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Estimationof Depth and Amplitude Coefficient by Linearization of Equations Due to Least-Square Method via Microgravity Data

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Abstract: The gravity anomaly caused by contrast densities of geologic structure can be represented by a continuous function of both shape and depth related variables with an amplitude coefficient related to mass. For the determination of the Amplitude coefficient and the depth of gravity anomaly we used data acquired along profiles. The gravity anomalies due to spherical and horizontal structures can be represented by an analytical formula which involves both anomaly depth and amplitude coefficient variables. The amplitude coefficient depends on radius and density contrast of structures. In this study, gravity data acquired from each profile are fitted over functions obtained from spherical and horizontal structures by linear inverse least square method; hence, obtaining the variables mentioned above. Validity of this method is tested by applying synthetic data without and with random noise. This method has also been applied to real gravity data. The agreement between the results obtained by the proposed method and other interpretation methods is good. Moreover, the depth obtained by the proposed approach is found to be in a very good agreement with the depth obtained from drilling information.

Key words: Gravity interpretation, inversion of field gravity data, Linearization, depth and amplitude coefficient determination.

INTRODUCTION

The problem of ambiguity in the interpretation of potential field data cannot be solved by any processing or interpretation technique (Roy, 1962). However, a unique solution may be obtained by incorporating some a priori information such as assigning a simple geometry to the causative source (Roy, *et al*., 2000). Although simple models may not be geologically realistic, they are usually sufficient to analyze sources of many isolated anomalies (Nettleton, 1976; Abdelrahman and El-Araby, 1993). In most cases, the shape of the model is assumed, and the depth variable may then be obtained by graphical method (Nettleton, 1976); ratio techniques (Bowin, *et al*., 1986;Abdelrahman, *et al*., 1989), transformation techniques (Odegard and Berg, 1965; Mohan, *et al*., 1986) least-squares approaches (Gupta, 1983; Lines and Treitel, 1984; Abdelrahman, *et al*., 1991) and Euler deconvolution (Thompson, 1982). However, a few methods have been developed to determine the shape of the buried structure from residual gravity anomaly such as Walsh transformation technique (Shaw and Agarwal, 1990) the least-squares methods (Abdelrahman and Sharafeldin, 1995; Abdelrahman, *et al*., 2001) and the use of correlation factor between successive least-squares residuals (Abdelrahman and El-Araby, 1993). Here, we present a method for determining the amplitude coefficient (A), and depth of the buried structures from residual gravity anomaly along profile. The validity of the proposed method is tested on synthetic examples and field examples from the IRAN.

Method:

Gravity Equations Linearization due to a Sphere Model:

The general gravity anomaly produced by a sphere, an infinite long horizontal cylinder, and a semi-infinite vertical cylinder is given in (Abdelrahman, *et al*., 1989) as:

$$g(x_i) = A\frac{z^m}{({x_i}^2 + z^2)^q}(i = 1,2,3,\dots N) \qquad (1)$$

where q is the shape (shape factor), z is depth, x_i is the position coordinate, and A is an amplitude coefficient related to the radius and density contrast of the buried structure. Examples of the shape factor for semi-infinite vertical cylinder, horizontal cylinder, and sphere are 0.5, 1.0, and 1.5, respectively.

For simplification, g_i is used in the rest of this paper instead of $g(x_i)$ (i = 1, . . ., N). Multiplying the two sides of equation (1) for sphere by the mathematical term $({x_i}^2 + z^2)^{1.5}$, it can be found:

$$Az = g({x_i}^2 + z^2)^{1.5}(i = 1,2,3,\dots N) \qquad (2)$$

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Squaring both sides of equation (2), the following equation is obtained.

$$(g_i^2x_i^2 + g_i^2z^2)(x_i^4z^4 + 2x_i^2z^2) = A^2z^2 (i = 1,2,3, \ldots N) \quad (3)$$

Arranging equation (3), it can result:

$$(g_i^2x_i^6 + 3g_i^2x_i^2z^4) + (3g_i^2x_i^4z^2 + g_i^2z^6) = A^2z^2 (i = 1,2,3, \ldots N) \quad (4)$$

Equation (4) is not linear in the function of parameters z, and A. In order to avoid this non linearity, new variables h_1, h_2, h_3, h_4 are introduced and defined as follows:

$h_1=z^2$, (5)
$h_2=z^4$, (6)
$h_3=z^6$, (7)
$h_4=z^2A^2$, (8)

Introducing these new variables into equation (4), it can be found.

$$(g_i^2x_i^6 + 3g_i^2x_i^4h_1) + (3g_i^2x_i^2h_2 + g_i^2h_3 - h_4) = 0 \qquad (i = 1,2,3, \ldots N) \quad (9)$$

Equation (9) is now linear in function of variables h_1, h_2, h_3, h_4.

The global optimal solution of the linear system of equations (9) is found by minimizing the following mathematical objective function onto the real space $\mathbb{R}^4$: In mathematical form, it can be written

$$h = \arg\ \min \varphi(h) = \sum_{i=1}^{N} [(g_i^2x_i^6 + 3g_i^2x_i^4h_1) + (3g_i^2x_i^2h_2 + g_i^2h_3 - h_4)]^2 \quad (10)$$

This mathematical nonlinear program is simply solved by finding the unique solution of the following system of linear equations: $\partial\varphi(h)/\partial h_i = 0$. This system of linear equations could be written in matrix form as:

$$\lambda h = f, \quad (11)$$

where λ is a squared matrix of 4 × 4 dimensions given as follows

$$= \begin{bmatrix} 3 \times \sum_{i=1}^{N} x_i^8 g_i^4 & 3 \times \sum_{i=1}^{N} x_i^6 g_i^4 & \sum_{i=1}^{N} x_i^4 g_i^4 & -\sum_{i=1}^{N} x_i^4 g_i^2 \\ 3 \times \sum_{i=1}^{N} x_i^6 g_i^4 & 3 \times \sum_{i=1}^{N} x_i^4 g_i^4 & \sum_{i=1}^{N} x_i^2 g_i^4 & -\sum_{i=1}^{N} x_i^2 g_i^2 \\ 3 \times \sum_{i=1}^{N} x_i^4 g_i^4 & 3 \times \sum_{i=1}^{N} x_i^2 g_i^4 & \sum_{i=1}^{N} g_i^4 & -\sum_{i=1}^{N} g_i^2 \\ 3 \times \sum_{i=1}^{N} x_i^4 g_i^2 & 3 \times \sum_{i=1}^{N} x_i^2 g_i^2 & \sum_{i=1}^{N} g_i^2 & -N \end{bmatrix}$$

h and f are vectors of four dimensions given as:

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} \text{and} f = \begin{bmatrix} -\sum_{i=1}^{N} x_i^{10} g_i^4 \\ -\sum_{i=1}^{N} x_i^8 g_i^4 \\ -\sum_{i=1}^{N} x_i^6 g_i^4 \\ -\sum_{i=1}^{N} x_i^6 g_i^2 \end{bmatrix}$$

The linear system of algebraic equations (11) could be easily solved by one of the iterative methods for solving nonlinear equations such as Jacobi method, Gauss-Seidel method, and Jacobi and Gauss-Seidel method (Press, *et al*., 1986). Here, it is solved by jacobi iteration method.

The system (11) has a unique solution h, and the causative body of the anomaly g(x) can probably be represented by a spherical model. The parameters related to the causative sphere body are computed as follows. From equations (5), (6) and (7), it can be easily found that the depth (z) from the surface to the sphere center is given by:

$$z = \frac{|h_1|^{1/2} + |h_2|^{1/4} + |h_3|^{1/6}}{3} \tag{12}$$

Using equations (8) and (12), the amplitude coefficient (A) can be given by:

$$A = \pm\frac{\sqrt{|h_4|}}{z} \tag{13}$$

Gravity Equations Linearization due to a Vertical cylinder Model:

Multiplying the two sides of equation (1) for vertical cylinder by the term $(x^2_i+z^2)^{0.5}$and squaring thetwo sides, it can be found:

$$A^2 = g_i({x_i}^2 + z^2)(i = 1,2,3, \ldots N) \tag{14}$$

Arranging equation (14), it can be concluded:

$${g_i}^2{x_i}^2 + {g_i}^2 z^2 - A^2 = 0 \qquad (i = 1,2,3, \ldots N) \tag{15}$$

Equation (4) is not linear in the function of parameters z, and A. In order to avoid this non linearity, new variables h_1, h_2, are introduced and defined as follows:

$$h_1 = z^2 \tag{16}$$
$$h_2 = A^2 \tag{17}$$

Introducing these new variables into equation (15), it can be found.

$$({g_i}^2{x_i}^2 + {g_i}^2 h_1 - h_2) = 0 \qquad (i = 1,2,3, \ldots N) \tag{18}$$

The global optimal solution of the linear system of equations (18) is reached by minimizing the following mathematical objective function onto the real space $\mathbb{R}^2$: In mathematical form, it can be written:

$$h = \arg\ \min\varphi(h) = \sum_{i=1}^{N}({g_i}^2{x_i}^2 + {g_i}^2 h_1 - h_2)^2 \tag{19}$$

This mathematical nonlinear program is simply solved by finding the unique solution of the following system of linear equations:$\partial\varphi(h)/\ \partial h_i = 0$. This system of linear equations could be written in matrix form as:

$$\lambda h = f \tag{20}$$

where λ is a squared matrix of 2 × 2 dimensions given as follows:

$$\lambda = \begin{bmatrix} -\sum_{i=1}^{N} {g_i}^4 & \sum_{i=1}^{N} {g_i}^2 \\ -\sum_{i=1}^{N} {g_i}^2 & N \end{bmatrix}$$

h and f are vectors of two dimensions given as:

$$h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \text{and} f = \begin{bmatrix} \sum_{i=1}^{N} {x_i}^2 {g_i}^4 \\ \sum_{i=1}^{N} {x_i}^2 {g_i}^2 \end{bmatrix}$$

The linear system of algebraic equations (20) could be easily solved byjacobi iteration method.

The system (20) has a unique solution h, and the causative body of the anomaly g(x) can probably be represented by a vertical cylinder model. The parameters related to the causative vertical cylinder body are computed as follows. From equation (16), the depth to the top of the body is found and given by:

$$z = \sqrt{|h_1|} \tag{21}$$

From equations (16) and (17) the amplitude coefficient can be given by

$$A = \pm\sqrt{|h_2|} \tag{22}$$

Gravity Equations Linearization due to a Horizontal cylinder Model:

Multiplying the two sides of equation (1) for horizontal cylinder by the term $(x_i^2 + z^2)$and squaring the two sides, it can be found:

$$g_i{}^1 x_i^2 + g_i z^2 - Az = 0 \qquad (i = 1,2,3, \dots N) \tag{23}$$

The nonlinearity of equation (23) is avoided by introducing new variables h_1, h_2 defined as follows:

$$h_1 = z^2 \tag{24}$$
$$h_2 = Az \tag{25}$$

Introducing these new variables into equation (23), it can be found.

$$(g_i x_i^2 + g_i h_1 - h_2) = 0 \qquad (i = 1,2,3, \dots N) \tag{26}$$

Equation (26) is now linear in function of variables h_1, h_2.

The global optimal solution of the linear system of equations (26) is reached by minimizing the following mathematical objective function onto the real space $\mathbb{R}^2$: mathematically, it can be written:

$$h = \arg\ \min \varphi(h) = \sum_{i=1}^{N} (g_i x_i^2 + g_i h_1 - h_2)^2 \tag{27}$$

This mathematical nonlinear program is simply solved by finding the unique solution of the following system of linear equations:$\partial\varphi(h)/\,\partial h_i = 0$. This system of linear equations could be written in matrix form as:

$$\lambda h = f \tag{28}$$

where λ is a squared matrix of 2×2 dimensions given as follows

$$\lambda = \begin{bmatrix} -\sum_{i=1}^{N} g_i^2 & \sum_{i=1}^{N} g_i \\ -\sum_{i=1}^{N} g_i & N \end{bmatrix}$$

h and f are vectors of two dimensions given as:

$$h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad \text{and} \quad f = \begin{bmatrix} \sum_{i=1}^{N} x_i^2 g_i^2 \\ \sum_{i=1}^{N} x_i^2 g_i \end{bmatrix}$$

The linear system of algebraic equations (28) could be easily solved by jacobi iteration method.

The system (28) has a unique solution h, and the causative body of the anomaly g(x) can probably be represented by a horizontal cylinder model. The parameters related to the causative vertical cylinder body are computed as follows. From equation (24), the depth to the top of the body is found and given by:

$$z = \sqrt{|h_1|} \tag{29}$$

From equations (24) and (25) the amplitude coefficient can be given by

$$A = \frac{h_2}{Z} \tag{30}$$

Synthetic Examples:

Horizontal cylinder and sphere models with different depth are applied as synthetic models. Equations (12) and (31) are applied to estimate the depth of the models both with and without the presence of random noises and the results are demonstrated in Table (1). The results of Table 1 show good agreement between assumed and evaluated depth, which obviously indicates the high efficiency of the proposed method.

Table 1: Interpretation of a synthetic gravity anomaly due to sphere model and horizontal cylinder model with an additional± 5% random error.

		The sphere		The horizontal cylinder	
Model depth (km)	using synthetic data	Using data with random errors ±%5	Percentage of error (σ)	Using data with random errors ±%5	Percentage of error (σ)
0.5	0.5	0.4953	-0.94	0.5353	7.07
1	1	0.9902	-0.98	1.1049	10 49
1.5	1.5	1.5225	1.50	1.4401	-3.99
2	2	2.0251	1.25	1.9421	-2.90
2.5	2.5	2.4895	-0.42	2.5171	0.68
3	3	2.9678	-1.07	3.0965	3.22
3.5	3.5	3.4553	-1.28	3.5640	1.83
4	4	4.1703	4.26	3.6171	-9.57
4.5	4.5	4.7129	4.73	4.3021	-4.40
5	5	5.1763	3.53	4.6988	-6.02
5.5	5.5	5.3051	-3.54	5.5854	1.55
6	6	5.6957	-5.07	6.2110	3.52

Field Examples:

To examine the applicability of the proposed method, the following two field examples are presented.

Qanat (Sub terrains) in Institute of Geophysics:

The proposed method has been adapted for interpreting residual gravity anomalies related to three different types of structures, e.g., a sphere, a vertical cylinder, and a horizontal cylinder. The Percentage of erroris used in this paper as a criteria in order to compare the observed and evaluated values.

The microgravity data was collected over top of a Qanat in northwest of Institute of Geophysics, Tehran university (hajian, 2008). This anomaly is interpreted by applying the proposed method and assuming a priori a spherical model, a vertical cylinder model, and a horizontal cylinder model, respectively.The interpretation yields to the estimation of the amplitude coefficient and the depth from the surface to the body as shown in Table 2.

Table 2: Interpretation of gravity field anomaly over the Qanat in northwest of Institute of Geophysics, Tehran university, Iran.

Gravity parameters	Horizontal cylinder model	Vertical cylinder model	Sphere model
Depth (m)	3.02	6.78	4.76
A (mGal)	4.79	2.34	11.45
Percentage of error (σ)	0.02	0.6	1.9

Table 3: Comparative study of depths.

Depth (m)	Method
2.83	Euler method (Ardestani, 2009)
3	Neural network method (Hajian, 2008)
3.2	horizontal and vertical gradient method (Ardestani, 2009)

The depth obtained (z = 3.02 m) in this case is found to be in very good agreementwith obtained other method (table 3). Table 2 shows that the highest value of σ = 1.9 is obtained for the sphere, meaning that, the residual gravity anomaly cannot be modeled as a sphere. The second highest value of σ = 0.6 is obtained for the Vertical cylinder, which means the residual gravity anomaly is also not preferably to be modeled as a Vertical cylinder.

The smallest value of σ= 0.02 is obtained for the horizontal cylinder, meaning that the gravity anomaly is preferably to be modeled as a horizontal cylinder.

Ajichai Salt Dome:

Geological scheme of the study area is shown in Figure 1 and residual gravity anomaly measured over a salt dome in east azarbayjan province, iran is shown in Figure 2 respectively. The results showed that a sphere model located at a depth of 41 m probably approximates the source of this anomaly.

The application of the proposed method to these field data by using a sphere model, a vertical cylinder, and a horizontal model respectively, yields to the estimation of the amplitude coefficient and the depth from the surface to the body as shown in Table 4.

The depth obtained (z = 41 m) in this case is found to be in very good agreement with that obtained by Euler depths of the anomaly computed through GEOSOFT software (z= 39 m) and drilling informations by geology and mineral explorations organization of iran (samimi, 2008). also Table 4 shows that the smallest value of σ = 1.06 is obtained for the sphere, meaning that the gravity anomaly is preferably to be modeled as a sphere.

Table 4: Interpretation of gravity field anomaly over the Ajichai salt dome, East Azarbayjan province, Iran.

Gravity parameters	Horizontal cylinder model	Vertical cylinder model	Sphere model
Depth (m)	34	39	41
A (mGal)	4.79	2.34	11.45
Percentage of error (σ)	1.06	2.66	3.56

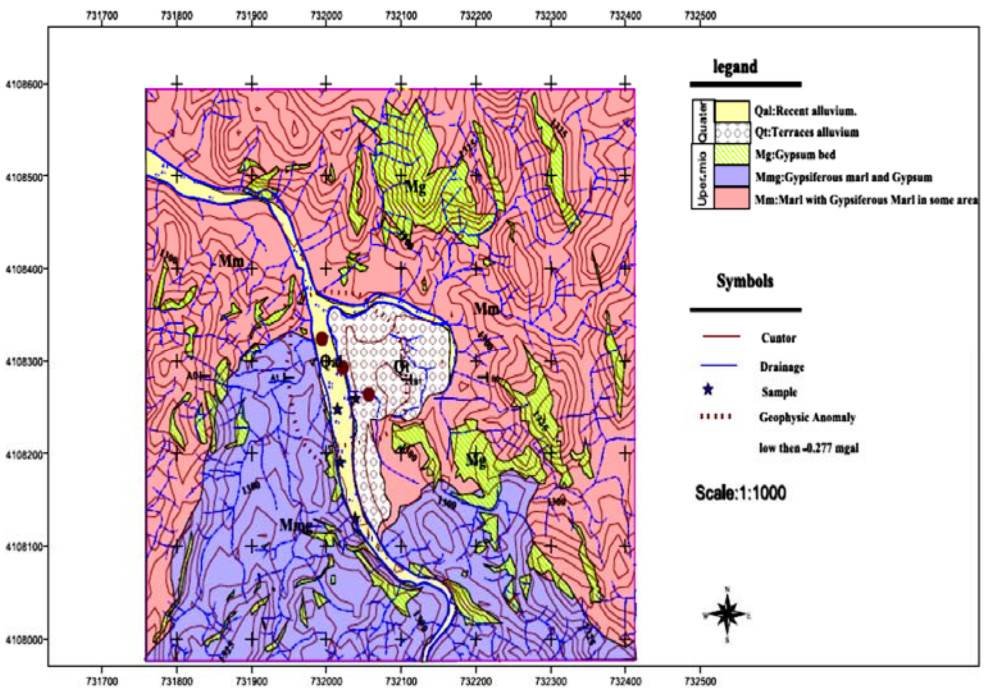

Fig. 1: geological scheme of the study area.

Conclusion:

The method which has originally been usedfor the determination of the depth and shape (Amplitude Coefficient) of the gravity anomalies is quitecapable in the case of the magnetic anomalies.The method is quite feasible and is applicablewith a few numerical computations andworks for narrow anomalies such as dikes inthe presence of random noises.

This method is therefore recommended for routine analysis of gravity anomalies in an attempt to determine the parameters related to the studied structures, and could also extended to be applicable to interpret any gravity anomaly if it is described by a bellshaped function.

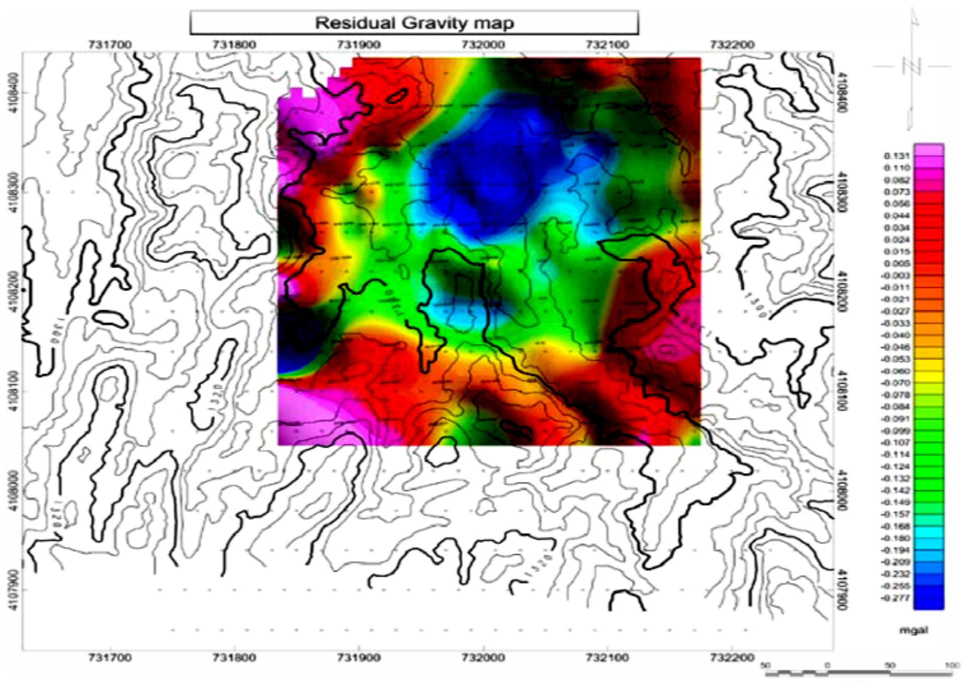

Fig. 2: Residual gravity anomalies of the study area (mGal).

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REFERENCES

Abdelrahman, E.M. and S.M. Sharafeldin, 1995. A least-squares minimization approach to shape determination from gravity data. Geophysics, 60: 589-590.

Abdelrahman, E.M. and T.M. El-Araby, 1993. A least-squares minimization approach to depth determination from moving average gravity anomalies. Geophysics, 59: 1779-1784.

Abdelrahman, E.M., A.I. Bayoumi, Y.E. Abdelhady, M.M. Gobashy and H.M. El-Araby, 1989. Gravity interpretation using correlation factors between successive least-squares residual anomalies. Geophysics, 54: 1614-1621.

Abdelrahman, E.M., H.M. El-Araby, T.M. El-Araby and E.R. Abo-Ezz, 2001. Three leastsquares minimization approach to depth, shape, and amplitude coefficient determination from gravity data. Geophysics, 66: 1105-1109.

Abdelrahman, E.M., A.I. Bayoumi and H.M. El-Araby, 1991. A least-squares minimization approach to invert gravity data.Geophysics, 56: 115-118.

Alvandi, A., 2011. Shape and Depth estimation of gravity anomalies using depth curves method and inversion magnetic data, M.S. Thesis, Islamic azaduniv, Hamedan.

Ardestani, v., 2009. Using relation figures of horizontal and vertical gradient in quantitative interpretation of gravity data. Journal of the Earth and Space Physics, 35: 51-62.

Bowin, C., E. Scheer and W. Smith, 1986. Depth estimates from ratios of gravity, geoid, and gravity gradient anomalies. Geophysics, 51: 123-136.

Gupta, O.P., 1983. A least-squares approach to depth determination from gravity data. Geophysics, 48: 357-360.

Hajian, A., 2008. Estimation of subsurface qanats depth by multi layer perceptron neural network via microgravity data. Advances in Geosciences, 20: 247-256.

Lines, L.R. and S. Treitel, 1984. A review of least-squares inversion and its application to geophysical problems. Geophysics, 32: 159-186.

Mohan, N.L., L. Anandababy and S. Roa, 1986. Gravity interpretation using Mellin transform. Geophysics, 52: 114-122.

Nettleton, L.L., 1976. Gravity and Magnetics in Oil Prospecting. McGraw-Hill Book Co., New York, pp: 462.

Odegard, M.E. and J.W. Berg, 1965. Gravity interpretation using the Fourier integral. Geophysics, 30: 424-438.

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, 1986. Numerical Recipes: The Art of Scientific Computing. CambridgeUniversity Press.

Roy, A., 1962. Ambiguity in geophysical interpretation. Geophysics, 27: 90-99.

Roy, L., B.N.P. Agarwal and R.K. Shaw, 2000. A new concept in Euler deconvolution of isolated gravity anomalies. Geophysics, 16: 77-93.

Samimi, S., 2008. Depth estimation of gravity anomalies using least square, M.S. Thesis, Islamic azaduniv, science and researchbranch, Tehran.

Shaw, R.K. and B.N.P. Agarwal, 1990. The application of Walsh transform to interpret gravity anomalies due to some simple geometrically shaped causative sources: A feasibility study. Geophysics, 55: 843-850.

Thompson, D.T., 1982. EULDPH-a new technique for making computer-assisted depth estimates from magnetic data. Geophysics, 47: 31-37.