Australian Journal of Basic and Applied Sciences, 5(7): 863-873, 2011
ISSN 1991-8178

Determine the Optimal Order Quantity in Multi-items's EOQ Model with Backorder

[1]Babak Khabiri, [2]Hadi Nasseri, [3]Ehsan Ehsani and [4]Nima Kazemi

[1]Department of Mathement, Islamic Azad University, Jouybar Branch, Jouybar, Iran.
[2]Department of Mathematical Sciences, Mazandaran University, Babolsar, Iran.
[3,4]Department of Industrial Engineering, Alghadir University, Tabriz, Iran.

Abstract: In this paper we survey the inventory problem for item's which delay occurred for supply them. In this situation the shortage of inventory happen and order which encounter shortage, will increase and make backorders and impose new costs on inventory system and impact on optimal quantity of order in EOQ model. Since in practice and in real situations we don't have accurate value for much data of model, we determine the optimal order quantity in fuzzy sense. For this objective first we consider an EOQ model with fuzzy costs and then we consider EOQ model with fuzzy backorder and for deffuzify of these two fuzzy models we employ signed distance, a defuzzification method for fuzzy number. At the end of this paper for imply the application of proposed model we mention a numerical example and furthermore we compare the result of fuzzy case with those of crisp case for one item.

Key words: Inventory; Backlog Demands; Fuzzy theory; Signed distance; Multi item's EOQ

INTRODUCTION

In the classical Economic Order Quantity (EOQ) models, it's assumed that the systems don't encounter the shortage of inventory and ever order deliver to customer on time and without delay. While this capability don't always exists in the inventory systems. In production process problems such as; impossibility of on time supply of raw material for entrance in production flow, unforcastable defect rate of machines and impossibility of precise forecast of market demand with due to the term of society that lead to the shortage of inventory.

In the inventory control, orders that can't satisfy in order to mentioned reasons called backorders. These orders impose many costs on inventory system that called shortage costs. Shortage cost is consisting of two costs: a fixed cost of inventory shortage and shortage which happen in the cycle of time. Ordinary in systems which are face to the shortage of inventory due to backlog orders, cost coefficient, annual demand, take crisp into account in multi items EOQ model. In the classical inventory models, demands, costs and etc describe with crisp statistical distributions. While in real situations and in real world most of these parameters are vague and imprecise. For these kinds of uncertainties it is possible to use fuzzy numbers instead of probabilistic approaches. In recent years, several researchers have applied the fuzzy sets theory and technique to develop and solve inventory problems. For example Change, (2004) worked out fuzzy modifications of the model of Salameh and Jabber, (2000) which took the defective rate of goods into account Björk, (2009). Yao and Lee, (1996) fuzzified the order quantity to the triangular fuzzy number, while the backorder quantity is an ordinary variable Lin, (2008). Yao and Lee, (1999) and Yao and Lee, (1996) fuzzified the order quantity q as a triangular fuzzy number and trapezoid fuzzy number and kept shortage quantity as a crisp real variable in the total cost of inventory with backorder Chiang, *et al.,* (2005). Other relevant contributions in this field are Roy and Maiti, (1997) and Vijayan and Kumaran, (2008), for instance. Finally Björk and Carlsson, (2005) solved the case, where the lead times are fuzzy, but the demand kept crisp. This paper addresses an EOQ-model with backorders, where the demand and the lead times are kept fuzzy (general triangular fuzzy numbers) Björk, (2009).

In the past research the authors had less attention on multi items model while in real world most of cases have more than one item. The purpose of the research is to determine the optimal quantity of order in multi item case with backorder. The paper is organized as follows; in section 2 the classical Economic order quantity (EOQ) model in multi item case with backorders is introduced. In section 3 some definitions and properties about fuzzy number and sign distance method are introduced. In section 4, model with fuzzy costs is presented and solved. In the next section a numerical example is provided to illustrate the results of proposed model accompanied by compare between fuzzy and crisp case for one item and sensitivity analysis.

Corresponding Author: Hadi Nasseri, Department of Mathematical Sciences, Mazandaran University, Babolsar, Iran
E-mail: nasseri@umz.ac.ir

2. Multi Item EOQ Model with Backorders:

In situation that we face to shortage of inventory, arrived orders add to each other and make backlog orders. Whenever order come to depository, before satisfy any other order, first backlog orders must satisfy. It is trivial that if backlog order doesn't have any cost then the optimal way is that the inventory in hand shouldn't be bigger than Zero. On the other hand it may this cost be so expensive, meanwhile if backlogged orders be between these two borders the optimal way is that some backlog order must exist at the end of T cycle. The cost of each backlog request contain a fixed cost $(\hat{\pi})$ plus proportionate cost with time that order can't satisfy. The behavior of this model is shown in fig .1. Also parameters of this model are:

$\hat{\pi}$: Fixed cost of each unit shortage

π : Time Dependent cost for each unit shortage
T_1: Time period that we don't have shortage
T_2: Time period that we encounter shortage
S ; Quantity of shortage .
Q : Quantity of each order .
W : Cost of rent depository that depend on the quantity of order .
O : Annual demand .
T : Time period of cycle
C_o: Cost order in each order
C_h : Holding cost in time cycle per each unit.

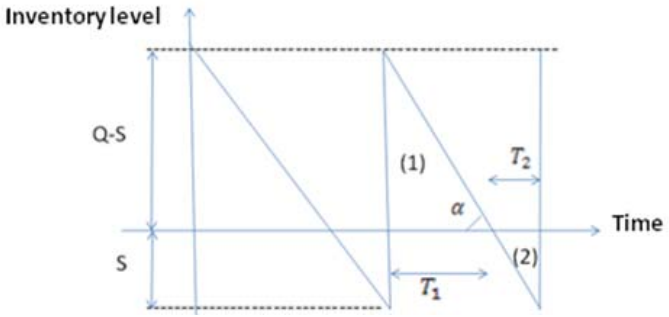

Fig. 1: The behavior of inventory model in length cycle

Form triangular (1) we have:

$$Tan\alpha = \frac{Q-s}{T_1} = D \Rightarrow T_1 = \frac{Q-s}{D}$$

From triangular (2) we have:

$$Tan\alpha = D = \frac{S}{T_2} \Rightarrow T_2 = \frac{S}{D}$$

The mean inventory in a cycle is equivalent to the bottom surface of inventory bent divide to T. It's mean that:

$$\text{Mean inventory} = \frac{\frac{Q-S}{2}\cdot\frac{Q-S}{D}}{T} = \frac{\frac{(Q-S)^2}{2D}}{\frac{Q}{D}} = \frac{(Q-S)^2}{2Q} \quad (1)$$

The mean backlog demand in a cycle is triangular's backlog demand's surface divide to T given by :

Mean backlog demand= $\frac{T_2.\frac{S}{2}}{T}=\frac{\frac{S}{D}.\frac{S}{2}}{T}=\frac{\frac{S^2}{2D}}{\frac{Q}{D}}=\frac{S^2}{2Q}$ (2)

Shortage cost = constant cost + variable with time cost = $\frac{\hat{\pi}s}{T}+\frac{\pi.\frac{S^2}{2Q}}{T}=\frac{\hat{\pi}.S.D}{Q}+\frac{\pi S^2}{2Q}$ (3)

The total cost of a system in a cycle is:

K = total cost = set up cost + Holding inventory cost + shortage cost + rent cost + purchase cost

$$K(Q,S)=\frac{D}{Q}C_O+\frac{(Q-S)^2}{2Q}C_h+(\frac{S..D}{Q}\hat{\pi}+\frac{S^2}{2Q}\pi)+Q.W+C..D \quad (4)$$

Since the goal is determine the quantity of economic order shortage is optimal quantity ore as follows:

$$\frac{\partial K(Q,S)}{\partial(Q)}=\frac{\partial K(Q,S)}{\partial(S)}=0 \quad (5)$$

$$Q^*=\sqrt{\frac{D^2\hat{\pi}^2-2DC_O(\pi+C_h)}{C_h{}^2-(\pi+C_h)(2w+C_h)}} \quad (6)$$

$$S^*=\frac{\sqrt{\frac{C_h{}^2(D^2\hat{\pi}^2-2DC_O(\pi+C_h))}{C_h{}^2-(\pi+C_h)(2w+C_h)}}-D\hat{\pi}}{\pi+C_h} \quad (7)$$

If system be containing of multi item instead of one item (6), (7) change as follows:

$$Q_i^*=\sqrt{\frac{D_i^2\hat{\pi}_i^2-2D_iC_{Oi}(\pi_i+C_{hi})}{C_{hi}{}^2-(\pi_i+C_{hi})(2w_i+C_{hi})}} \quad (8)$$

$$S_i^*=\frac{\sqrt{\frac{C_{hi}{}^2\left(D_i^2\hat{\pi}_i^2-2D_iC_{Oi}(\pi_i+C_{hi})\right)}{C_{hi}{}^2-(\pi_i+C_{hi})(2w_i+C_{hi})}}-D_i\hat{\pi}_i}{\pi_i+C_{hi}} \quad (9)$$

which *i* shows the *i*-th item in a inventory system.

Definitions and property about fuzzy number:

Some basics from fuzzy set theory need to be introduced in order to make the following model development self – Contained. Much of these can be found in Chang, (2004).

Definition 1.:

Consider the fuzzy set $A=(a,b,c)$ where $a<b<c$ and defined on R, which is called a triangular fuzzy number if the membership $\tilde{A}$ is givent by:

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-a)\big/(b-a) & a\le x\le b\\ (c-x)\big/(c-b) & b\le x\le c\\ 0 & other\ wise\end{cases} \quad (10)$$

Definition 2.:

Let $\tilde{B}$ be a fuzzy set on R and 0 ≤ α ≤ 1. The α - cut B(a) of $\tilde{B}$ is all the point x such that

$\mu_{\tilde{B}}(x) \geq \alpha$, i.e.

$$B(\alpha) = \{x \mid \mu_{\tilde{B}}(x) \geq \alpha\} \tag{11}$$

Definition 3.:

For 0 ≤ α ≤ 1, the fuzzy set $_{[a_\alpha, b_\alpha]}$ defined on R is called an a - level fuzzy interval if the membership function of $_{[a_\alpha, b_\alpha]}$ is given by

$$\mu_{[a_\alpha, b_\alpha]}(x) = \begin{cases} \alpha & a \leq x \leq b \\ 0 & other\ wise \end{cases} \tag{12}$$

Decomposition Principle:

Let $\tilde{B}$ be a fuzzy set on R and 0 ≤ α ≤ 1. Suppose the a-cut of $\tilde{B}$ to be a closed interval $[B_L(\alpha)B_U(\alpha)]$, that is, $B(\alpha)=[B_L(\alpha)B_U(\alpha)]$. Then we have:

$$\tilde{B} = \bigcup_{0 \leq \alpha \leq 1} \alpha B(\alpha) \tag{13}$$

or

$$\mu_{\tilde{B}}(x) = \bigvee_{0 \leq \alpha \leq 1} \alpha C_{B(\alpha)}(x) \tag{14}$$

Where

(i) α B (α) is a fuzzy set with membership function

$$\mu_{\alpha B(\alpha)}(x) = \begin{cases} \alpha, & x \in B(\alpha) \\ 0, & other\ wise \end{cases}$$

(ii) $C_{B(\alpha)}(x)$ is a characteristic function of B(α), that is

$$C_{B(\alpha)}(x) = \begin{cases} 1, & x \in B(\alpha) \\ 0, & x \notin B(\alpha) \end{cases}$$

Remark 1.:

From the decomposition Principle and (11), we obtain

$$\tilde{B} = \bigcup_{0 \leq \alpha \leq 1} \alpha B(\alpha) = \bigcup_{0 \leq \alpha \leq 1} [B_L(\alpha)_\alpha, B_U(\alpha)] \tag{15}$$

or

$$\mu_{\tilde{B}}(x) = \bigvee_{0 \leq \alpha \leq 1} \alpha C_{B(\alpha)}(x) = \bigvee_{0 \leq \alpha \leq} \mu_{[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]}(x) \tag{16}$$

For any $a,b,c,k \in$R, $a < b$ and $c<d$, the interval operation are as follows:

$(i)\ [a,b](+)[c,d] = [a+c, b+d]$

$(ii)\ [a,b](-)[c,d] = [a-d, b-c]$

$$(iii)\ k(.)[a,b] = \begin{cases} [ka\ , kb]\ , & k \geq 0, \\ [kb, ka]\ , & k < 0, \end{cases}$$

Further, for $a>0$ and $c>0$.

$(iv)\ [a,b]\ (.)\ [c,d] = [ac, bd].$

$(v)\ [a,b]\ (\div)\ [c,d] = [\frac{a}{d}, \frac{b}{c}].$

In order to find non – fuzzy values for model in the next section we need to use some distance measures as in Cheng [1], we will use signed distance method.

Definitions 3:

For any a and 0MBOL206\f"Symbol"\s12R ,define the sign distance from a to 0 as $d_0(a,0)$. If $a>0$, the distance from a to 0 is $d_0(a,0)=a$; if $a<0$, the distance from a to 0 is $-a=-d_0(a,0)$. Hence $d_0(a,0)=a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R for which the a - cut are continuous functions on $\alpha\in[0,1]$. Then, for any $\tilde{B}\in\Omega$, we have:

$$\tilde{B} = \bigcup_{0\le\alpha\le1} [B_L(\alpha)_\alpha, B_\cup(\alpha)_\alpha] \tag{17}$$

From Chang [1] it can be finally stated how to calculate the signed distance .

Definition 4.:

For $\tilde{B}\in\Omega$, define the signed distance of $\tilde{B}$ to $\tilde{0}_1$ (i.e , y – axis) as

$$d(\tilde{B},\tilde{0}_1) = \int_0^1 d([B_L(\alpha)_\alpha, B_\cup(\alpha)_\alpha],\tilde{0}_1)d\alpha = \frac{1}{2}\int_0^1 (B_2(\alpha)+B_\cup(\alpha))d\alpha \tag{18}$$

According to definition 4, we obtain the following property:

Property 1:

For the triangular fuzzy number $\tilde{A}$= (a, b, c) , the a - cut $\tilde{A}$ is A_U (a) = c – (c-b)a. The signed distance of $\tilde{A}$ to $\tilde{0}_1$ is:

$$d(\tilde{A},\tilde{0}_1) = \frac{1}{4}(a+2b+c) \tag{19}$$

Fuzzy Multi Item EOQ Model with Backorders:

In this section we fuzzify multi item case of (4). In this model fuzzy cost is consist of : holding cost of each unit item i (C_{hi}), set up cost in each order for item i (C_{oi}), fixed shortage cost for each unit item i ($\hat{\pi}_i$), time dependent shortage cost for each unit item i (πi), rent cost for each unit item i (w_i) and purchase cost of each unit item (C_i). The multi item of (4) is:

$$X(Q_i,S_i) = \sum_{i=1}^{m}\left(\frac{D_i}{Q_i}C_{oi} + \frac{(Q_i-S_i)^2}{2Q_i}C_{hi} + \frac{S_iD_i}{Q_i}\hat{\pi}_i + \frac{S_i^2}{2Q_i}\pi_i + w_iQ_i + C_iD_i\right) \tag{20}$$

The Cost coefficient take consider as triangular fuzzy number and inter in (20). The result given by (21)

$$\tilde{X} = \tilde{X}(Q_i,S_i) = \sum_{i=1}^{m}\left(\frac{D_i}{Q_i}\tilde{C}_{oi} + \frac{(Q_i-S_i)^2}{2Q_i}\tilde{C}_{hi} + \frac{S_iD_i}{Q_i}\tilde{\hat{\pi}}_i + \frac{S_i^2}{2Q_i}\tilde{\pi}_i + Q_i\tilde{W}_i + D_i\tilde{C}_i\right) \tag{21}$$

Where

$$\tilde{C}_{oi} = (C_{oi}-\Delta_{1i}, C_{oi}, C_{oi}+\Delta_{2i}) \tag{22}$$

$$\tilde{C}_{hi} = (C_{hi} - \Delta_{3i}, C_{hi}, C_{oi} + \Delta_{4i}) \quad (23)$$

$$\tilde{\hat{\pi}}_i = \hat{\pi}_i - \Delta_{5i}, \hat{\pi}_i, \hat{\pi}_i + \Delta_{6i} \quad (24)$$

$$\tilde{\pi}_i = \pi_i - \Delta_{7i}, \pi_i, \pi_i + \Delta_{8i} \quad (25)$$

$$\tilde{W}_i = W_i - \Delta_{9i}, W_i, W_i + \Delta_{10i} \quad (26)$$

$$\tilde{C}_i = C_i - \Delta_{11i}, C_i, C_i + \Delta_{12i} \quad (27)$$

Also

$0 < \Delta_{1i} < C_{oi}, \Delta_{2i} > 0$ *and* $0 < \Delta_{3i} < C_{hi}, \Delta_{4i} > 0$ *and* $0 < \Delta_{5i} < \hat{\pi}_i, \Delta_{6i} > 0$ *and* $0 < \Delta_{7i} < \pi_i$,

$\Delta_{8i} > 0$ *and* $0 < \Delta_{9i} < W_i, \Delta_{10i} > 0$ *and* $0 < \Delta_{11i} < C_i, \Delta_{12i} > 0$

and Δ_{ji}, j=1,2,…,12 are determined by decision makers. Now we deffuzify (21) and for this objective we used signed distance method based on property 2, the signed distance of $\tilde{X}$ to $\tilde{0}_1$ is given by:

$$d(\tilde{X}, \tilde{0}_1) = \sum_{i=1}^{m} [\frac{D}{Q_i} d(\tilde{C}_{oi}, \tilde{0}_1) + \frac{(Q_i - S_i)^2}{2Q_i} d(\tilde{C}_{hi}, \tilde{0}_1) + \frac{S_i D_i}{Q_i} d(\tilde{\hat{\pi}}_i, \tilde{0}_1) + \frac{S_i^2}{2Q_i} d(\hat{\pi}_i, \tilde{0}_1) + Q_i d(\tilde{W}_i, \tilde{0}_1) + D_i d(\tilde{C}_i, \tilde{0}_1)] \quad (28)$$

Where $d(\tilde{C}_{oi}, \tilde{0}_1), d(\tilde{C}_{hi}, \tilde{0}_1), d(\tilde{\hat{\pi}}_i, \tilde{0}_1), d(\tilde{\pi}_i, \tilde{0}_1), d(\tilde{W}_i, \tilde{0}_1), d(\tilde{C}_i, \tilde{0}_1)$ Obtain according to property 1 calculated as follows:

$$d(\tilde{C}_{oi}, \tilde{0}_1) = \frac{1}{4}[(C_{oi} - \Delta_{1i}) + 2C_{oi} + (C_{oi} + \Delta_{2i})] = C_{oi} + \frac{1}{4}(\Delta_{2i} - \Delta_{1i}) \quad (29)$$

$$d(\tilde{C}_{hi}, \tilde{0}_1) = \frac{1}{4}[(C_{hi} - \Delta_{3i}) + 2C_{hi} + (C_{hi} + \Delta_{4i})] = C_{hi} + \frac{1}{4}(\Delta_{4i} - \Delta_{3i}) \quad (30)$$

$$d(\tilde{\hat{\pi}}_i, \tilde{0}_1) = \frac{1}{4}[(\hat{\pi}_i - \Delta_{5i}) + 2\hat{\pi}_i + (\hat{\pi}_i + \Delta_{6i})] = \hat{\pi}_i + \frac{1}{4}(\Delta_{6i} - \Delta_{5i}) \quad (31)$$

$$d(\tilde{\pi}_i, \tilde{0}_1) = \frac{1}{4}[(\pi_i - \Delta_{7i}) + 2\pi_i + (\pi_i + \Delta_{8i})] = \hat{\pi}_i + \frac{1}{4}(\Delta_{8i} - \Delta_{7i}) \quad (32)$$

$$d(\tilde{W}_i, \tilde{0}_1) = \frac{1}{4}(W_i - \Delta_{9i}) + 2W_i + (W_i + \Delta_{10i}) = W_i + \frac{1}{4}(\Delta_{10i} - \Delta_{9i}) \quad (33)$$

$$d(\tilde{C}_i, \tilde{0}_1) = \frac{1}{4}[(C_i - \Delta_{11i}) + 2C_i + (C_i + \Delta_{12i})] = C_i + \frac{1}{4}(\Delta_{12i} - \Delta_{11i}) \quad (34)$$

Based on (29) , (30), (31), (32), (33), (34) and substitiotion in (28) we Will have the following :

$$X^*(Q_i, S_i) = d(\tilde{X}, \tilde{O}_1) = \sum_{i=1}^{m} [\frac{D_i}{Q_i}(C_{oi} + \frac{1}{4}(\Delta_{2i} - \Delta_{1i})) + \frac{(Q_i - S_i)^2}{2Q_i}(C_{hi} + \frac{1}{4}(\Delta_{4i} - \Delta_{3i}))$$
$$+ \frac{S_i D_i}{Q_i}(\hat{\pi}_i + \frac{1}{4}(\Delta_{6i} - \Delta_{5i})) + \frac{S_i^2}{2Q_i}(\pi_i + \frac{1}{4}(\Delta_{8i} - \Delta_{7i})) + Q_i(W_i + \frac{1}{4}(\Delta_{10i} - \Delta_{9i})) \quad (35)$$
$$+ D_i(C_i + \frac{1}{4}(\Delta_{12i} - \Delta_{11i})]$$

In order to obtain Q_i^* we must improve that $X^*(Q_i,S_i)$ is convex in Q_i^* point. It mean that we must improve the second derivative of $X^*(Q_i,S_i)$ is greater than zero. Since the cost function has tow variable we must examine its convexity through the hessian matrix. Therefore first we have to compute the first and second derivative.

$$A=\begin{vmatrix}\frac{\partial^2X^*(Q_i,S_i)}{\partial^2S_i} & \frac{\partial^2X^*(Q_i,S_i)}{\partial S_i\partial Q_i}\\ \frac{\partial^2X^*(Q_i,S_i)}{\partial S_i\partial Q_i} & \frac{\partial^2X^*(Q_i,S_i)}{\partial^2Q_i}\end{vmatrix}=\frac{\partial^2X^*(Q_i,S_i)}{\partial^2S_i}\cdot\frac{\partial^2X^*(Q_i,S_i)}{\partial^2Q_i}-\left(\frac{\partial^2X^*(Q_i,S_i)}{\partial S_i\partial Q_i}\right)^2 \tag{36}$$

$$\frac{\partial X^*(Q_i,S_i)}{\partial Q_i}=[-\frac{D_i}{Q_i^2}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})+(\frac{1}{2}-\frac{S_i^2}{2Q_i^2})(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})-\frac{S_iD_i}{Q_i^2}(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4})$$
$$-\frac{S_i^2}{2Q_i^2}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4})+(W_i+\frac{\Delta_{10i}-\Delta_{9i}}{4})] \tag{37}$$

$$\frac{\partial^2X^*(Q_i,S)}{\partial Q_i^2}=\frac{2D_i}{Q_i^3}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})+\frac{S_i^2}{Q_i^3}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{2S_iD_i}{Q_i^3}(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4})$$
$$+\frac{S_i^2}{Q_i^3}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}) \tag{38}$$

$$\frac{\partial X^*(Q_i,S_i)}{\partial S_i}=\frac{S_i-Q_i}{Q_i}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{D_i}{Q_i}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)+\frac{S_i}{Q_i}\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right) \tag{39}$$

$$\frac{\partial^2X^*(Q_i,S_i)}{\partial^2S_i}=\frac{1}{Q_i}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{1}{Q_i}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}) \tag{40}$$

$$\frac{\partial^2X^*(Q_i,S_i)}{\partial S_i\partial Q_i}=-\frac{S_i}{Q_i^2}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{D_i}{Q_i^2}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)-\frac{S_i}{Q_i^2}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}) \tag{41}$$

$$A=[\frac{1}{Q_i}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{1}{Q_i}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4})][\frac{2D_i}{Q_i^3}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})+\frac{S_i^2}{Q_i^3}(C_{hi}$$
$$\frac{\Delta_{4i}-\Delta_{3i}}{4})+2\frac{S_iD_i}{Q_i^3}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)+\frac{S_i^2}{Q_i^3}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4})$$
$$[-\frac{S_i}{Q_i^2}\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)-\frac{D_i}{Q_i^2}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)-\frac{S_i}{Q_i^2}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4})]^2 \tag{42}$$

$$A=\frac{2D_i}{Q_i^4}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{S_i^2}{Q_i^4}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})^2+2\frac{S_iD_i}{Q_i^4}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+$$
$$\frac{S_i^2}{Q_i^4}(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4})(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{2D_i}{Q_i^4}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)+\frac{S_i^2}{Q_i^4}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)$$
$$+2\frac{S_iD_i}{Q_i^4}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)+\frac{S_i^2}{Q_i^4}\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)^2-\frac{S_i^2}{Q_i^4}\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)^2-$$
$$\frac{D_i^2}{Q_i^4}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)^2$$

$$-\frac{S_i^2}{Q_i^4}\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)^2-2\frac{S_iD_i}{Q_i^4}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}))-2\frac{S_i^2}{Q_i^4}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)-2\frac{S_iD_i}{Q_i^4}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)$$

$$A=\frac{2D_i}{Q_i^4}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{2D_i}{Q_i^4}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)-\frac{D_i^2}{Q_i^4}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)^2 \quad (44)$$

If we let $A>0$ have:

$$2\,(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})[(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4})]>D_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)^2 \quad (45)$$

It's mean that equation (46) must be true to $X^*(Q_i,S_i)$ be convex in Q_i^* point. While decision maker want to intend about Q_i^*, first they should examine obtained relation. Therefore for determine Q_i^*. we must solve this equation given by:

$$\frac{\partial X^*(Q_i,S_i)}{\partial Q_i}=\frac{\partial X^*(Q_i,S_i)}{\partial S_i}=0 \quad (46)$$

$$\frac{\partial X^*(Q_i,S_i)}{\partial S_i}=0$$

$$\Leftrightarrow \quad \frac{S_i-Q_i}{Q_i}(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\frac{D_i}{Q_i}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)+\frac{S_i}{Q_i}\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)=0$$

$$\Leftrightarrow \quad (S_i-Q_i)(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+D_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)+S_i\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)=0$$

$$\Leftrightarrow \quad S_i[(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)]=Q_i(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})-D_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)$$

$$\Leftrightarrow \quad S_i=\frac{Q_i(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})-D_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)}{(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)} \quad (47)$$

$$\frac{\partial X^*(Q_i,S_i)}{\partial Q_i}=0$$

$$\Longleftrightarrow -\frac{D_i}{Q_i^2}(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})+\frac{1}{2}(1-\frac{S_i^2}{Q_i^2})(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})-\frac{S_iD_i}{Q_i^2}\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)-\frac{S_i^2}{2Q_i^2}\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)+(W_i+\frac{\Delta_{10i}-\Delta_{9i}}{4})=0$$

$$\Leftrightarrow \quad 2D_i(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})-(Q_i^2-S_i^2)(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+2S_iD_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)+S_i^2\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)-2Q_i^2(W_i+\frac{\Delta_{10i}-\Delta_{9i}}{4})=0$$

$$\Longleftrightarrow \quad 2D_i(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4})-Q_i^2[(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4})+2(W_i+\frac{\Delta_{10i}-\Delta_{9i}}{4})]+S_i^2[(C_{hi}+ \quad (48)$$

$$\frac{\Delta_{4i}-\Delta_{3i}}{4}) + \left(\pi_i + \frac{\Delta_{8i}-\Delta_{7i}}{4}\right)] + 2S_iD_i\left(\hat{\pi}_i + \frac{\Delta_{6i}-\Delta_{5i}}{4}\right) = 0$$

we let (48) in (49) then we have:

$$2D_i(C_{oi} + \frac{\Delta_{2i}-\Delta_{1i}}{4}) - Q_i{}^2[(C_{hi} + \frac{\Delta_{4i}-\Delta_{3i}}{4}) + 2(W_i + \frac{\Delta_{10i}-\Delta_{9i}}{4})] + \frac{\left[Q_i\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)-D_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)\right]^2[(C_{hi} +}{\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)+\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)}$$

$$\frac{\Delta_{4i}-\Delta_{3i}}{4}) + \left(\pi_i + \frac{\Delta_{8i}-\Delta_{7i}}{4}\right)] + 2\frac{\left[Q_i\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)-D_i\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)\right]}{\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)+\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)}D_i\left(\hat{\pi}_i + \frac{\Delta_{6i}-\Delta_{5i}}{4}\right) = 0$$

After simplify, we can write:

$$Q_i{}^2 = \frac{D_i{}^2\left(\hat{\pi}_i + \frac{\Delta_{6i}-\Delta_{5i}}{4}\right)^2 - 2D_i\left(C_{oi} + \frac{\Delta_{2i}-\Delta_{1i}}{4}\right)\left[\left(C_{hi} + \frac{\Delta_{4i}-\Delta_{3i}}{4}\right) + \left(\pi_i + \frac{\Delta_{8i}-\Delta_{7i}}{4}\right)\right]}{\left(C_{hi} + \frac{\Delta_{4i}-\Delta_{3i}}{4}\right)^2 - \left[\left(C_{hi} + \frac{\Delta_{4i}-\Delta_{3i}}{4}\right) + \left(\pi_i + \frac{\Delta_{8i}-\Delta_{7i}}{4}\right)\right]\left[\left(C_{hi} + \frac{\Delta_{4i}-\Delta_{3i}}{4}\right) + 2\left(W_i + \frac{\Delta_{10i}-\Delta_{9i}}{4}\right)\right]}$$

$$Q_i = \left(\frac{D_i{}^2\left(\hat{\pi}_i+\frac{\Delta_{6i}-\Delta_{5i}}{4}\right)^2 - 2D_i\left(C_{oi}+\frac{\Delta_{2i}-\Delta_{1i}}{4}\right)\left[\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)+\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)\right]}{\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)^2 - \left[\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)+\left(\pi_i+\frac{\Delta_{8i}-\Delta_{7i}}{4}\right)\right]\left[\left(C_{hi}+\frac{\Delta_{4i}-\Delta_{3i}}{4}\right)+2\left(W_i+\frac{\Delta_{10i}-\Delta_{9i}}{4}\right)\right]}\right)^{0.5} \quad (49)$$

If we have closer look on (49), the optimal order quantity in fuzzy case is close to crisp case. If $\Delta_{1i} = \Delta_{2i} = \Delta_{3i} = \Delta_{4i} = \Delta_{5i} = \Delta_{6i} = \Delta_{7i} = \Delta_{8i} = \Delta_{9i} = \Delta_{10i} = \Delta_{11i} = \Delta_{12i} = \Delta \rightarrow$ ıen the fuzzy case is identical to the crisp case. Hence the fuzzy case is expanding of the crisp case.

5. Comparison and Sensitivity Analysis of Model's Parameters in Fuzzy and Crisp Case:

In the following some examples provide for comparison between the result of proposed model and those of crisp model in addition to sensitivity analysis. Since the proposed model is multi item, this comparison can apply for all items in an inventory system, but we perform operation for one item for example. Assume the item A is a product in a company and inventory control department want to order this product for company and obtain data are given by:

D_A=11056 *unit/year*, C_{OA} = 5200 *$/cycle*, C_{hA} = 2*$/unit/year*, $\hat{\pi}_A = 0.06\ \$/unit$, π_A = 3*$unit/year*, W_A= 5200*$/year* C_A = 0.33*$/unit/year*. Furthermore assume that expert's viewpoint about these values are respectively set as:

$\Delta_{1A} = 280, \Delta_{2A} = 320$ and $\Delta_{3A} = 0.05, \Delta_{4A} = 0.2$ and $\Delta_{5A} = 0.01, \Delta_{6A} = 0.02$ and $\Delta_{7A} = 0.5, \Delta_{8A} = 1.25$ and $\Delta_{9A} = 200, \Delta_{10A} 500$ and $\Delta_{11A} = 0.12, \Delta_{12A} = 0.35$

In Table 4 the optimal order quantity (Q_A^*) and optimal total cost in each year (TC_i^*) are calculate in both fuzzy and crisp case and variation between them are measured by $Rel\ Q = [(Q_A^* - Q_{AC}^*)/Q_{AC}^*] \times 100\%$ *and* $Rel\ TC = [(TC_A^* - TC_{AC}^*)/TC_{AC}^*] \times 100\%$. The results in Table 4 indicate that there is an increase with 0.21% in (Q_A^*) and 0.158% in (TC_i^*) in fuzzy case rather than crisp case.

Now in order to perform sensitivity analysis we take consider two parameters of model in each table. In table 5 tow parameters, annual demand D_A which is consider as crisp, and order cost C_{oA} which is one the fuzzy parameters of proposed model, are consider. If we decrease with 25% in D_A and C_{oA} result show that there is a decrease respectively 13.3% and 13.38% with D_A and C_{oA}. Keeping this If we increase with 25% in D_A and C_{oA} result show that there is an increase respectively 11.79% and 11.79% with D_A and C_{oA}. Hence

variation in these two parameters is almost identical. It's mean that if we increase the mentioned values, the optimal order quantity will increase and inversely if we decrease these values the optimal quantity will decrease. The same result could be deducted for optimal total cost (see Table 5). According to analysis of Table 5, the other tables can be analyzed.

Table 1: Comparison between fuzzy and crisp case

	$Q_A^*(\%)$	$TC_A^*(\%)$
Crisp	338.757	343077.8905
Fuzzy	339.45	343622.1412
Increase(%)	0.21	0.158

Table 2: Sensitivity analysis of two parameters

$\pi_A(\%)$	$Q_A^*(\%)$	$TC_A^*(\%)$	$W_A(\%)$	$Q_A^*(\%)$	$TC_A^*(\%)$
-25	-13.3	-13.54	-25	-13.38	-13.20
+25	11.79	11.96	+25	11.79	11.63

Table 3: Sensitivity analysis of two parameters

$\pi_A(\%)$	$Q_A^*(\%)$	$TC_A^*(\%)$	$W_A(\%)$	$Q_A^*(\%)$	$TC_A^*(\%)$
-25	0.00008	-2.3×10^{-7}	-25	15.49	-13.23
+25	0.00006	0.054	+25	-10.56	11.66

Table 4: Sensitivity analysis of two parameters

$\pi_A(\%)$	$Q_A^*(\%)$	$TC_A^*(\%)$	$W_A(\%)$	$Q_A^*(\%)$	$TC_A^*(\%)$
-25	0.00008	-2.3×10^{-7}	-25	15.49	-13.23
+25	0.00006	0.054	+25	-10.56	11.66

Conclusions:

In many problems in inventory context, existence vague in many classical model causes to unreliability to result of models, therefore use of fuzzy models sound more desirable. In this paper we fuzzify classical EOQ model with backorders in case which costs are fuzzy and other value are crisp. In this case we obtain optimal quantity of order for each item. Meanwhile for deffuzify proposed model we apply signed distance method. One cases of the model with six cost component fuzzy and all others crisp are considered for composition between fuzzy and crisp case and sensitivity study. The conclusion from the comparisons was that there is an increase to 0.21% in optimal order quantity while this value is 0.158% for total annual cost. Future research includes the task to cover the combination of more fuzzy and crisp parameters and also composition between different methods of diffuzification.

ACKNOWLEDGMENT

The first authors thanks to the Research Center of Algebraic Hyperstructures and Fuzzy Mathematics, Babolsar, Iran for its support.

REFERENCES

Björk, K.M., 2009. An analytical solution to a fuzzy economic order quantity problem, International Journal of Approximate Reasoning, 50: 485-493.

Björk, K-M., C. Carlsson, 2005. The outcome of imprecise lead times on the distributors, in: Proceedings of the 38th Annual Hawaii International Conference on System Sciences (HICSS'05), Track 3, HICSS, pp: 81-90.

Chang, H-C., 2004. An application of fuzzy sets theory to the EOQ model with imperfect quality items, Computers and Operations Research, 31: 2079-2092.

Chiang, J., J-S. Yao, H-M. Lee, 2005. Fuzzy inventory with backorder defuzzification by signed distance method, Journal of Information Science and Engineering, 21: 673-694.

Lin, Y.J., 2008. A periodic review inventory model involving fuzzy expected demand short and fuzzy backorder rate, Computers & Industrial Engineering, 54: 666-676.

Roy, T.K., M.A. Maiti, 1997. Fuzzy EOQ model with demand dependent unit cost under limited storage facility, European Journal of Operational Research, 99: 425-432.

Salameh, M.K., M.Y. Jaber, 2000. Economic production quantity model for items with imperfect quality, International Journal of Production Economics, 64: 59-64.

Vijayan, T., M. Kumaran, 2008. Inventory models with a mixture of backorders and lost sales under fuzzy cost, European Journal of Operational Research, 189: 105-119.

Yao, J.S., & H.M. Lee, 1996. Fuzzy inventory with or without backorder for fuzzy order quantity, Information Sciences, 93: 283-319.

Yao, J.S. and H.M. Lee, 1996. "Fuzzy inventory with backorder for fuzzy order quantity" Information Sciences, 93: 283-319.

Yao, J.S. and H.M. Lee, 1999. "Fuzzy inventory with or without backorder for fuzzy order quantity with trapezoid fuzzy number," Fuzzy Sets and Systems, 105: 311-337.