables labeled young, not young, very young, not very young, etc. rather than the numbers 0, 1, 2, 3.. (Bellman & Zadeh, 1977). The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms. The main applications of the linguistic approach lie in the realm of humanistic systems—especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas (Zadeh, 1975).

2.2 Fuzzy Numbers:
A fuzzy number is a convex normalized fuzzy set of the real line R such that (Zimmermann, 1992):
- It exists such that one \( x_0 \in R \) with \( \mu (x_0) = 1 \) (\( x_0 \) is called mean value of \( \tilde{X} \))
- \( \mu(x) \) is piecewise continuous.

It is possible to use different fuzzy numbers according to the situation. In applications it is often convenient to work with triangular fuzzy numbers (TFNs) because of their computational simplicity, and they are useful in promoting representation and information processing in a fuzzy environment. In this study TFNs are adopted in the fuzzy AHP and fuzzy TOPSIS methods. Triangular fuzzy numbers can be defined as a triplet \((l, m, u)\). The parameters l, m, and u respectively, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. A triangular fuzzy number is shown in Fig. 1 (Deng, 1999).

There are various operations on triangular fuzzy numbers. But here, only important operations used in this study are illustrated. If we define, two positive triangular fuzzy numbers \((l_1, m_1, u_1)\) and \((l_2, m_2, u_2)\) then: (K is a positive real number)

1. \((l_1, m_1, u_1) + (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)\) \( (1) \)
2. \((l_1, m_1, u_1) \cdot (l_2, m_2, u_2) = (l_1l_2, m_1m_2, u_1u_2)\) \( (2) \)
3. \((1/l_1, 1/m_1, 1/u_1)\) \( \Rightarrow \) \((1/ul, 1/ml, 1/l\) \) \( (3) \)
4. \((l_1, m_1, u_1) . K = (l_1k, m_1k, u_1k)\) \( (4) \)

The distance between two triangular fuzzy numbers can be calculated by vertex method (Chen, 2000):

\[
d_p(\tilde{m}, \tilde{n}) = \sqrt{1/3[(1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]} \quad (5)
\]

3. Fuzzy Analytic Hierarchy Process:
First proposed by Thomas L. Saaty, the analytic hierarchy process (AHP) is a widely used multiple criteria decision-making tool (Saaty, 1980). The analytic hierarchy process, since its invention, has been a tool at the hands of decision-makers and researchers, becoming one of the most widely used multiple criteria decision-making tools (Vaidya & Kumar, 2006). Although the purpose of AHP is to capture the expert’s knowledge, the traditional AHP still cannot really reflect the human thinking style (Kahraman, Cebeci, & Ulukan, 2003). The traditional AHP method is problematic in that it uses an exact value to express the decision-makers opinion in a comparison of alternatives (Wang & Chen, 2007). And AHP method is often criticized, due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process (Deng, 1999). To overcome all these shortcomings, fuzzy analytical hierarchy process was developed for solving the hierarchical problems. Decision-makers usually find that it is more accurate to give interval judgments than fixed value judgments. This is because usually he/she is unable to make his/her preference explicitly about the fuzzy nature of the comparison process (Kahraman, Cebeci, & Ulukan, 2003). The first study of fuzzy AHP is proposed by Van Laarhoven and Pedrycz (Van Laarhoven & Pedrycz, 1983), which compared fuzzy ratios described by triangular fuzzy numbers. Buckley initiated trapezoidal fuzzy numbers to express the decision-makers evaluation on alternatives with respect to each criterion (Buckley, 1985). Chang introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons (Chang, 1996).
In this study the extent fuzzy AHP is utilized, which was originally introduced by Chang (Chang, 1996). Let X = {x₁, x₂, x₃,..., xₙ} be an object set, and G = {g₁, g₂, g₃,......., gₙ} be a goal set. Then, each object is taken and extent analysis for each goal is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

\[ M^{1}_{gi}, M^{2}_{gi},..., M^{m}_{gi}, i=1,2,3,...,n \]

Where \( M^{j}_{gi} \) (j=1, 2, 3,.., m) all are TFNs. The steps of Chang (Chang, 1996) extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the its object is defined as

\[ S_{i} = \sum_{j=1}^{m} M^{j}_{gi} \otimes \left[ \sum_{j=1}^{m} M^{j}_{gi} \right]^{-1} \tag{6} \]

To obtain \( \sum_{j=1}^{m} M^{j}_{gi} \) (Fuzzy Summation of Row), the fuzzy addition operation of m extent analysis values for a particular matrix is performed such as:

\[ \sum_{j=1}^{m} M^{j}_{gi} = \left( \sum_{j=1}^{m} l_{j}, \sum_{j=1}^{m} m_{j}, \sum_{j=1}^{m} u_{j} \right) \tag{7} \]

And to obtain \( \left[ \sum_{j=1}^{m} M^{j}_{gi} \right]^{-1} \), the fuzzy addition operation of \( M^{j}_{gi} \) (j=1, 2,.., m) values is performed such as:

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} M^{j}_{gi} = \left( \sum_{i=1}^{n} l_{i}, \sum_{i=1}^{n} m_{i}, \sum_{i=1}^{n} u_{i} \right) \tag{8} \]

(Summation of Column)

And then the inverse of the vector above is computed, such as:

\[ \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M^{j}_{gi} \right]^{-1} = \left( 1/\sum_{i=1}^{n} u_{i}, 1/\sum_{i=1}^{n} m_{i}, 1/\sum_{i=1}^{n} l_{i} \right) \tag{9} \]

Step 2: As \( M_{1} = (l_{1}, m_{1}, u_{1}) \) and \( M_{2} = (l_{2}, m_{2}, u_{2}) \) are two triangular fuzzy numbers, the degree of possibility of \( M_{2} \geq M_{1} \) is defined as:

\[ V (M_{2} \geq M_{1}) = \sup_{Y \geq x} \left[ \min \left( u_{M_{1}(x)} \right), u_{M_{2}(Y)} \right] \tag{10} \]

And can be expressed as follows:

\[ V (M_{2} \geq M_{1}) = hgt (M_{1} \cap M_{2}) = u_{M_{2}(a)} \tag{11} \]

\[ (M_{2} \geq M_{1}) = \begin{cases} 1 & \text{IF } m_{2} \geq m_{1} \\ 0 & \text{IF } l_{1} \geq v_{2} \\ 1 - \frac{u_{2}}{(m_{2} - u_{2})} - (m_{1} - l_{1}) & \text{OTHERWISE} \end{cases} \tag{12} \]
That: \( V(M_2 \geq M_1) = \) Bigness degree, \( M_2 = \) First S, \( M_1 = \) secondary S

Figure 2 illustrates Eq. (11) where \( d \) is the ordinate of the highest intersection point \( D \) between \( \mu_{M_1} \) and \( \mu_{M_2} \). To compare \( M_1 \) and \( M_2 \), we need both the values of \( V(M_i \geq M_j) \) and \( V(M_j \geq M_i) \).

**Fig. 2:** The intersection between \( M_1 \) and \( M_2 \).

**Step 3:** The degree possibility for a convex fuzzy number to be greater than \( k \) convex fuzzy \( M_i \) \((i=1, 2, k)\) numbers can be defined by

\[
V(M \geq M_1, M_2, \ldots, M_k) = V((M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and } (M \geq M_k)) = \min (M \geq M_i), i=1, 2, 3, \ldots, k.
\]

(13)

For \( k=1, 2, \ldots, n; k \neq i \). Then the weight vector is given by

\[
d'(A_i) = \min V(S_i \geq S_k)
\]

(14)

Where \( A_i = (i=1, 2, \ldots n) \) are \( n \) elements.

**Step 4:** Via normalization, the normalized weight vectors are:

\[
W^* = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T
\]

(15)

That \( W^* \) is a non-fuzzy number.

**Step 5:** Determination alternatives final weight

\[
A_1 = (A_1 \text{ to } C1 \times C1 \text{ to GOAL}) + (A_1 \text{ to } C2 \times C2 \text{ to GOAL}) + (A_1 \text{ to } C3 \times C3 \text{ to GOAL}) + \ldots + (A_1 \text{ to } C_n \times C_n \text{ to GOAL})
\]

(16)

\( n \) = number of criteria

4. Fuzzy TOPSIS Method:

The TOPSIS method was firstly proposed by Hwang and Yoon (Hwang & Yoon, 1981). The basic concept of this method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from negative ideal solution. Positive ideal solution is a solution that maximizes the benefit criteria and minimizes cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria (Wang & Elhag, 2006). In the classical TOPSIS method, the weights of the criteria and the ratings of alternatives are known precisely and crisp values are used in the evaluation process. However, under many conditions crisp data are inadequate to model real-life decision problems. Therefore, the fuzzy TOPSIS method is proposed where the weights of criteria and ratings of alternatives are evaluated by linguistic variables represented by fuzzy numbers to deal with the deficiency in the traditional TOPSIS. In this paper, the extension of TOPSIS method is considered which was proposed by Chen (Chen, 2000) and Chen et al. (Chen, Lin, & Huang, 2006) the algorithm of this method can be described as follows:

Step 1: First of all a committee of decision-makers is formed. In a decision committee that has \( K \) decision-makers; fuzzy rating of each decision-maker \( D_k = (k=1, 2, \ldots K) \) can be represented as triangular fuzzy number \( \tilde{R}_K \) = (K=1, 2, \ldots, K) with membership function \( \mu_{\tilde{R}_K}(x) \).
Step 2: Then evaluation criteria are determined.
Step 3: After that, appropriate linguistic variables are chosen for evaluating criteria and alternatives.
Step 4: Then the weights of criteria are aggregated (Chen, Lin, & Huang, 2006).

If the fuzzy ratings of all decision-makers are described as triangular fuzzy numbers $\tilde{R}_k^e = (a_k, b_k, c_k), k=1, 2, \ldots, K$, then the aggregated fuzzy rating can be determined as $\tilde{R} = (a, b, c), k=1, 2, \ldots, K$. Here, $a = \min \{ a_k \}$

$$B = \frac{1}{K} \sum_{k=1}^{K} b_k, \ c = \max \{ c_k \}$$ (17)

If the fuzzy rating and importance weight of the $k$th Decision-maker are $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ and $\tilde{w}_{jk} = (w_{jk1}, w_{jk2}, w_{jk3}), i=1, 2, \ldots, m, j=1, 2, \ldots, n$ respectively, then the aggregated fuzzy ratings $\tilde{x}_{ij}$ of alternatives with respect to each criterion can be found as $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$

Here, $a_{ij} = \min \{ a_{ijk} \}$, $b_{ij} = \frac{1}{K} \sum_{k=1}^{K} b_{ijk}$, $c_{ij} = \max \{ c_{ijk} \}$

Then the aggregated fuzzy weights $\tilde{w}_{ij}$ of each criterion are calculated as:

$$\tilde{w}_{ij} = (w_{j1}, w_{j2}, w_{j3})$$

Here, $w_{j1} = \min \{ w_{jk1} \}$, $w_{j2} = \frac{1}{K} \sum_{k=1}^{K} w_{jk2}$, $w_{j3} = \max \{ w_{jk3} \}$

(19)

Step 5: Then the fuzzy decision matrix is constructed as:

$$\tilde{D} = \begin{bmatrix}
\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\
\tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn}
\end{bmatrix}, \quad \tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$$

Here $(\tilde{x}_{ij}) = (a_{ij}, b_{ij}, c_{ij})$ and $(\tilde{w}_j) = (w_{j1}, w_{j2}, w_{j3}); \ i=1, 2, \ldots, m, j=1, 2, \ldots, n$ can be approximated by positive triangular fuzzy numbers.

Step 6: After constructing the fuzzy decision matrix, it is normalized. Instead of using complicated normalisation formula of classical TOPSIS, the linear scale transformation can be used to transform the various criteria scales into a comparable scale. Therefore, we can obtain the normalized fuzzy decision matrix $\tilde{R}$ (Chen, 2000).

$$\tilde{R} = \left[ \tilde{r}_{ij} \right]_{m \times n}, \quad i=1, 2, \ldots, m; \quad j=1, 2, \ldots, n$$

(20)

Here:

$$\tilde{r}_{ij} = \left( \frac{a_{ij}}{c_{ij}}, \frac{b_{ij}}{c_{ij}}, \frac{c_{ij}}{c_{ij}} \right), \quad c_{ij}^+ = \max_j c_{ij}$$

Step 7: Considering the different weight of each criterion, the weighted normalized decision matrix is computed by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix. The weighted normalized decision matrix $\tilde{V}$ is defined as:

$$\tilde{V} = \left[ \tilde{V}_{ij} \right]_{m \times n}, \quad i=1, 2, \ldots, m; \quad j=1, 2, \ldots, n$$

(21)

$$\tilde{V}_{ij} = \tilde{r}_{ij} \cdot \tilde{w}_j$$

Here $\tilde{V}_{ij}$ represents the importance weight of criterion $C_j$. According to the weighted normalized fuzzy decision matrix, normalized positive triangular fuzzy numbers can also approximate the elements $\tilde{V}_{ij}, \forall i,j$. 1108
Step 8: Then, the fuzzy positive ideal solution (FPIS, $A^+$) and fuzzy negative ideal solution (FNIS, $A^-$) are determined as (Chen, Lin, & Huang, 2006):

$$A^+ = (\bar{V}_1^+, \bar{V}_2^+, \ldots, \bar{V}_n^+)$$  \hspace{1cm} (22)

$$A^- = (\bar{V}_1^-, \bar{V}_2^-, \ldots, \bar{V}_n^-)$$  \hspace{1cm} (23)

Where $V_{ij}^+ = \max \{\bar{V}_{ij}\}$ and $V_{ij}^- = \min \{\bar{V}_{ij}\}; \text{ } i=1,2,\ldots, m; \text{ } j=1,2,\ldots, n$

Step 9: Then the distance of each alternative from FPIS and FNIS are calculated as:

$$d_i^+ = \sqrt{\frac{1}{m} \sum_{j=1}^{m} (\bar{V}_{ij}^+ - V_{ij}^+)^2}$$  \hspace{1cm} (24)

$$d_i^- = \sqrt{\frac{1}{m} \sum_{j=1}^{m} (\bar{V}_{ij}^- - V_{ij}^-)^2}$$  \hspace{1cm} (25)

Step 10: A closeness coefficient of each alternative (CC) is defined to rank all possible alternatives. The closeness coefficient represents the distances to the fuzzy positive ideal solution ($A^+$) and fuzzy negative ideal solution ($A^-$) simultaneously. The closeness coefficient of each alternative is calculated as (Chen, 2000):

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+}, \text{ } i=1,2,\ldots, m$$  \hspace{1cm} (26)

Step 11: According to the closeness coefficient, the ranking of the alternatives can be determined. Obviously, according to Eq. (26) an alternative $A_i$ would be closer to FPIS and farther from FNIS as $CC_i$ approaches to 1.

5. Results of Application in Sungun Copper Mine Reclamation:

Our application is related with the plant species selection problem for mine reclamation of a sungun copper mine. Sungun copper mine is located in northern east of Iran (East Azerbaijan province). The deposit’s proved ore reserve is about 380 and total probable and possible reserves is about 1000 million tons. The estimation of mined land of Sungun copper mine is about 38 square kilometers which will complete until the end of mining activities. Region of the mine is a mountain area. There is the highest and lowest elevation respecting free seas equal 2460 and 1700 m. For that reason there are big differences in height (about 750 m) and topography Sungun is an open pit mine with mountain climate. Temperature is cold till moderate, moderately humid. There are various flora and fairly compact natural vegetation (Bangian & Osanloo, 2008). Initial selection of plant Species which is studied for mine reclamation must be done based on the primary factors. The Plant Species should be harmonize with the primary factors which are Type of post mining land use, zone climate, Nature of soil. These are six alternatives ($A_1$ = maple, $A_2$= Oak, $A_3$= Ash, $A_4$= Barberry, $A_5$= Paliurus spina –Christi, and $A_6$= Sloe). Then evaluation criteria (secondary factors) are determined as C1) Perspective of the region, C2) resistance against disease and insects, C3) strength and method of growth, C4) availability to plant Species, C5) economic efficiency, C6) Protection of soil and storing water, C7) prevention from pollution.

5.1 Application with Fuzzy AHP Method:

In this section, fuzzy AHP method is proposed for the same problem of the mine. Here proposed a group decision based on fuzzy AHP. Firstly decision-makers prepared questionnaires forms and then with division against other importance carry out pair-wise comparison. Decision-makers use the linguistic variables, to evaluate the ratings of alternatives with respect to each criterion and they converted into triangular fuzzy numbers. Fuzzy numbers defined as very low (1, 2, 3), low (2,3,5), medium (3,5,7), high (5,7,9), very high (7,9,9). For example, criteria questionnaire respect to goal is shown in Table 1(Alavi, Akbari, & Parsaei, 2011). Then, a comprehensive pair-wise comparison matrix is built as in Table 2.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linguistic variables</td>
<td>Very high</td>
<td>High</td>
<td>low</td>
<td>Medium</td>
<td>low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>fuzzy numbers</td>
<td>7,9,9</td>
<td>5,7,9</td>
<td>2,3,5</td>
<td>3,5,7</td>
<td>2,3,5</td>
<td>5,7,9</td>
<td>5,7,9</td>
</tr>
</tbody>
</table>
Table 2: Matrix of criteria respect to goal.

<table>
<thead>
<tr>
<th>goal</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>0.778</td>
<td>1.4</td>
<td>1</td>
<td>1.4</td>
<td>0.778</td>
<td>0.778</td>
<td>0.193</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.285</td>
<td>3</td>
<td>1.8</td>
<td>3</td>
<td>1.285</td>
<td>1.285</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.8</td>
<td>4.5</td>
<td>3</td>
<td>4.5</td>
<td>1.8</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>0.556</td>
<td>1</td>
<td>1</td>
<td>0.714</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.169</td>
</tr>
<tr>
<td>0.778</td>
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<td>3.333</td>
<td>1.4</td>
<td>2.333</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>1.285</td>
<td>1</td>
<td>4.5</td>
<td>3</td>
<td>4.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>C3</td>
<td>0.222</td>
<td>0.222</td>
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<td>1.4</td>
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<td>C4</td>
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</tr>
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<td>3.5</td>
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<td>1.4</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>C5</td>
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<td>0.222</td>
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<td>0.285</td>
<td>1</td>
<td>0.222</td>
<td>0.222</td>
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<tr>
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<td>1.667</td>
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<td>1</td>
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<td></td>
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<tr>
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<td>1</td>
<td>1</td>
<td>0.714</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.169</td>
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<tr>
<td>0.778</td>
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<td>3.333</td>
<td>1.4</td>
<td>2.333</td>
<td>1</td>
<td>1</td>
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<tr>
<td>1.285</td>
<td>1</td>
<td>4.5</td>
<td>3</td>
<td>4.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>0.556</td>
<td>1</td>
<td>1</td>
<td>0.714</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.169</td>
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<tr>
<td>0.778</td>
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<td>1.285</td>
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<td>3</td>
<td>1</td>
<td>1</td>
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<td>1</td>
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</tr>
</tbody>
</table>

From Table 2, according to extent analysis synthesis values respect to main goal are calculated like in Eq. (6): 
S1 = 8.734, 12.655, 18.400 0.011, 0.017, 0.027 = 0.092, 0.220, 0.492, also 
S2 = 0.066, 0.171, 0.435, S3 = 0.033, 0.073, 0.197, S4 = 0.037, 0.122, 0.353, S5 = 0.033, 0.073, 0.197, S6 = 0.066, 0.171, 0.435, S7 = 0.066, 0.171, 0.435. These fuzzy values are compared by using Eq. (12), and next values are obtained: Then priority weights (Min) are calculated by using Eq. (13, 14):

\[ v(S1 \geq S2) = 1.000, v(S1 \geq S3) = 1.000, v(S1 \geq S4) = 1.000, v(S1 \geq S5) = 1.000, v(S1 \geq S6) = 1.000, v(S1 \geq S7) = 1.000, \]
\[ v(S2 \geq S1) = 0.876, v(S2 \geq S3) = 1.000, v(S2 \geq S4) = 1.000, v(S2 \geq S5) = 1.000, v(S2 \geq S6) = 1.000, v(S2 \geq S7) = 1.000, \]
\[ v(S3 \geq S1) = 0.419, v(S3 \geq S2) = 0.574, v(S3 \geq S4) = 0.766, v(S3 \geq S5) = 1.000, v(S3 \geq S6) = 0.574, v(S3 \geq S7) = 0.574, \]
\[ Min v(S4) = 0.728, Min v(S5) = 0.419, Min v(S6) = 0.876, Min v(S7) = 0.876. \]

After the normalization of these values priority weight respect to main goal is calculated as (0.193, 0.169, 0.081, 0.140, 0.081, 0.169, 0.169). After the priority weights of the criteria are determined, the priority of the alternatives will be determined for each criterion. From the pair-wise comparisons matrixes of the decision-makers for six alternatives, evaluation matrixes are formed too. Then, priority weights of alternatives for each criterion are determined by making the same calculation. The weight vector from Matrix of alternatives respect to C1 is calculated as (0.196, 0.161, 0.196, 0.161, 0.161, 0.125). Respect to C2 is calculated as (0.196, 0.161, 0.196, 0.161, 0.161, 0.125). Respect to C3 is calculated as (0.267, 0.197, 0.197, 0.113, 0.113, 0.113). Respect to C4 is calculated as (0.196, 0.129, 0.175, 0.175, 0.196, 0.129). Respect to C5 is calculated as (0.257, 0.195, 0.231, 0.105, 0.105, 0.105). Respect to C6 is calculated as (0.229, 0.165, 0.229, 0.105, 0.105, 0.165). Respect to C7 is calculated as (0.211, 0.112, 0.188, 0.188, 0.151, 0.151).

By using of eq. (16), Alternative A1 which has the highest priority weight is selected as a best plant species selection for the mine reclamation. The ranking order of the alternatives with fuzzy AHP method is A1>A3>A2>A4>A5>A6 that are shown in figure 3.

Fig. 3: Priority of plants Species in sungun mine reclamation plan by F.AHP method.
5.2 Application With Fuzzy TOPSIS Method:

In this section fuzzy TOPSIS method is proposed for the plant species selection problem of the sungun mine complex. The algorithm of this method can be described as follows:

1. Construct the fuzzy decision matrix:

Fuzzy numbers defined as very low (1, 2, 3), low (2, 3, 5), medium (3, 5, 7), high (5, 7, 9), very high (7, 9, 9). Linguistic variables converted into triangular fuzzy numbers to form fuzzy decision matrix as shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3: Fuzzy decision matrix.</th>
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</thead>
<tbody>
<tr>
<td>C1</td>
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<tr>
<td>-------</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
<tr>
<td>A5</td>
</tr>
<tr>
<td>A6</td>
</tr>
</tbody>
</table>

2. Normalize the fuzzy decision matrix

Because in this research, all criteria are positive, for positive criteria, selected the highest number in each column, then all numbers are divided thereon.

3. Normalized fuzzy decision matrix is formed by eq. (20).

4. Construct weighted normalized fuzzy decision matrix: it is formed by eq. (21).

Weight vector (0 till 1) is obtained from to normalize the importance coefficient that by division the fuzzy numbers of importance coefficient on their total accounts of table1. (1 = 0.037, 2 = 0.074, 3 = 0.111, 5 = 0.185, 7 = 0.259, 9 = 0.333)

5. Determine FPIS and FNIS: the fuzzy positive ideal solution (FPIS, A+) and fuzzy negative ideal solution (FNIS, A−) are determined as:

After a weighted normalized fuzzy decision matrix is formed, fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are determined as in the following:

\[ V_j^+ = \max \{ \tilde{V}_{ij} \} = (0.333, 0.333, 0.185, 0.259, 0.185, 0.333, 0.333) \]

\[ V_j^- = \min \{ \tilde{V}_{ij} \} = (0.086, 0.062, 0.016, 0.037, 0.016, 0.041, 0.041) \]

Then the distance of each alternative from FPIS and FNIS with respect to each criterion are calculated by using eq. (24 and 25) that the results of all alternatives’ distances from FPIS and FNIS are shown in Tables 4 and 5.

<table>
<thead>
<tr>
<th>Table 4: Distances between Ai (i=1, 2, 3,4,5,6) and A+ with respect to each criterion.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distances</td>
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<tr>
<td>-------</td>
</tr>
<tr>
<td>d(A1,A+)</td>
</tr>
<tr>
<td>d(A2,A+)</td>
</tr>
<tr>
<td>d(A3,A+)</td>
</tr>
<tr>
<td>d(A4,A+)</td>
</tr>
<tr>
<td>d(A5,A+)</td>
</tr>
<tr>
<td>d(A6,A+)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5: Distances between Ai (i=1, 2, 3,4,5,6) and A− with respect to each criterion.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distances</td>
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<td>-------</td>
</tr>
<tr>
<td>d(A1,A−)</td>
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<tr>
<td>d(A2,A−)</td>
</tr>
<tr>
<td>d(A3,A−)</td>
</tr>
<tr>
<td>d(A4,A−)</td>
</tr>
<tr>
<td>d(A5,A−)</td>
</tr>
<tr>
<td>d(A6,A−)</td>
</tr>
</tbody>
</table>

\( d_i^+ \) and \( d_i^- \) of six alternatives are shown in Tables 4 and 5 too. Then closeness coefficients of six alternatives are calculated by eq. (26). Closeness Coefficient are Calculated for the alternatives that are shown in figure 4.
According to the closeness coefficient of six alternatives, the ranking order of six alternatives is determined as A1>A3>A4>A5>A2>A6. The first alternative is closer to the FPIS and farther from the FNIS.

There have reached the same result with fuzzy AHP approximately and maple plant is selected as first alternative. Fuzzy AHP and fuzzy TOPSIS methods are both appropriate for the selection of a plant species or other multi-criteria decision-making problems of the mine. But these two methods have some limitations and advantages. According to the problem the most appropriate method should be chosen. In this research can summarize the differences and similarities between fuzzy AHP and fuzzy TOPSIS methods as follows:

- **When these two methods are compared with respect to the amount of computations, fuzzy AHP requires more complex computations than fuzzy TOPSIS.**
- **Pair-wise comparisons for criteria and alternatives are made in fuzzy AHP, while there is no pair-wise comparison in fuzzy TOPSIS and are based on their relative distances to positive ideal solution and negative ideal solutions.**
- **TOPSIS has been proved to be one of the best methods addressing rank reversal issue that is the change in the ranking of the alternatives when a non-optimal alternative is introduced.**
- **In the extent analysis of fuzzy AHP, the priority weights of criterion or alternative can be equal to zero. In this situation, we do not take this criterion or alternative into consideration. This is the one of the disadvantages of this method.**
- **Both in fuzzy AHP and fuzzy TOPSIS can adopt linguistic variables.**
- **In this research, the ranking results of the fuzzy AHP and fuzzy TOPSIS are the same approximately. This shows that when the decision-makers are consistent with him/her in determining the data, two methods independently, the ranking results will be same.**

**Conclusions:**

Decision-makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision-making process. By using fuzzy AHP and fuzzy TOPSIS, uncertainty and vagueness from subjective perception and the experiences of decision-maker can be effectively represented and reached to a more effective decision. In this study plant Species selection with fuzzy AHP and fuzzy TOPSIS method has been proposed. Although two methods have the same objective of selecting the best plant Species for the mine reclamation, they have differences. In fuzzy TOPSIS decision-makers used the linguistic variables to assess the importance of the criteria and to evaluate the each alternative with respect to each criterion. These linguistic variables converted into triangular fuzzy numbers and fuzzy decision matrix was formed. Then normalized fuzzy decision matrix and weighted normalized fuzzy decision matrix were formed. After FPIS and FNIS were defined, distance of each alternative to FPIS and FNIS were calculated. And then the closeness coefficient of each alternative was calculated separately. According to the closeness coefficient of three alternatives, the best alternatives has been determined as A1= maple. In fuzzy AHP, decision-makers made pair-wise comparisons for the criteria and alternatives under each criterion. Then these comparisons integrated and decision-makers’ pair-wise comparison values are transformed into triangular fuzzy numbers. The priority weights of criteria and alternatives are determined by Chang extent analysis. According to the combination of the priority weights of criteria and alternatives, the best alternative is determined. According to the fuzzy AHP, the best alternative is A1 that the same as fuzzy TOPSIS. Companies should choose the appropriate method for their problem according to the situation and the structure of the problem they have. In future studies, other multi-criteria methods like fuzzy PROMETHEE and ELECTRE can be used to handle plant Species selection problems.

However, due to the dire environmental situation around of the mine, planting the plant Species should be placed in Sungun mine according to priority, to absorb different pollutions and harmful elements. It also gives
certain beauty to the surrounding landscape. Technicality Notice that, all selective plants are native and have consistency with sungun local environment, soil, climate condition.

REFERENCES


