Australian Journal of Basic and Applied Sciences, 5(12): 1685-1682, 2011
ISSN 1991-8178

An Analysis for Optimization of Heat Transfer for Various Heat Sink Cross-section and Length

[1]Mehdi Nafar and [2]Mohammad Tavassoli

[1]Department of Electrical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran.
[2]Department of Mechanical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran.

Abstract: An analytical simulation model is presented in this study to predicting and optimizing the thermal performance, maximum thermal dissipation and the least material cost in electrical devices. According to general derivation, the longitudinal fin arrays on a heat sink can have either square, rectangular, equilaterally triangular, or cylindrical cross-section. It is observed that actual convection flow velocity through fins is usually unknown to designers. By input of the Biot number Bi, heat transfer coefficient ratio, H and the shape parameter, the heat transfer equation which is expressed in implicit form can be solved by iterative method to calculate the optimum fin length and fin thickness. Optimization of heat- sink designs and typical parametric behaviors are discussed based on the sample simulation results. Also the thermal resistance of a heat sink can be obtained to illustrate the cooling performance under various design conditions.

Key word: Analytical Model, thermal Performance, electrical device.

INTRODUCTION

The several kinds of heat sink assembly commonly used. With the increase in dissipation from microelectronic devices and the reduction in overall form factors, it become an essential practice to optimize heat-sink designs with least trade-offs in material and manufacturing costs. The heat-sink assembly is powerful apparatus for heat removal in today's thermal engineering. This apparatus is mainly designed to remove heat effectively from the equipment to environmental. The augmentation of heat dissipation in heat sink has attracted the attention of researchers for several decades. For all types of heat sink assembly, the size of each fin and the arrangement of fin array play a very important role to promote heat transfer. Kern and Kraus (1972) studied the optimum design for fin and spine subjected to a constant heat transfer coefficient. Saski and kishimoto (1986) optimized, with a criterion of fin to channel thickness ratio of unity, the dimensions of water cooled micro-channel at a given pressure loss. An analytical method of optimizing forced convection heat sinks was proposed by knight *et al*. (1991)(1992) for fully developed flow in closed finned channels. They presented normalized non-dimensional thermal resistances as a function of the number of channels again for a fixed pressure drop. Kraus (1988) surveyed the whole progress of extended surface. During the process for designing fins, material weight and manufacturing availability are also the important factors to be concerned. Hence, Bar-Cohen and Jelinek (1985) presented a procedure to establish optimum arrays of longitudinal rectangular fins for the least material optimization. The optimum ratio of fin thickness to fin length can be found based on the maximum heat flow per unit width from the fin. Computational techniques were also employed in investigating the thermal performance of extruded heat sinks. (Metrol, A., 1993; Mansingh, V. and K. Hassur, 1993) Yeh and Chang (1995) optimized the longitudinal convective fin arrays that include various profiles. Rectangular, triangular, convex-parabolic and concave- parabolic profiles were taken into consideration . For vertical rectangular fins protruding from a vertical rectangular base, leung and probert (1989) used experimental measurement to find the optimum spacing. Teertstra *et al*. (1999) carried out the numerous methods to predict the average heat transfer rate for plate fin heat sinks used in the design and selection of heat sinks for electronic applications . A composite solution was developed for the limiting cases of fully developed and developing flolw between isothermal parallel plates and fluid velocity. Poulikakos and Bejan (1982) have published extensively in the area of optimization according to an entropy minimization approach. Kou *et al*. (2003) derived a transcendental equation that include only three variables to find the optimum longitudinal fin length and fin number in a heat sink.

In the optimum thermal design of a heat sink for the least material cost there are usually two approaches for its realistic application. For the first approach, the shape and the cross- sectional area of fin array and the total volume of the fins in a heat sink are prescribed to ascertain its optimum fin length accompanied by the total fin number (Kou, *et al*., 2003). For the second approach the number of fins and the volume of each fin are fixed to find its optimal fin length and fin cross-section area for each particular fin shape (Bar- Cohen, A. and M. Jelinek, 1985; Yeh. R.H. and M. Chang, 1995), where the shape of the longitudinal fin is limited by the

Corresponding Author: Dr. Mehdi Nafar Department of Electrical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran.
Tel:+989173018583; E-mail: mehdinafar@yahoo.com

rectangular cross-section and the width or height of the fin is assumed to be a fixed constant, the cross-section of the longitudinal fin in study can cover square, rectangular, equilateral triangular and cylindrical fin arrays because of thegeneralized mathematical formulation. Instead of assuming a fixed width in a rectangular fin, this study assumes a known ratio of fin thickness to fin width in advance to match the requirement of the present analysis. In this paper by knowing the values of Biot number, Bi, heat transfer coefficient, H and shape parameter, the optimum equation with maximum heat dissipation can be solved to find the optimum specification of fins in a heat sink. Finally the performance of heat sink for squre, rectangular triangular and cylindrical fin arrays are demonstrated by thermal resistance, which is the most important factor for a heat sink in designing an electronic cooling system.

Heat sink Categories:

One way to categorize heat sinks is by the cooling mechanism employed to remove heat from the heat-sinks. It can be largely divided into five categories:

Passive Heat Sinks are used in either natural convection or in applications where heat dissipation does not rely on designated supply of air flows.

Semi-Active Heat Sinks leverage off existing fans in the system.

Active Heat Sinks employ designated fans for its own use such as fan heat sinks in either impingement or vertical flows. This type of heat sinks usually involves mechanically moving component and its reliability depends on heavily on the reliability of the moving parts.

Liquid Cooled Cold Plates typically employ tubes in –block designs or pumped water, oil, or other liquids.

Phase Change Recirculating System includes two-phases system that employ a set of boiler and condenser in a passive self driven mechanism. Heat pipe system incorporate either no wicks in a gravity fed arrangement or wicks that do not require gravity feeds. This category also includes solid –to- liquid systems but those are usually used to moderate transient temperature gradient rather than for the purpose of dissipating heat.

Heat Sink Types:

Heat sinks can be classified in terms of manufacturing methods and their final form shapes. The most common types of air-cooled heat sinks include

1. Stampings: Copper or aluminum sheet metals are stamped into desired shapes. They are used in traditional air cooling of electronic components and offer a low cost solution to low density thermal problems. Suitable for a high volume production and advanced tooling with high speed stamping would lower costs. Additional labor-saving options, such as taps, clips and interface materials, can be factory applied to help reduce the board assembly costs.
2. Extrusions: Allow the formation of elaborate two-dimensional shapes capable of dissipating large wattage loads. They may be cut machined and incorporating serrated fins improves the performance by approximately 10 to 20 % at the expense of extrusion rate. Extrusion limits, such as the fin height-to-gap aspect ratio, minimum fin thickness-to-height and maximum base to fin thickness usually dictate the flexibility in design options. As the aspect ratio increases, the extrusion tolerance needs to be compromised.
3. Bonded / Fabricated Fins: Most air cooled heat sinks are convection limited and the overall thermal performance of an air cooled heat sink can often be improved significantly if more surface area exposed to the air stream can be provided even at the expense of conduction paths. This process allow for a much greater fin height-to-gap aspect ratio of 20 to 40 %, greatly increasing the cooling capacity without increasing volume requirements.
4. Castings: Sand, lost core and die casting processes are available with or without vacuum assistance, in aluminum or copper-bronze. This technology is used in high density pin which provide maximum performance when using impingement cooling.
5. Folded Fin: Corrugated sheet metal in either aluminum or copper increases surface area and hence the volumetric performance. The heat sink is then attached to either a base plate or directly to the heating surface via epoxying or brazing. It is not suitable for high profile heat sinks due to the availability and from the fin efficiency point of view. However it allows to obtain high performance heat sinks in applications where it is impractical or impossible to use extrusions or bonded fins.

Theoretical Analysis:

The exact solution of heat transfer rate for single fin can be obtained in the form as following :

$$q = \theta_b \sqrt{h_f kap} \tan h \left(\sqrt{\frac{h_f p}{ka}} l \right) \tag{1}$$

Where a is fin cross-section and p is perimeter of longitudinal fin

Total heat loss of a heat sink can be determined by assuming the thermal contribution from the fin surface and the fin spacing on the heat sink surface according to:

$$Q = n\theta_b\sqrt{h_f kap}\tan h\left(\sqrt{\frac{h_f p}{ka}}\,l\right) + h_b(A - na)\theta_b \tag{2}$$

Where n is the number of fins on heat sink and A is the sink base area .
The fin cross – section area and perimeter can be assumed in a general form as :

$$a = \alpha t^2 \tag{3}$$

$$p = \beta t \tag{4}$$

Eqs.(3) and (4) can include many types of fin cross-section . In this study four different types of fin array such as square, rectangular equilateral and cylindrical fin are introduced in Figure 1.

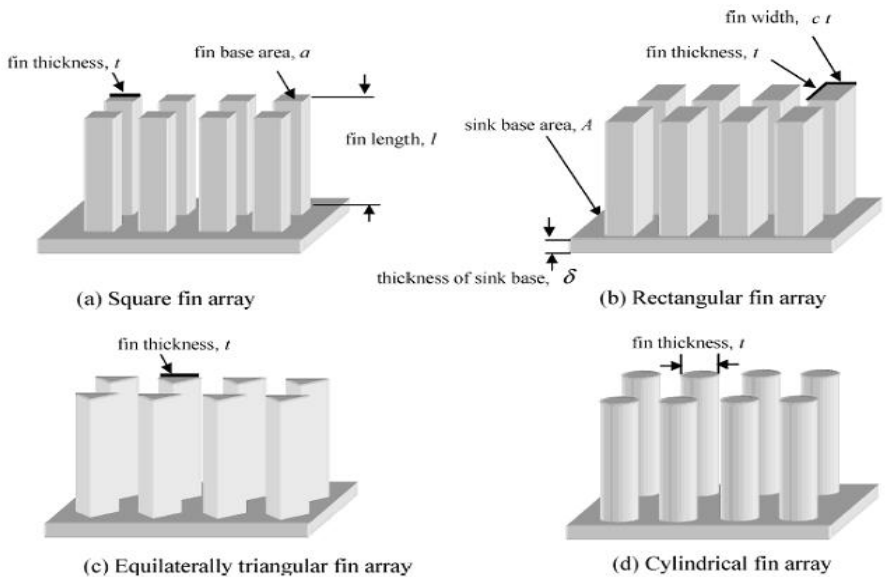

Fig. 1: Various types of heat sinks and fin array.

The Biot number Bi, the dimensionless parameter, D, the dimensionless length of fin, S, the ratio of heat transfer coefficients, H, the dimensionless shape parameter, γ and the effectiveness of a heat sink, ε, are defined as follow:

$$Bi = \frac{h_f v^{\frac{1}{3}}}{k} \tag{5}$$

$$D = \frac{n v^{\frac{2}{3}}}{A} \tag{6}$$

$$S = \frac{l}{v^{\frac{1}{3}}} \tag{7}$$

$$H = \frac{h_b}{h_f} \tag{8}$$

$$\gamma = \frac{\beta^{\frac{1}{2}}}{\alpha^{\frac{1}{4}}} \tag{9}$$

$$\varepsilon = \frac{Q}{h_b A\theta_b} \tag{10}$$

Dimensionless parameters H^* and Bi^* are defined as:

$$H^* = \frac{H}{\gamma^2} \tag{11}$$

$$Bi^* = Bi\gamma^2 \tag{12}$$

Also :

$$\epsilon(S) = \frac{D}{(S)^{\frac{3}{4}}\sqrt{Bi^*}H^*}\tanh\left[\sqrt{\mathrm{Bi}^*}(\mathrm{S})^{\frac{3}{4}}\right] + 1 - \frac{D}{S} \tag{13}$$

The effectiveness of a heat sink, ε, is defined as the ratio of the total heat transfer rate from the from the heat sink to the heat transfer rate that takes place from the sink base area only. That is,

$$\varepsilon = \frac{Q}{h_b A\theta_b} > 1 \tag{14}$$

The total fin cross-section area to the heat sink base area:

$$\emptyset = \frac{D}{S} = \frac{nv}{Al} = \frac{na}{A} \tag{15}$$

The effectiveness of a heat sink can be rewritten as:

$$\epsilon(\emptyset) = \frac{D^{\frac{1}{4}}\emptyset^{\frac{3}{4}}}{\sqrt{Bi^*}H^*}\tanh\left[\sqrt{Bi^*}\left(\frac{D}{\emptyset}\right)^{\frac{5}{4}}\right] + 1 - \emptyset \tag{16}$$

The efficiency of a heat sink η, is defined as follows:

$$\eta = \frac{Q}{Q_{ideal}} < 1 \tag{17}$$

The heat transfer rate of an ideal heat sink is

$$Q_{ideal} = \left[nplh_f + (A - na)h_b\right]\theta_b \tag{18}$$

The efficiency of a heat sink can be written in dimensionless form as:

$$\eta = \frac{\varepsilon H^*}{\left[S^{\frac{3}{2}}\phi + (1-\phi)H^*\right]} \tag{19}$$

To find the maximum heat dissipation at one set of fixed values of D,H^* and Bi^*, the optimum value of S can be found by differentiating Eq.(13) relative to S and then setting this derivative to zero.
The results a re as follows:

$$3\gamma(S_{opt})^{\frac{1}{4}}\tanh\left[\gamma\sqrt{\mathrm{Bi}}(\mathrm{S_{opt}})^{\frac{5}{4}}\right] - 5\gamma^2\sqrt{Bi}(S_{opt})^{\frac{3}{2}}sech^2\left[\gamma\sqrt{Bi}(S_{opt})^{\frac{3}{4}}\right] = 4\sqrt{Bi}H \tag{20}$$

From Eq. (7) the optimum length of fins on a heat sink can be obtained as:

$$l_{opt} = v^{\frac{1}{3}}.S_{opt} \tag{21}$$

In designing an electronic cooling system thermal resistance is the most important factor for a heat sink . The thermal resistance of a heat sink including the thickness effect of sink base is defined as:

$$R = \frac{1}{A}\left(\frac{1}{\varepsilon h_b} + \frac{\delta}{k}\right) \tag{22}$$

The term of effective heat transfer surface area is also introduced here as:

$$\sigma = npl + A(1 - na) \tag{23}$$

The of the heat transfer coefficient is closely related to the fin shape, fin spacing, fin cross-sectional area and flow velocity among other things.

Results and Conclusions:

The relationship between effectiveness, ε and Bi^* and ϕ is showed in Figure 2. When D=0.5 or 1 and H^*=0.2 or 0.4 . It can be observed that the effectiveness, ε, increases as the Bi^* decreases. This figure also show that the optimum value of ϕ becomes a larger value with the increase of Bi^*.

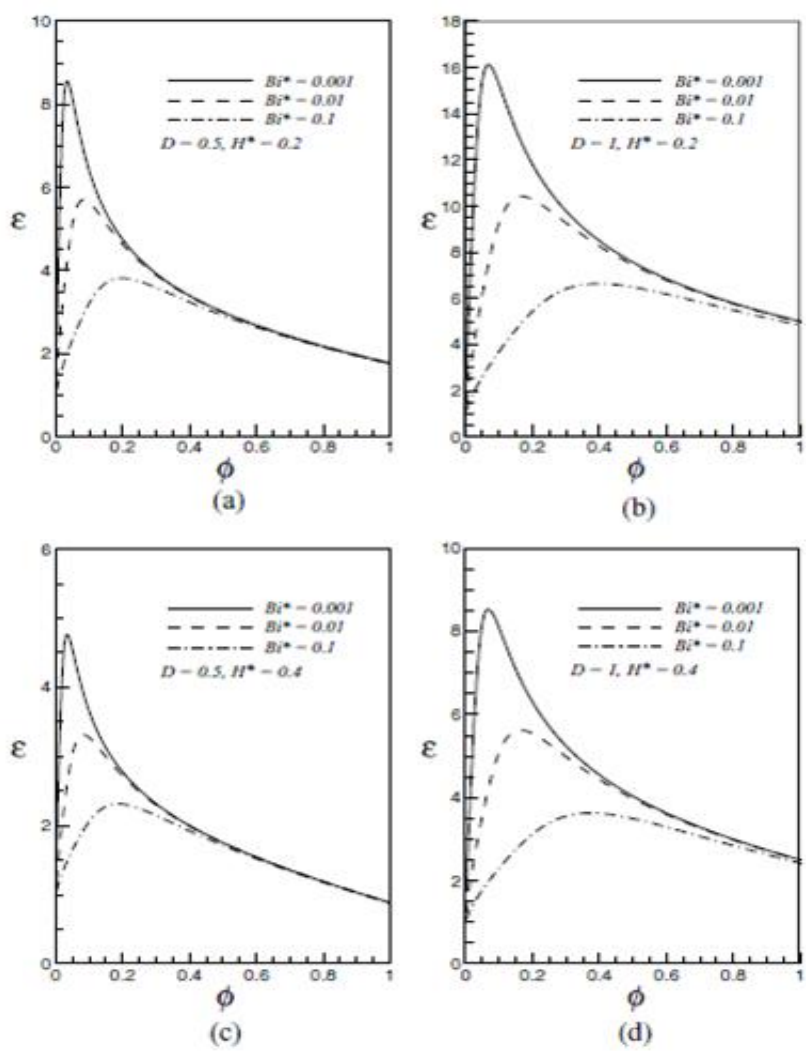

Fig. 2: Relationship between ε *and* ϕ.

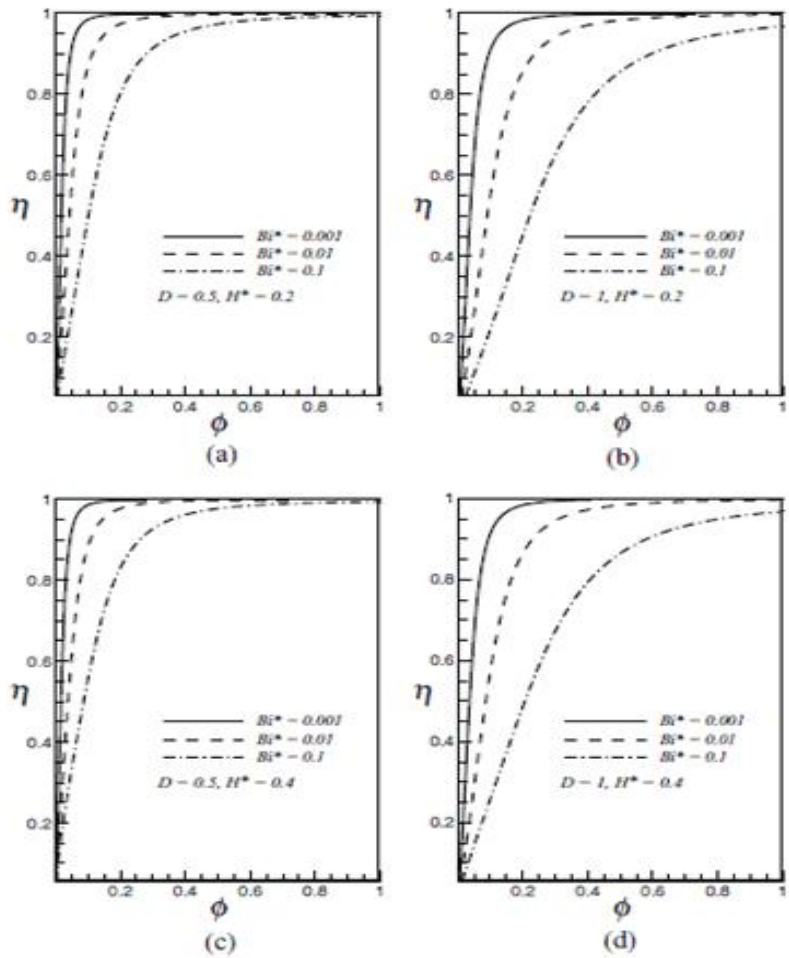

Fig. 3: Relationship between η *and* ϕ.

The optimum fin length can be described by setting the volume of a fin . Figure3. Indicates the dependence of the efficiency, η, of a heat sink on Bi^* and ϕ for D=0.5 or 1 and $H^* = 0.2$ or $H^* = 0.4$. The efficiency increases with the rising of ϕ and also increases as Bi and S decreases. The relationship between S_{opt}, H^* and Bi^* is demonstrated in Figure 3. The optimum dimensionless fin length, S_{opt} becomes longer as H^* increases and Bi^* decreases . Because the rectangular fin can have distinct values in its length width and thickness this study assumes a known ratio of fin thickness to fin width in advance to match the requirement of Eqs.(3) and (4) . Table 1 Shows the comparison of heat sink performance for square, rectangular equilateral triangular and cylindrical fin arrays based on the same input values of heat sink base area thermal conductivity heat transfer coefficients number of fins and volume of a fin. The material chosen for the heat sink is aluminum with a thermal conductivity of 164 $W/_{m^2K}$. These cases evection Therefore, heat transfer coefficients are chosen as $h_f = h_b = 40\frac{W}{m^2K}$ and 80 $\frac{W}{m^2k}$. For the comparison of heat sink performance among square equilateral triangular and cylindrical fin arrays the equilateral triangular fin array has the smallest value of the optimum fin length and the maximum effective heat transfer surface area.

The cylindrical fin arrays have the longest optimum fin length and the least effective heat transfer surface area, including the largest thermal resistance. For a lower ratio of fin thickness to fin with, the thermal resistance can approach a lower value and is accompanied with the difficulty of manufacturing .Another numerical simulation about the heat the heat sink with the optimum rectangular fin array is carried out in table 2. The

material chosen for the sink is duralumin, with a thermal conductivity of 164 $\frac{W}{m^2 K}$ and aluminum, with a thermal conductivity of 204 $\frac{W}{m^2 K}$. The thermal resistance decreases as the ratio of fin thickness to fin width decreases when the other input parameters stay constant.

A transcendental equation has been derived to find the optimum value of S_{opt}. by giving the dimensionless values of Bi^* and H^*. The thermal resistance of a heat sink can be obtained as the reference for the thermal management of electronic equipment. The heat sink with the maximum effective heat transfer surface area has the lowest thermal resistance when the ratio of heat transfer coefficient is set as 1. For the heat sink with rectangular fin array, a low ratio of fin thickness to fin width has the advantage of lower thermal resistance. The smaller value of aspect ratio in the cross – section for the rectangular fin array may result in a weaker structure and increase the difficulty of manufacturing.

Table 1: Optimum fin arrays for cylindrical square equilateral triangular and rectangular fins.

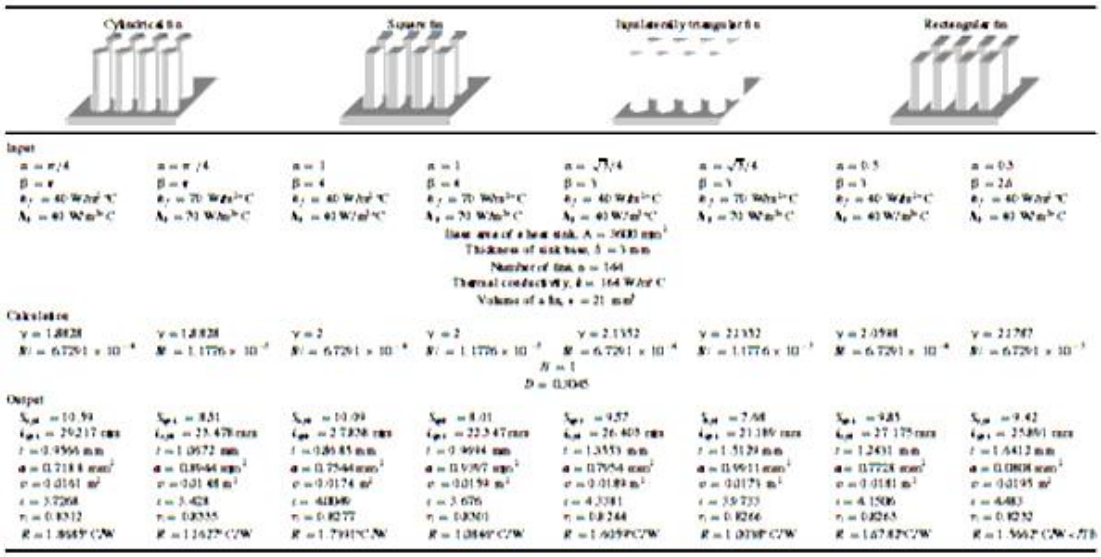

Table 2: Optimum fin arrays for rectangular fins with various the ratio of fin width thickness and thermal conditions.

Geometric profile						
Input data						
Ratio of fin width to fin thickness	$\alpha = 0.1$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.01$
Thermal conductivity	k =164 W/m°C	k =204 W/m°C	k =164 W/m°C	k =204 W/m°C	k =164 W/m°C	k =204 W/m°C
	Base area of a heat sink, A = 5712 mm^2					
	Thickness of sink base, δ= 4 mm					
	Number of fins, n = 25					
	Heat transfer coefficient, $h_f = h_b$= 40 W/m^2°C					
	Volume of a fin, v= 1000 mm^3					
Calculation						
	$\gamma = 2.6376$	$\gamma = 2.6376$	$\gamma = 3.0646$	$\gamma = 3.0646$	$\gamma = 4.4944$	$\gamma = 4.4944$
	$Bi = 2.439 \times 10^{-3}$	$Bi = 1.9608 \times 10^{-3}$	$Bi = 2.439 \times 10^{-3}$	$Bi = 1.9608 \times 10^{-3}$	$Bi = 2.439 \times 10^{-3}$	$Bi = 1.9608 \times 10^{-3}$
	$H = 1$					
	$D = 0.4377$					
Output						
$S_{opt.}$	4.87	5.3	4.31	4.7	3.16	3.44
$l_{opt.}$	48.7 mm	53 mm	43.1 mm	47 mm	31.6 mm	34.4 mm
t	14.3296 mm	13.7361 mm	21.5415 mm	20.6284 mm	56.2544 mm	53.9164 mm
a	20.5339 mm^2	18.8679 mm^2	23.2019 mm^2	21.2766 mm^2	31.6456 mm^2	29.0698 mm^2
σ	0.0436 m^2	0.0453 m^2	0.0539 m^2	0.0561 m^2	0.0947 m^2	0.0986 m^2
ε	6.1603	6.4021	7.5733	7.8796	13.18	13.7412
η	0.8074	0.8076	0.8029	0.8026	0.795	0.7954
R	0.7147°C/W	0.6871°C/W	0.5822°C/W	0.5589°C/W	0.3363°C/W	0.3219°C/W

REFERENCES

Bar- Cohen, A. and M. Jelinek, 1985. Optimum Arrays of Longitudinal Rectangular Fins in Convection Heat Transfer Heat Transfer Engineering, 6: 68-78.

Kern, D. and A.D. Kraus, 1972. Extended Surface Heat Transfer, McGraw Hill New York 1972.

Knight, R.W., D.J. Hall, J.S. Goodling and R.C. Jaeger, 1992. Heat Sink Optimization with Application to Microchannels IEEE Transaction on Components Hybrids and Manufacturing Technology, 15(5): 832-842.

Knight, R.W., J.S. Goodling and D.J. Hall, 1991. Optimal Thermal Design of Forced Convection Heat Sinks Analytical. ASME Journal of Electronic Packaging, 113: 313-321.

Kou, H.S., J.J. Lee and C.Y. Lai, 2003. Thermal Analysis and Optimum Fin Length of a Heat Sink Heat Transfer Engineering, 24(2): 18-29.

Kraus, A.D., 1988. Sixty-Five Years of Extended Surface Technology 1922-1987, Appl. Mech. Rev., 41: 321-364.

Leung, C.W. and S.D. Probert, 1989. Heat-Exchanger Performance : Effect of Orientation Applied Energy, 33: 235-252.

Mansingh, V. and K. Hassur, 1993. Thermal Analysis of a Pin Grid Array Packing Proceedings of the International Electronics Packaging Conference, 1: 410-419.

Metrol, A., 1993. Optimization of Extruded Type External Heat Sink for Mutichip Module ASME Journal of Elctronic Packaging, 115: 440-444.

Proulikakos, D. and A. Bejan, 1982. Fin Geometry for Minimum Entropy Generation in Forced Convection . ASME J. Heat Transfer, 104: 616-623.

Sasaki, S. and T. Kishimoto, 1986. Optimal Structure for Microgrooved Cooling Fin for High Power LSI Devices Electronics Letters, 22(25): 1332-1334.

Teetsra, P.M., M.M. Yovanovich, J.R. Culham and T.F. Lem czyk, 1999. Analytical Forced Convection Modeling of Plate Fin Heat Sinks. Proc. 15th Annu. IEEE Semicon. Thermal Meas .Manag . Symp., San Diago, Calif., pp: 34-41.

Yeh. R.H. and M. Chang, 1995. Optimum Longitudinal Convective Fin Arrays Int . Comm. Heat Mass Transfer, 22(3): 445-460.