Students' Procedural and Conceptual Understanding of Mathematics

1Nor Hasnida Che Ghazali and 2Effandi Zakaria

1,2Department of Educational Methodology and Practice, Faculty of Education, Universiti Kebangsaan Malaysia, Bangi, Selangor.

Abstract: The goal in mathematics teaching has shifted towards an emphasis on both procedural and conceptual understanding. The importance of gaining procedural and conceptual understanding is aligned with the objective of mathematics education in Malaysia. This study sought to investigate students' procedural and conceptual understanding of algebra. It also examined the relationship between mathematics procedural understanding and conceptual understanding. Using a survey method, the study was carried out on 132 students from secondary schools. The instrument used was the algebra test containing 14 conceptual and procedural items. The data were analyzed descriptively based on percentage, mean, and standard deviation to determine students' procedural and conceptual understanding of mathematics. Pearson's correlation was used to determine the relationship between procedural and conceptual understanding. The findings revealed that the students' level of procedural understanding is high whereas the level of conceptual understanding is low. They had a higher procedural understanding gain compared to conceptual understanding gain. Furthermore, there was a significant positive relationship between mathematics procedural understanding and conceptual understanding.

Key word: Procedural understanding, conceptual understanding, algebra, mathematics.

INTRODUCTION

Recently, the teaching process has shifted its focus towards a balance between procedural and conceptual understanding. This is aligned with the mathematics education objectives in Malaysia, which are to equip the teachers and students with mathematics skills and to provide explanations. The National Council of Teachers of Mathematics hope that students are able to present, analyze, and make generalizations using graphs, tables, words, or symbolic criteria (NCTM, 2000). Every student should learn mathematics with understanding (Hope, 2006). The same goes with the Turkish National Education System that focuses on both procedural and conceptual understanding in high school mathematics education (Isleyen, 2003). Procedural mathematics understanding is knowledge that focuses on skills and step-by-step procedures without explicit reference to mathematical ideas (Hope, 2006). Mere procedural skills often fail to provide readily applicable methods to solve mathematics problems. Conceptual mathematics understanding is knowledge that involves a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics (Hope, 2006). Thus, it involves a situation where students are able to recreate formulas and proofs without the rote process. Moreover, students are allowed to make choices and apply their understanding through active engagement (Boaler, 2000). Students must have an understanding of both if they are to understand mathematics in depth (Wilkins, 2000). Students learning a topic like algebra are facing problems with topics related to concepts and not with those involving algorithms and procedures. In a study on matriculation students dealing with algebra such as number system, quadratics, polynomials, matrices, and functions, Yaakob (2007) found that 11 students achieved an A and 14 students failed. Students are prone to using procedures rather than knowing how the procedures are achieved. They focus more on calculation procedures than conceptual ones. Beth (2001) found that most students in the United States of America are at a medium level in procedural knowledge but a low level in conceptual knowledge. This finding concurred with previous studies by Abd. Rahman (2006) and Mary and Heather (2006). To successfully complete an algebra problem, students must develop both procedural and conceptual understanding (Mary & Heather, 2006). In fact, until now, many observations are focusing on issues about the relationship between procedural and conceptual teaching. Some schools only emphasize procedural understanding and do not directly focus on conceptual understanding as they do on passing examinations alone.
According to Lim (2002), procedural understanding can aid in understanding conceptual understanding. Until today, mathematics education researchers are continuing studies to understand the balance between the two understandings. Some researchers agree that both are important and that integrating both of them is important to increase students' understanding (Mary & Heather, 2006).

Recent analysis conducted in Malaysia focus on the level of procedural and conceptual understanding involved pre-service teachers in teacher training colleges. Zakaria and Zaini (2009) examined the conceptual and procedural knowledge of 105 teachers in three teacher training colleges in Malaysia. The instruments used are instruments from Faulkenberry (2003) involving 17 subjective items involving fractions. The study showed that the level of conceptual and procedural knowledge is high-average with a mean score of 44.72 points from the overall score of 68 points. The trainees displayed competence in representing a fraction as part of a set, a region, and a ratio. They also demonstrated conceptual knowledge in sketching one whole when given a fraction and in solving word problems that involved fractions. However, they were too dependent on algorithms and on the memorization of formulas, tips, and rules, and they were unable to provide explanations or justifications on how they obtained a particular answer. Similar findings by Faulkenberry (2003) determined the level of procedural and conceptual knowledge and pedagogical content knowledge of rational numbers. The study involved 15 middle school mathematics teachers in the United States who were registered for the mathematics methods course. Of the 15 teachers, 3 teachers were selected to undergo an interview. The study found that the respondents' level of procedural knowledge was high and conceptual knowledge was moderate to high. In contrast, respondents who were involved with the interviews showed a moderate level of conceptual knowledge. The three respondents also rely on standard algorithms. In fact, they also indicated the low level of pedagogical content knowledge. Walter (2007) conducted a study on the concept of perimeter, area, the relationship between them and volume. Respondents consisted of 110 teacher trainees from the Faculty of Education at Lakehead University, Ontario, and three groups of students of the Primary/Junior (P/J), Junior/Intermediate (J/I), and Intermediate/Senior (I/S) categories. Respondents were given a test and an interview. The study found that regardless of group or background of mathematics, teachers generally had less understanding of the concepts in the given questions except for the four broad concepts. Most of the respondents used their procedural skills to enable them to memorize the formula.

There are studies that look upon the effectiveness of teaching methods to improve students' level of understanding. For example, a study by Zulnaidi & Zakaria (2009) indicated that there were significant differences in mathematics conceptual knowledge between the information mapping strategy group and the traditional group. It was also found that there is a positive relationship between conceptual understanding and mathematics achievement. There are also studies conducted on students' conceptual knowledge or understanding (Dede, 2004; Amato, 2005; Abd. Rahman, 2006; Zakaria et al., 2010). Zakaria et al. (2010) conducted a study on the level of conceptual mathematical knowledge from topic sequences and series. The study also examined the relationship between conceptual knowledge and student achievement in mathematics. It was conducted through a survey of 250 college students from one matriculation center in Kedah. The results showed that conceptual knowledge of accounting students was at a low level when compared to that of physical and life science students and there was a significant relationship between conceptual knowledge and mathematics achievement of students. This study was supported by Abd. Rahman (2006), who found that students' concepts of algebra were very low. The study was conducted on 200 students from three schools in Seremban, Negeri Sembilan. It also determined the relationship between students' mastery of mathematical concepts and their attitudes and anxiety. The results showed that the relationship between students' mastery of concepts and attitudes was low, while anxiety did not correlate significantly with students' mastery of concepts. Dede (2004) conducted a study on algebraic concepts of 120 students from grade 8. The study used 17 open-ended questions answered by students followed by an interview. The results showed that students had difficulties in understanding the concept for various reasons, such as lack of information about operations and errors prior to transferring knowledge to new situations and were unsure about their prior knowledge.

**Purpose and Objectives of the Study:**

The purpose of this study was to determine students' procedural and conceptual understanding in algebra. Its specific objectives were:

1. To determine the level of procedural and conceptual understanding of algebra in lower secondary school students.
2. To determine the procedural and conceptual understanding of algebra in lower secondary school students.
3. To determine whether there is a statistically significant relationship between students' procedural and conceptual understanding of mathematics.
MATERIALS AND METHOD

Sample:
The sample consisted of 132 respondents from lower secondary schools in Malaysia who were selected through a process of cluster sampling. From these, 82 (62.12%) were female respondents and 50 (37.88%) were male. The overall age of the respondents were fourteen years old. These students had learned mathematics at a formal school from the age of seven years.

Instrumentation:
An algebra test was administered to measure their procedural and conceptual understanding. The test is a modified test from Ross (2006). The test has been modified and translated by the researchers using the "back-to-back translation" method. The test consists of questions on algebra as taught in secondary schools according to the Malaysia Education Syllabus, Ministry of Education. The test consists of 14 items, 8 items measuring procedural understanding and 6 items measuring conceptual understanding. Of all the items, 6 items are objectives items and the rest are subjective items. The items were scored according to a rubric by Ross (2006). The procedural items were given 10 marks and the conceptual items were given 8 marks. The overall total score was 18. The students scored in the range of 0 to 18. The test was given to experts in mathematics education for validation. A few changes were made on the basis of comments and suggestions in order to suit the syllabus of mathematics in Malaysia. Internal consistency with Cronbach's Alpha was used. The reliability coefficient for the test was found to be 0.804. An alpha value must be in the range of 0.65 to 0.95 for the instrument to be reliable. Therefore, the test was found to be reliable. The reliability coefficient for the procedural item was 0.73, whereas that for the conceptual item was 0.67.

Results:
Level of Procedural and Conceptual Understanding:
Seventy-seven (58.3%) students achieved a score of 7.5 to 10 and were categorized as having a high level of procedural understanding. Meanwhile, 29 (22%) were considered average, with scores ranging from 5.3 to 7.4, and 25 (18.9%) scored 2.6 to 5.2 and were considered low achievers. One (0.8%) was considered very low, with scores from 1.1 to 2.5 out of a full score of 10 (Table 1). Only three (2.2%) students achieved a score of 6.0 to 8.0 and were categorized as having a high level of conceptual understanding. Meanwhile, seven (5.3%) were considered average with scores ranging from 4.3 to 5.9, and 72 (54.5%) scored 2.0 to 4.2 and were considered low achievers. Fifty (37.9%) were considered very low with scores from 1.0 to 1.9, out of a full score of 8.0.

Table 1: Students’ Level of Procedural and Conceptual Understanding.

<table>
<thead>
<tr>
<th>Score</th>
<th>Level of Procedural Understanding</th>
<th>Number in Sample</th>
<th>Score</th>
<th>Level of Conceptual Understanding</th>
<th>Number in Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5-10</td>
<td>High</td>
<td>77(58.3%)</td>
<td>6.0-8.0</td>
<td>High</td>
<td>3(2.2%)</td>
</tr>
<tr>
<td>5.3-7.4</td>
<td>Average</td>
<td>29(22.0%)</td>
<td>4.3-5.9</td>
<td>Average</td>
<td>7(5.3%)</td>
</tr>
<tr>
<td>2.6-5.2</td>
<td>Low</td>
<td>25(18.9%)</td>
<td>2.0-4.2</td>
<td>Low</td>
<td>72(54.5%)</td>
</tr>
<tr>
<td>1.1-2.5</td>
<td>Very Low</td>
<td>1(0.8%)</td>
<td>1.0-1.9</td>
<td>Very Low</td>
<td>50(37.9%)</td>
</tr>
</tbody>
</table>

Descriptive statistics were also calculated over procedural and conceptual gains. The mean gains for procedural items were higher than the mean gains for conceptual items. The mean difference was 0.47. The standard deviation for both mean gains was low, with less than +1.0 standard deviation. Thus, students gain higher marks for factual questions rather than conceptual questions.

Table 2: Students Procedural and Conceptual Gains.

<table>
<thead>
<tr>
<th>Types of Question</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural Questions</td>
<td>0.75</td>
<td>0.21</td>
</tr>
<tr>
<td>Conceptual Questions</td>
<td>0.28</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Procedural and Conceptual Understanding of Students:
The second objective of this study is to investigate the students' procedural and conceptual understanding. Questions 1, 4, 6 (7a), 7b), 9, 10, and 12 were analyzed to determine their procedural knowledge. Table 3 shows the responses to multi-choice questions 1, 4, and 6.
Table 3: Responses to Multi-choice Questions.

<table>
<thead>
<tr>
<th>Responds</th>
<th>Question 1</th>
<th>Question 4</th>
<th>Question 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14.4%</td>
<td>1.5%</td>
<td>11.4%</td>
</tr>
<tr>
<td>B</td>
<td>3.8%</td>
<td>6.8%</td>
<td>25.0%</td>
</tr>
<tr>
<td>C</td>
<td>2.3%</td>
<td>22.7%</td>
<td>52.3%</td>
</tr>
<tr>
<td>D</td>
<td>79.5%</td>
<td>68.9%</td>
<td>11.4%</td>
</tr>
</tbody>
</table>

**Question 1:**
What is the value of □ in the equation 43 = □ - 28?
For this question, the majority of students, about 79.5% (105 students) answered D) 71, which is the correct answer, and gained a full score of 1.0. The mean score was 0.8 and its standard deviation was 0.405. The most popular wrong answer was A), which was chosen by 19 out of 132 students.

**Question 4:**
What is the rule used in the table to get the numbers in column B from the numbers in column A?
The mean score was 0.69 out of a full score of 1.0 and its standard deviation was 0.465. About 68.9% (91 students) gave the correct answer, which is D) Divide the number in column A by 4. The most popular wrong answer was C) Multiply the number in column A by 4, which was chosen by 30 out of 132 students.

**Question 6:**
Table shows values for Y = 2X + 5. Which sentence describes the change in the y values compared to the change in the x values?
About 52.3% (69 students) gave the answer C) The y values increase by 2 as the x values increase by 1. The mean score was 0.52 out of a full score of 1.0. The most popular wrong answer was B), which was chosen by 33 out of 132 students.

**Question 7a:**
How many langsat trees with 2 rows of rambutan trees?
The mean score was 0.92 out of 1.0 and its standard deviation was 0.277. About 91.7% (121 students) answered 16 langsat trees. Four students answered 84; three students answered 4; and the rest answered 3, 5, 18, and 32.

**Question 7b:**
Complete the table. (n = number of rows of rambutan trees).
About 90.2% (119 students) answered correctly and scored a full score of 2.0 as in Table 4. There were 9.1% (12 students) that filled in the table wrongly or just left it blank. There were a few students that could fill in number of trees correctly for n = 1, 2, 3, and 4, but they got it wrong for n = 5. Only 0.8% (1 student) just left it blank and gained a score of 0 out of a full score of 2.

Table 4: Number of rambutan trees and langsat trees.

<table>
<thead>
<tr>
<th>N</th>
<th>Number of rambutan trees</th>
<th>Number of langsat trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>40</td>
</tr>
</tbody>
</table>

**Question 9:**
a = b - 2 is a true statement when a = 3 and b = 5. Find a different pair of values for a and b that also make this a true statement.
The mean score was 0.70 and the standard deviation was 0.458. About 70.5% (93 students) gave the correct answer, such as a = 4 and b = 6 or a = 8 and b = 10. There were some students who answered wrongly, such as b = a + 2 and 2 = b - a. Fifteen students just left it blank.

**Question 10:**
The table represents a relationship between A and B. Based upon this relationship, what is the missing number in column A? ___.
The mean score was 0.63 out of 1.0 and its standard deviation was 0.476. About 65.9% (87 students) answered correctly, which was 48. The rest of the students, 34.1% (45 students) had written the answer wrongly, such as 28, 32, 44, 40, 52, 56, 58, or other answers. Fifteen students just left it blank.

**Question 12:**
Find the value(s) of \( y \) that make the equation true. Show how you got your answer for \( 19 = 3 + 4y \).
About 56.8% (75 students) answered correctly, \( y = 4 \). The rest of the students gave various answers, such as -4, 26, or 12. The workings are as follows:

\[
19 = 3 + 4y \\
19 - 3 = 4y \\
16 = 4y \\
4 = y \\
\]
About 43.2% (57 students) had answered wrongly and gained the value of \( y \) equal to -4.

\[
19 = 3 + 4y \\
4y = 3 - 19 \\
4y = -16 \\
y = -16/4 = -4 \\
\]
This shows that the respondents could not perform the algebraic operation. Some students may also have written as

\[
19 = 3 + 4y \\
19 = 7y \\
19 + 7 = y \\
26 = y \\
\]
Some even answered it as \( y = 19 - 3 + 4 = 12 \).

The third objective of this study is to investigate students' conceptual understanding. The students' conceptual understanding in solving algebraic tests is shown in the following 6 items involving 3 objective questions and 3 subjective questions. Questions 2, 3, 5, 7c), 8, and 11 were analyzed to determine the students' conceptual understanding. Table 5 showed the responses gained from multiple-choice questions for questions 2, 3, and 5.

<table>
<thead>
<tr>
<th>Table 5: Responds from Multi-choice Questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responses</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

**Question 2:**
Sofiah has some trading cards. Hanani has 3 times as many trading cards as Sofiah. They have 36 trading cards altogether. Which of these equations represents their trading card collection?
About 57.6% (76 students) answered correctly, which was C) \( x + 3x = 36 \). The mean score was 0.58 and the standard deviation was 0.496. The most popular wrong answer was A) \( 3x = 36 \), which was chosen by 41 out of 132 students. From the answers given, it is shown that not all students have strong conceptual understanding in stating the equation from a statement. For example, the answer A) \( 3x = 36 \) showed that the students could not interpret assumptions and correlations that included mathematical concepts in that sentence. They may read the phrase "3 times" in the question and interpret it as \( 3x \). The sentence "36 trading cards altogether" shows that the students are supposed to solve the mathematics relation as \( 3x = 36 \).

**Question 3:**
Baharin writes the following rule: If a and b represents any two numbers, \( a + b = b + a \). Which of the following describes Baharin's rule in words?
About 39.4% (52 students) answered B) Order doesn't matter when adding two numbers. The mean score was 0.39 and its standard deviation was 0.490.
Question 5:
Which of the following statements is NOT TRUE about the equation \( y = 2t \), if \( t \) is a positive number?
About 25% (33 students) answered correctly, which was C) it shows that the value of \( y \) is independent of the value of \( t \). The mean score was 0.25 and its standard deviation was 0.435.

Question 7c:
Look at the table. You might notice that the number of apple trees can be found by using the formula \( n \times n \). The number of pine trees can be found by using the formula \( 8 \times n \). Remember, \( n \) is the number of rows of apple trees. There is a value of \( n \) for which the number of apple trees equals the number of pine trees. Find that value of \( n \). Explain how you found that answer.
Only 3.8% (5 students) answered correctly, which was \( n = 8 \), because \( 8 \times 8 = 64; n \times n = 8 \times n; \) and \( 8 \times 8 = 64 \).

Question 8:
Thariq is exactly one year older than Farid. Let \( T \) stand for Thariq's age and \( F \) stand for Farid's age. Write an equation to compare Thariq's age to Farid's age.
About 32.6% (43 students) answered correctly and scored 1, such as \( T = F + 1 \) or \( T - F = 1 \) or \( T - 1 = F \).

Question 11:
Circle the correct answer (linear/not linear) BELOW and give reasons for your answer. The relationship between the age of the car and the value of the car is/is not linear because ________.
Only 6.8% (9 students) answered correctly and scored 2. The answer was "not linear because linear means the changes at a constant rate." Some of the possible wrong answers are "not linear because the car's age and the car's price are from a different year," or "linear because the reduction is constant."

Relationship Between Mathematics Procedural Understanding and Conceptual Understanding:
Pearson's correlation test was used to observe the relationship between procedural and conceptual understanding. The result of the analysis showed that the relationship between students' procedural and conceptual understanding was average \( r = 0.512, p < 0.05 \). The higher the level of procedural understanding, the higher the level of students' conceptual understanding, and vice versa.

Discussion:
The response given by the students in the Procedural and Conceptual Understanding Test showed a high level of procedural understanding but a low level of conceptual understanding. The mean score of procedural understanding was 7.46 out of a full score of 10.0, whereas the mean score of conceptual understanding was 2.24 out of a full score of 8.0. In answering the first question, a number of students provided a wrong answer A) 15. This showed that the students operated wrongly as \( 43 - 28 = 15 \) due to their weak procedural understanding. This finding was similar to that of Ross (2006), where the students scored a mean correct answers score of 0.65 across 16 teaching slots. The same goes with research done by Zakaria and Zaini (2009) who found that the students did the division operation wrongly when they were to change the numerator and the denominator when the division operation is changed to multiplication operation.

In answering Question 7a) about the number of langsat trees, many students were confused. The chosen answer, 4, showed that the students had wrongly calculated them. The chosen answer as 80 showed that the students calculated all the langsat trees in the question paper for \( n = 1, 2, 3, \) and 4, which was \( 8 + 16 + 24 + 32 = 80 \). For the last procedural type of question, the students could not give reasons for the non-linear answer that they had chosen. This may be due to the fact that the teachers did not focus on defining linearity concepts to students. Answering the second question of conceptual understanding resulted in misconceptions when they could not interpret correlation that involved mathematical concepts in the sentence, for example, the wrong answer, which was A) \( 3x = 36 \). The students read the phrase "3 times" in the question and interpreted it as \( 3x \). According to Mary and Heather (2006), vocabulary plays an important role in learning mathematics concepts.

There was a moderate correlation between procedural and conceptual understanding of mathematics. This correlation occurs because procedural understanding enables them to acquire mathematics concepts and yet increase their conceptual understanding. The same goes the other way round where the students' conceptual knowledge enables them to master procedural understanding more. Rittle et al. (2001), indicate that the correlation between the two mathematics understandings were really understood.
The procedures used by the students while solving mathematical problems shows the various levels of students' conceptual understanding. These findings are also similar to that of Star (2002), in which the conceptual knowledge is highly correlated to procedural knowledge. The same goes for the findings by Johann et al. (2005), which showed that the correlation between the conceptual and procedural achievement index is significant. These findings also correspond with Rittle et al. (2001), which showed that the early conceptual knowledge of students predicts the increment in their procedural knowledge, whereas the increment in procedural knowledge predicts the improvement in their conceptual knowledge. Furthermore, it showed that conceptual and procedural knowledge developed iteratively. These findings also contradicted the findings by Byrnes and Wasik (1991) who discovered that the correlation between the procedural and conceptual understanding was inconsistent and depended on the assignments given.

**Conclusion:**

Research reveals that most students display high levels of procedural understanding but a low level of conceptual understanding. Descriptive statistics showed that the procedural gains scores were higher than the conceptual gains scores. The respondents gained the highest score for questions that asked them to fill in the table with values from the given figure and the lowest for questions that asked them to choose the best statement to explain the changes of certain variables compared to other variables. They also gained the highest score when asked to choose the suitable equations. They gained the lowest for the question asking them about conceptual understanding of the number of trees and in determining the linear relationship using conceptual understanding. Students should be exposed to procedural and conceptual understanding. In addition to that, both understandings are highly correlated. Conceptual understanding enables students to solve mathematical problems in various forms and novel settings. Students with high levels of conceptual knowledge are capable of solving problems that they have never come across before. Hence, a reformation in teaching is needed to boost conceptual understanding among students in order to minimize the use of algorithms and memorization. The results of this study corroborated and supplemented earlier studies conducted on secondary students.

**REFERENCES**


