Robust Fuzzy MIMO Bang-Bang Controller for two Links Robot Manipulators

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Abstract: In this paper a new fuzzy controller for multi-input-multi-output (MIMO) systems has been proposed. The new MIMO self-tuning robust controller is called as Fuzzy Bang-Bang Controller (FBBBC) and is used for rigid-type robot which has two links manipulators. The controller operation is demonstrated by simulation of manipulators movement from any initial position to up and downward positions. The comparison between the proposed controller and Slide Mode Controller (SMC) is carried out for to tracking performance ability. The comparison is based on: the speed of convergences, maximum payload, amplitude and frequency. It was concluded that based on the tables and the simulation results proposed controller is better than slide mode controller (SMC) in handle big load, amplitude and frequency.

Keywords: Bang-Bang, robotic manipulators, self-tuning controller, MIMO systems control.

INTRODUCTION

Dynamics of rigid robot manipulators are highly nonlinear and contain uncertain elements such as friction. Many efforts have been made in developing control schemes to achieve the precise control of robot manipulators (Feng Lin, 2007; Weibing Gao and James C. Hung, 1993; Elbrous, M.J, et al., 2002; Mahdi, S.A, et al., 2009; -A.,Saad, 1992; Shinya, A., 2009). Among available options, fuzzy control has a great potential since it is able to compensate for the uncertain nonlinear dynamics using the programming capability of human control behavior. In control theory, a bang–bang on–off controller, switches abruptly between two states. They are often used to control a plant that accepts a binary input, for example a furnace that is either completely on or completely off. Most common residential thermostats are bang–bang controllers. The Heaviside step function in its discrete form is an example of a bang–bang control signal. Due to the discontinuous control signal, systems that include bang–bang controllers are variable structure systems, and bang–bang controllers are thus variable structure controllers. Artificial intelligence technique such as fuzzy logic has provided the means to develop flexible fuzzy bang-bang controller. One of the earliest fuzzy bang-bang controllers (FBBC) was developed by Chiang and Jang (Richard, Y., et al., 1994). It made debut in Cassini spacecraft’s deep space exploration project.

Nowadays, digital processors are readily available for the industrial implementation of the fuzzy controllers. The fuzzy controllers are flexible, simple to build and provide robustness to the bang-bang controller. As the application of fuzzy controller widens, more complex systems are attempted and solutions have expanded to neural networks. As a result, fuzzy neural networks bang-bang controllers have gained the attention of researchers. Grantham and Mesbah (Grantham K.H., 1996) have demonstrated how a fuzzy bang-bang system can be converted to fuzzy neural network. The fuzzy set parameters, which are based on human logic rules, cannot be trained to meet the specific requirement. Neural networks, on the other hand are provided with learning abilities. The purpose of modifying the fuzzy controller to fuzzy neural network controller is to train the controller to meet the desired response.

The idea of fuzzy controller is not new. (A. Kendall, 1991) and Mamdani (W.J.M. Kichert and E.H. Mamdani, 1978) were first to point out that with mean of maxima (MOM) defuzzification, the fuzzy controller is identical to a multilevel relay. Application of the fuzzy relay in power control was first presented by Panda and Mishra (G. Panda, R.R. Mishra, 2000). Hard limiter was used in this work to convert the defuzzified output to two-level control. New controllers are not acceptable to control community, unless their stability is proven with the existing stability techniques. In case of fuzzy controllers, the heuristic approach of fuzzy rules results in partitioning of decisions space (phase plane) into two semi-planes by means of a sliding (switching) line. Similarity between fuzzy bang-bang controller and sliding mode controller (SMC) can be used to redefine the diagonal from of fuzzy logic controller in terms of an SMC, with boundary limits, to verify the stability of the proposed bang-bang controller (Chung-Chun K. and Chia-Chang L, 1994; Rainer Palm, et al., 1997). SMC is a robust control method (Slotine, J.J.E and Weiping Li, 1991) and its stability is proven with lyapunov’s direct method. So in lieu of SMC, the fuzzy bang-bang control stability can be easily established.

In optimal control engineering, the optimal time problem is to find the best possible control technique to transfer the state of the system from a given initial stat to a specified final state in the shortest possible time. The minimum time control is highly desirable for bang-bang controller design, especially for satellite attitude control. The Pontrygain’s minimum Principle (PMP) has been extensively used to design time optimal control.
is the membership function of antecedent part defined as
\[ jA = jA \]

The set \( A \) is the membership functions of fuzzy bang-bang control is indeed a robust control system.

Fuzzy bang-bang Controller has been successfully applied in various areas for nearly two decades. Some of these are in spacecraft satellite attitude control systems (Thongchet S. and Kuntanapreeda, S., 2001) the servo systems VIA (S.Y. Chang, et al., 1995) Control of Water Tank System (K. Tanaka, M. Sugeno, 1992) in the reduction of harmonic current pollution (B. Mazari and F. Mefri, 2005) crane hosting and lowering operation (S. Yasunobu, S. Miyamoto, 1985) the elevator control (F. Fujitec, 1988) and in process control valves operation (R. Ushiyama and H. Walden, 1991). FBBBC has become a popular tool after these successful applications. Recently, many analysis results and design methodologies of fuzzy control have been reported. However, most of these reported research only focused on single-input-single-output (SISO) systems. Multi-input-multi-output (MIMO) fuzzy controllers are desirable in many situations.

In this paper the proposed FBBBC consists of many sets of linguistic rules. Each set of linguistic rules is designed as a suction controller. Its shows that there is a switching manifold in the MLFLC. The consequent part of the fuzzy rules has only three linguistic values, while the premise parts are freely chosen. It is structurally simple due to three membership functions in its fuzzy output set and rule matrix. An important feature of FBBRC is its flexibility and its optimal time response, which is independent of initial conditions. This paper is organized as follows. In section 2, the MLFBBBC is reviewed and the membership functions for a complement type of a control structure for MIMO systems are defined. In section 3, a two-link manipulators robot system is studied to demonstrate the FBBBC. Simulation results of the two controllers are compared and analyzed in section 4. Finally, conclusions are presented in section 6.

**The membership functions for a complement type of a control structure for MIMO systems:**

The FBBBC is reviewed in this section. In addition, a special type of membership functions, the complement type, is defined. The ranges of state input and output variables it is necessary for any fuzzy controller, which are considered to be a reasonable representation of situations that controller may face and yield to stability and optimality conditions. Figure 6 shows the structure of FBBBC with the self-tuning optimization scheme which described below.

**Linguistic Description:**

The inputs and output parameters, as well as the partitions and spread of the controller membership functions are initially selected to match the dynamic response of a manipulators robotics system. The inputs \( X_i \), where \( X_i \) is the universe of discourse of the four inputs, \( i = 1, 2, 3, 4 \). For input variable, \( x_1 \) = "error angle for link 1", \( x_2 \) = "error angle for link 2", the universe of discourse, \( X_{1,2} = [-20, 20] \) deg., which represents the range of perturbation angle about the zero reference. Index \( k \) is assigned to tally the input membership functions. For input variable \( x_3 \) = "error angle rate for link 1", and \( x_4 \) = "error angle rate for link 2", the universe of discourse \( X_{3,4} = [-10, 10] \) deg./sec. The output universe of discourse \( y_{1,2} = [-200, +200] \) represents the bang-bang output. The set \( A_i^k \) is the membership function of antecedent part defines as

\[
A_i^k = [\tilde{A}_i^1 = LN, \tilde{A}_i^2 = Z, \tilde{A}_i^3 = LP]
\]

Similar values are selected for inputs \( x_1, x_2, x_3, x_4 \), \( \tilde{A}_i^2 = \tilde{A}_i^3 = \tilde{A}_i^4 = \tilde{A}_i^1 \). The set \( B_i^k \) which denotes the membership function values for outputs variable \( y_{1,2} \) are defined as

\[
B_i^k = [B_i^1 = J2, B_i^2 = Z, B_i^3 = J1]
\]
\[
B_i^2 = [B_i^1 = J2, B_i^2 = Z, B_i^3 = J1]
\]
\[
J_{1,2}, Z_{1,2}, J_{1,2} \text{ are on/off- firing command for the robot manipulators link1 and link2, respectively.}
\]

**Fuzzy Rules:**

The Fuzzy bang-bang action is a nonlinear control technique that is based on heuristic human logic rules. An initial setting of inference rules is conducted first. The initial value of \( a_{i,j} \), is so set that the domain of input \( x_j \) is divided equally. The initial value of width \( b_{i,j} \) is set to allow overlaps of membership functions. These rules are based on four inputs variables, each with three values, thus there are at most \( N^3 = 81 \) regular rule partitions) are defined as (LN, Z, LP). Where LN = Large Negative, Z = Zero, LP = Large Positive. The tuning rules-partitions are heuristically chosen to reset the links smoothly over the universe of discourse. In other
words, the input are expressed by $x_1, x_2, ..., x_n$, and the output is expressed by $y$, the inference rule of simplified fuzzy reasoning [24] can be expressed by the following.

Rule i:

If $x_i$ is $A_i$; and $x_{i+1}, ..., x_{i+m-1}$ is $A_i$, THEN $y$ is $w_i$ ($i = 1, ..., n$)

The fuzzy rules can be illustrated as a representation of the matrix of 81 rows and 3 columns (the number of membership function is depicted. Figure 1 ((a) & (b)) respectively show the surface for the rules of the FBBC before and after tuning.

![Fig. 1: viewer surface for the rules of the FBBC: (a) before tuning (b) after tuning.](image)

The symmetry of the rules matrix is expected as it arises from the symmetry of the system dynamics. The decomposition of the $j^{th}$ rule from the FBBC’s inputs to the output is given by

$$
\mu (y)_{B,j} = \prod_{i=1}^{2} \left\{ \mu (x_i) A_{i,j} \right\}
$$

(2)

Where $j = 1, 2, 3, 4,..., n$ is the index of $n$ matching rules, which are applicable from inferences of inputs. Conventional Fuzzy FBBC uses the standard decomposition technique (Richard, Y., et al., 1994; W.J.M. Kichert and E.H. Mamdani, 1978; S.Y. Chang, et al., 1995).

**Fuzzy Set Membership Functions:**

The input variables and values assigned to fuzzy set membership functions are shown in Figure 3. Triangular shape membership functions are used in this work. These membership functions are sensitive to small changes that occur in the vicinity of their centers. A small change across the central membership function $A_1^2$, located at the origin, can produce abrupt switching of control command $u$ between the +ve and –ve halves of the universe of discourse, resulting in chattering. The overlapping of the central membership function membership functions $A_1^2$ with the neighboring membership functions $A_1^1$ and $A_1^3$ reduce the sensitivity of the bang-bang control action. The membership function $A_i$ of the antecedent part is expressed by an isosceles triangle shown in Figure 2.

Where $\mu_{A_i}$ is a membership value of the antecedent part.

![Fig. 2: Membership Function of Antecedent Part.](image)
Triangular membership functions in Figure 2 are based on mathematical characteristics given in Table 1. In Table 1 the $a_i$ and $b_i$ are the tuning parameters for range and central location of membership functions respectively and shown in Figure 3. Smooth transition between the adjacent membership functions is achieved with higher percentage of overlap, which is commonly set to 50%.

**Table 1 Mathematical Characterization of Triangular Membership Functions**

<table>
<thead>
<tr>
<th>Linguistic value</th>
<th>Triangular Membership functions</th>
</tr>
</thead>
</table>
| $A_i^{k=1}$      | $\mu A_i^1(x) = \begin{cases} 
1 & x = -b_i \\
\frac{2|x - a_i|}{b_i} & -b_i < x \leq -a_i \\
\frac{-2|x - a_i|}{b_i} & -a_i < x \leq 0 \\
\frac{2|x - a_i|}{b_i} & 0 < x \leq a_i \\
1 & x = b_i 
\end{cases}$ |
| $A_i^{k=2}$      | $\mu A_i^2(x) = \begin{cases} 
1 & a_i \leq x \leq b_i \\
\frac{2|x - a_i|}{b_i} & -a_i < x \leq a_i 
\end{cases}$ |
| $A_i^{k=3}$      | $\mu A_i^3(x) = \begin{cases} 
1 & x = b_i 
\end{cases}$ |

The input- output data ($x_1, ..., x_m, y^r$) is inputted. Fuzzy reasoning is performed for the input data ($x_{m+1}, x_m$) by using Equations 4 and 6. The membership value $\mu_i$ of each inference rule and the output of fuzzy reasoning $y$ are derived.

$$
\mu_i = A_i^{k=1}(x_1) \cdot A_i^{k=2}(x_2) \cdot ... \cdot A_i^{k=m}(x_m)$$

$$
y = \frac{\sum_{i=1}^{n} \mu_i \cdot w_i}{\sum_{i=1}^{n} \mu_i}
$$

**Fig. 3:** Un-tuned membership functions of input $x_{1,2} =$ 'error angle', $x_{3,4} =$ 'error angle rate' for FBBC.
Tuning of a real number $w_i$ of the consequent part is performed offline by substituting the output of fuzzy reasoning $y$, membership value $\mu_i$, and output data $y'$ into Equation 16. Tuning of the center value $a_{ij}$ and the width $b_{ij}$ of membership functions of the antecedent part is conducted by substituting the changed real number $w_i$ of the consequent part in procedure of, the output of fuzzy reasoning $y$, membership value $\mu_i$ and output data $y'$ into Equations 7 and 8.

$$a_{ij}(t+1) = a_{ij}(t) - \frac{K_a \cdot \mu_i}{\sum_{i=1}^{n} \mu_i} \cdot (y - y') \cdot (w_i(t) - y) \cdot \text{sgn}(x_j \cdot a_{ij}(t)) \cdot \frac{2}{b_{ij}(t) \cdot A_{ij}(x_j)}$$  \hspace{1cm} (7)

$$b_{ij}(t+1) = b_{ij}(t) - \frac{K_b \cdot \mu_i}{\sum_{i=1}^{n} \mu_i} \cdot (y - y') \cdot (w_i(t) - y) \cdot \frac{1 - A_{ij}(x_j)}{A_{ij}(x_j)} \cdot \frac{1}{b_{ij}(t)}$$  \hspace{1cm} (8)

$$w_i(t+1) = w_i(t) - \frac{K_w \cdot \mu_i}{\sum_{i=1}^{n} \mu_i} \cdot (y - y')$$

The inference error $D(t)$ is calculated from Equation 9, and [Step 3] to [Step 6] are repeated until its change $D(t)-D(t-1)$ is less than a threshold value.

$$D(t) = \frac{1}{2} (y(t) - y')^2$$  \hspace{1cm} (9)

The membership functions of input $x_{1, 2} = \text{"error angle"}$, $x_{3, 4} = \text{"error angle rate"}$ for FBBC after tuning are showing in Figure 4.

**Fig. 4:** The tuned membership functions of input $x_{1, 2} = \text{"error angle"}$, $x_{3, 4} = \text{"error angle rate"}$ for FBBC.

**Structure of the fuzzy logic controller for bang-off-bang output (fuzzy bang-off-bang):**

The structure of the fuzzy logic controller for bang-off-bang output is similar to that in the largest value of the domain with maximal membership degree except for the addition of one output fuzzy set. The span for the fuzzy set $\text{off}$ is set to zero. The output is 200,-200 or 0 similar to bang-off-bang the Largest of Maximum...
LOM) Aggregation controller output. The outputs membership functions shown in Figure 5(a). Perturbation of the links from the zero reference acts on the output membership functions according to the rule matrix. The Bang-Bang firing action $J$ of output membership functions is shown in Figure 5(b).

![Fig. 5(a): FBBRC outputs $y_{1,2}$ membership functions. (b) FBBC two level Bang-Bang $y_{1,2}$ crisp outputs.](image)

In control theory, gain scheduling is an approach to control of non-linear systems that uses a family of linear controllers, each of which provides satisfactory control for a different operating point of the system. The linearization of outputs FBBC membership functions gains simply have been selected as presented in the equations 10 and 11 respectively to control the robot manipulators movements with different payloads.

$$G_1 = 180 + m_L \times 10 \quad (10)$$

$$G_2 = 80 + m_L \times 8 \quad (11)$$

Where $m_L$ is the payload value.

![Fig. 6: The structure of FBBC with the self-tuning optimization scheme in Matlab/Simulink environment.](image)
Case study: A two-link rigid-type robot manipulators:
In this section, we illustrate the performance of the FBBC by simulation of a two-joint rigid- type rigid robot. For the following robot no longer assume that the link masses are concentrated at the centers of masses. Hence, each link has a moment of inertia. All one, assume that there are frictions in the joints. The mechanical model of a robot manipulator is shown in Figure 7. Table 2 shows the parameters of the tow-link SCARA-type robot (Feng Lin, 2007).

![Fig. 7: Tow-link rigid-type robot manipulator.](image)

Table 2: Parameters of the tow-link rigid-type robot (Feng Lin, 2007).

<table>
<thead>
<tr>
<th>Parameters link 1</th>
<th>Parameters link 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 = 13.86 \text{oz} )</td>
<td>( m_2 = 3.33 \text{oz} )</td>
</tr>
<tr>
<td>( J_1 = 62.39 \text{ozin}^2 )</td>
<td>( J_2 = 110.70 \text{ozin}^2 )</td>
</tr>
<tr>
<td>( l_1 = 8 \text{in} )</td>
<td>( l_2 = 6 \text{in} )</td>
</tr>
<tr>
<td>( r_1 = 4.12 \text{in} )</td>
<td>( r_2 = 3.22 \text{in} )</td>
</tr>
<tr>
<td>( b_1 = 20 \text{ozins} )</td>
<td>( b_2 = 50 \text{ozins} )</td>
</tr>
<tr>
<td>( m_L \in [5,100] \text{oz} )</td>
<td></td>
</tr>
</tbody>
</table>

The system simulated with the goal of controlling the manipulator movement from any initial position to the up or downward position with different payload possible. That is \( \theta_1 = 90^\circ \text{ or } -90^\circ \), \( \theta_2 = 90^\circ \text{ or } -90^\circ \). The dynamic equation of the rigid robot manipulator as taken from reference [1] defines as below.

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_1 \\
\ddot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
+ \begin{bmatrix}
U_1 \\
U_2
\end{bmatrix}
+ \begin{bmatrix}
W_1 \\
W_2
\end{bmatrix}
= \begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}
\]

(11)

Where
- \( q \) is the generalized coordinate vector
- \( \tau \) is the generalized force vector
- \( M(q) \) is the inertia matrix
- \( V(q, \dot{q}) \) is the Coriolis/centripetal vector
- \( W(q) \) is the gravity vector
- \( U(q) \) is the friction vector

The elements in the above equation have been calculated as follows.

\[
q = \begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} = \begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix}, \quad \tau = \begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}
\]
\[ M_{11} = J_1 + J_2 + m_1 r_1^2 + m_3 l_1^2 + m_2 r_2^2 + 2m_2 l_1 r_2 \cos q_2 + m_4 l_1^2 + m_5 l_2^2 + 2m_5 l_1 l_2 \cos q_2 \]
\[ M_{12} = J_2 + m_2 r_2^2 + 2m_2 l_1 r_2 \cos q_2 + m_4 l_2^2 + m_5 l_1 l_2 \cos q_2 \]
\[ M_{21} = J_2 + m_2 r_2^2 + m_4 l_1 r_2 \cos q_2 + m_5 l_2^2 + m_5 l_1 l_2 \cos q_2 \]
\[ M_{22} = J_2 + m_2 r_2^2 + m_5 l_2^2 \]

\[ V_1 = (m_2 l_1 r_2 + m_1 l_1 l_2)(2\dot q_1 - \dot q_2) \sin q_2 \]
\[ V_2 = (m_1 l_1 r_2 + m_5 l_1 l_2) \dot q_2 \sin q_2 \]
\[ U_1 = b_1 \dot q_1 = -b_1 \dot q_1 \]
\[ U_2 = b_2 \dot q_2 = b_2 \dot q_2 \]
\[ W_1 = (m_1 g r_1 + m_2 g l_1 + m_3 g l_1) \sin q_1 + (m_2 g r_2 + m_5 g l_2) \sin(q_1 + q_2) \]
\[ W_2 = (m_2 g r_2 + m_5 g l_2) \sin(q_1 + q_2) \]

The control is given by FBBC, hence
\[ \tau = M(q)u + N(q, \dot q) \] (13)

Where:
\[ N(q, \dot q) = \begin{bmatrix} V_1 + U_1 + W_1 \\ V_2 + U_2 + W_2 \end{bmatrix} \]

Under the FBBC control; that is
\[ \dot \theta = M(q)^{-1} (\tau - N(q, \dot q)) \] (13)

**Simulation Studies:**

The section presents two studies: one is the application to a robot control problem, and the other a comparison study of performance between FBBC and SMC.

**Control of a Robot:**

Here, we demonstrate the proposed FBBC by the tracking control of a two-link robotic manipulator with two degree of freedom in the rotational angle described by angles \( q_1 \) and \( q_2 \) as shown in figure 6. To evaluating the performance of the proposed controller thus obtained, we simulate the actual response of the system for different \( m_L \). In these simulations, we use the following initial conditions to upward positions. \( q_1 = 90 = \frac{\pi}{2}, \quad q_2 = 90 = \frac{\pi}{2}, \quad \dot q_1 = 0, \quad \dot q_2 = 0. \)

FOR \( m_L = 10 \) oz, the angle positions, controller firing and angle velocities for 1st and 2nd link, with FBBC are shown in Figures 8 (a, b, c, d, e and f) respectively. For convenience, the angle positions are plotted in degrees.

FOR \( m_L = 20 \) oz, the angle positions, controller firing and angle velocities for 1st and 2nd link, with FBBC are shown in Figures 9 (a, b, c, d, e and f) respectively.

FOR \( m_L = 10 \) oz, the angle positions, controller firing and angle velocities for 1st and 2nd link, with FBBC are shown in Figures 10 (a, b, c, d, e and f) respectively.

FOR \( m_L = 20 \) oz, the angle positions, controller firing and angle velocities for 1st and 2nd link, with FBBC are shown in Figures 11 (a, b, c, d, e and f) respectively.

[(a) and (b)]
Fig. 8: The robot manipulators with \( m_2 = 10 \) oz under FBBC: (a) the angle 1 position, (b) the angle 2 position, (c) FBBC input for angle 1 (d) FBBC input for angle 2, (e) Angle velocity for link 1 (f) Angle velocity for link 2.

Fig. 9: The robot manipulators with \( m_2 = 20 \) oz under FBBC: (a) the angle 1 position, (b) the angle 2 position, (c) FBBC input for angle 1 (d) FBBC input for angle 2, (e) Angle velocity for link 1 (f) Angle velocity for link 2.
Fig. 10: The robot manipulators with \( m_l = 30 \) oz under FBBC: (a) the angle 1 position, (b) the angle 2 position, (c) FBBC input for angle 1, (d) FBBC input for angle 2, (e) Angle velocity for link 1, (f) Angle velocity for link.
Fig. 11: The robot manipulators with \( m_l = 40 \text{ oz} \) under FBBC: (a) the angle 1 position, (b) the angle 2 position, (c) FBBC input for angle 1 (d) FBBC input for angle 2, (e) Angle velocity for link 1 (f) Angle velocity for link 2.

From the results of the above simulations, Figures (8, 9, 10 and 11), (a, b, c, d, e and f) the controller is tested with different payloads and movements, theses results show the good behaviour of the FBBC. It can be observed that the performance is satisfactory for both joints, indicating that the controller is capable of handling the nonlinearities, couplings and uncertainties present in the system. The control input for link 1 and 2 generated switches indiscriminately very fast and smooth.

**Comparison Study:**

In order to compare the proposed controller FBBC with the SMC in this work both controllers used the same input and initial conditions, however, the output controller’s response are different. The complete design of the SMC controller is describing in reference (Weibing Gao and James C. Hung, 1993). The comparison is shown in tables and based on: the speed of convergences, maximum payload, amplitude and frequency.

### Table 3: At fixed payload \( (m_l) \) find the Convergence time, When \( m_l = 40 \text{ oz}, A = 0.1 \text{ unit} \) and \( \omega = 0.01 \text{ rad/sec} \)

<table>
<thead>
<tr>
<th>( \theta_1 ) &amp; ( \theta_2 ) (Degree)</th>
<th>( \dot{\theta}_1 ) (sec)</th>
<th>( \dot{\theta}_2 ) (sec)</th>
<th>( \dot{\theta}_1 ) (sec)</th>
<th>( \dot{\theta}_2 ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>9.2</td>
<td>4.1</td>
<td>8.5</td>
<td>4.7</td>
</tr>
<tr>
<td>180°</td>
<td>9.4</td>
<td>4.3</td>
<td>10.5</td>
<td>5.2</td>
</tr>
<tr>
<td>270°</td>
<td>10.1</td>
<td>5.3</td>
<td>12.2</td>
<td>6.6</td>
</tr>
<tr>
<td>360°</td>
<td>11.1</td>
<td>7.1</td>
<td>14.9</td>
<td>8.1</td>
</tr>
</tbody>
</table>

### Table 4: At fixed \( (\omega, A) \) find the maximum payload \( (m_l) \), When \( \omega = 0.01 \text{ rad/sec} \) and \( A = 0.1 \text{ unit} \)

<table>
<thead>
<tr>
<th>( \theta_1 ) &amp; ( \theta_2 ) (Degree)</th>
<th>( m_l ) (oz)</th>
<th>( m_l ) (oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>180°</td>
<td>44</td>
<td>95</td>
</tr>
<tr>
<td>270°</td>
<td>35</td>
<td>90</td>
</tr>
<tr>
<td>360°</td>
<td>30</td>
<td>80</td>
</tr>
</tbody>
</table>

### Table 5: At fixed payload \( (m_l, \omega, A) \) find the maximum Amplitude \( (A) \), When \( m_l = 40 \text{ oz}, \omega = 0.001 \text{ rad/sec} \)

<table>
<thead>
<tr>
<th>( \theta_1 ) &amp; ( \theta_2 ) (Degree)</th>
<th>( A ) (Unit)</th>
<th>( A ) (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0.95</td>
<td>5</td>
</tr>
<tr>
<td>180°</td>
<td>0.93</td>
<td>3.6</td>
</tr>
<tr>
<td>270°</td>
<td>0.65</td>
<td>2.7</td>
</tr>
<tr>
<td>360°</td>
<td>0.6</td>
<td>2</td>
</tr>
</tbody>
</table>

In Table 3 the both controllers is tested with same initials conditions payload, frequency and amplitude \( (m_l = 40 \text{ oz}, A = 0.1 \text{ unit} \) and \( \omega = 0.01 \text{ rad/sec} \) and seeking same desire positions this table showed that the FBBC could reset robot links from any initial condition and with satisfactory settling time comparing with fast controller as SMC. Table 4 shows the maximum payload can be handled by both controllers in same conditions, which clearly shows the FBBC can carry load heavier than SMC. Table 5 shows the FBBC can track higher amplitude then SMC. Form Table 6 shows the bang-bang controller can track high desired frequency better then SMC.
Table 6: At fixed payload (Ml) find the maximum frequency (ω), When mL=40 oz, and A=0.1 unit.

<table>
<thead>
<tr>
<th>mL=40 oz &amp; A=0.1 unit</th>
<th>Initial value</th>
<th>MSC</th>
<th>FBBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0.001</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>180°</td>
<td>0.001</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>270°</td>
<td>0.001</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>360°</td>
<td>0.001</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Conclusions:

This paper propose MIMO fuzzy bang-bang controller. We demonstrate its operation with a simulated model a real system application. The simulation model, representing robot no longer assume that the link masses are concentrated at the centers of masses. Hence, each link has a moment of inertia. We also, assume that there are frictions in the joints. Fuzzy controllers as known for absorbing the non-linearity of the systems and, as the results show, it works well for the real nonlinear MIMO system. The bang-bang control is inherently time-optimal, and this property is an important feature of FBBC.

Comparison between the FBBC and the SMC showed that FBBC is better than slide mode controller SMC in handle big load, amplitude and frequency. The stability and optimality of FBBC were satisfied by the SMC-lyapunov criterion and optimal bang-bang control theory respectively. Tuning of a new fuzzy bang-bang relay controller is presented. The proposed scheme has stability support of sliding mode control due to its proximity with non-linear Bang-Bang control theory. The new controller is simple in configuration with two level output, similar to Bang-Bang relay controls and yet has a fuzzy decision making capability on its inputs side. The front-end inputs are similar to standard fuzzy controllers based upon Mamdani implication but have a largest of Maxima defuzzification output. The simulation results confirm the dynamic control capabilities of the FBBC are superior to SMC under adverse conditions.

REFERENCES


